**DÉPARTEMENT DE PHYSIQUE DE
L'ÉCOLE NORMALE SUPÉRIEURE**

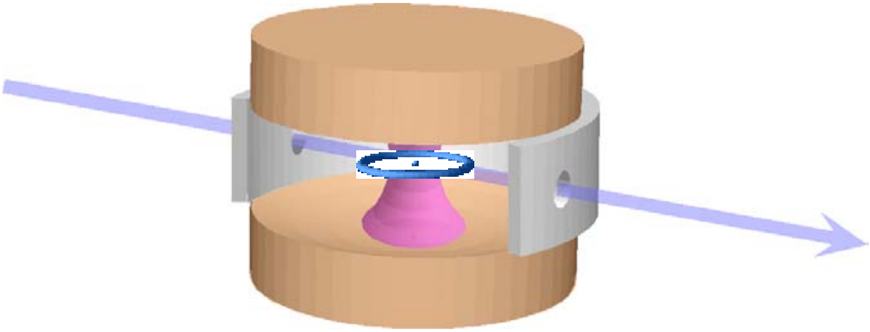
LABORATOIRE KASTLER BROSSEL

Cavity Quantum Electrodynamics
Lecture 1

Michel BRUNE

Quantum information and Cavity QED

- Principle of cavity QED experiments:



- Two level atoms interacting with a single mode of a high Q cavity

In practice:

✓ Rydberg atoms

✓ superconducting microwave cavity

- The "strong coupling" regime: coupling >> dissipation

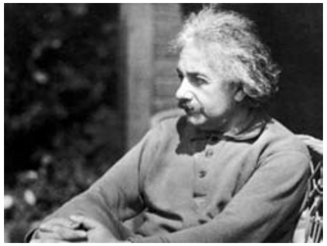
- Theoretical background: the Jaynes Cummings model

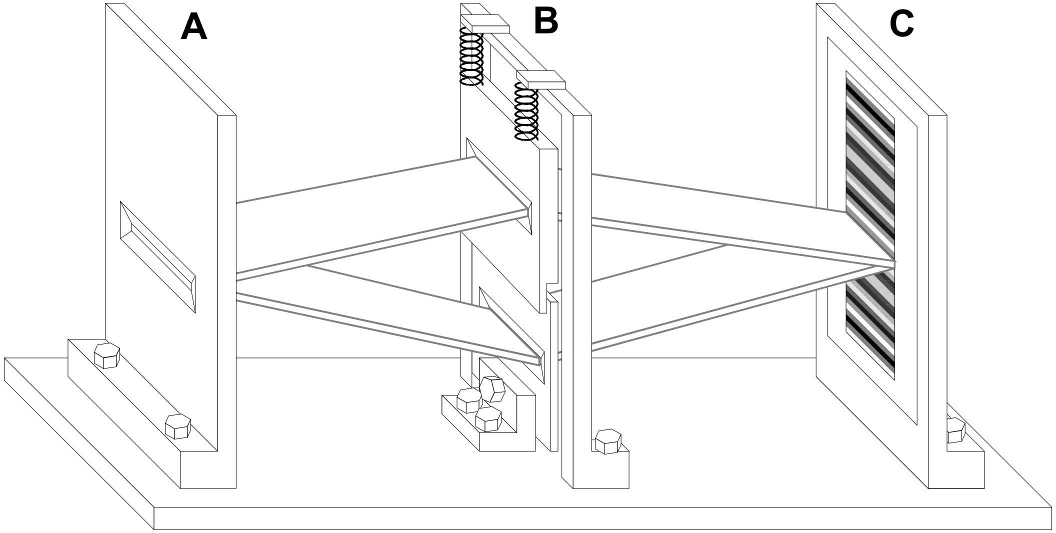
Simple but contains already a lot of physics

- Fields of interest:
  - ❑ Fundamental: better understanding of quantum theory.
  - ❑ "Applications": using quantum physics to manipulate quantum information.

Central role of "entanglement"

Probing the EPR atom-field entanglement





Illustrating Bohr-Einstein dialog

central concept: complementarity

Massive slits: insensitive to collisions with single particles

Interferences: mater behave as waves

Experiment performed with photons, electrons, atoms, molecules.

Light slits: recoil of the slit monitors which path information

No interferences: mater behave as particles

The "Schrödinger cat"

- Elementary formulation of the problem:

  Superposition principle in quantum mechanics:

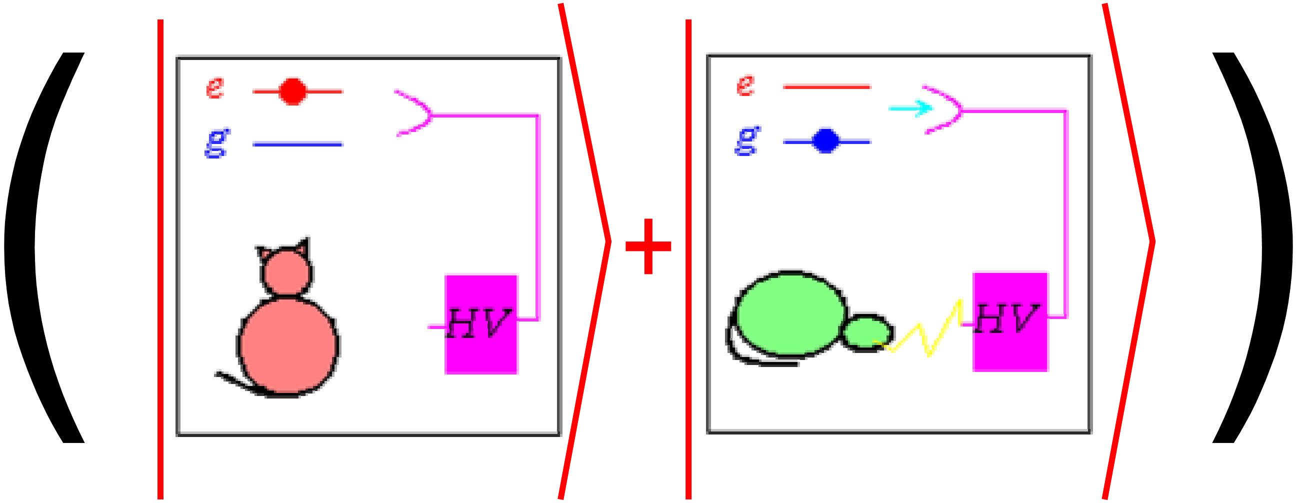
  - Any superposition state is a possible state
  - Schrödinger: this is obviously absurd when applied to macroscopic objects such as a cat !!

Up to which scale does the superposition principle applies?

"Schrödinger cat" and quantum theory of measurement

- Hamiltonian evolution of a microscopic system coupled to a measurement apparatus:

$$\left|\Psi_{at+\ldots+chat}\right\rangle = \frac{1}{\sqrt{2}}$$



→ Entangled atom-meter state

Problem: real meters provide one of the possible results not a superposition of the two

→ too much entanglement in QM?

Need to add something?

NO! "decoherence" does the work

Entanglement and quantum information

Can one do something "useful" from the strangeness of quantum logic?

- Yes:
  - Quantum cryptography: does work
  - Quantum communication: teleportation
  - Quantum algorithms:
    - Search problems (Grover)
    - Factorization (Shor)
- How?

  by manipulating qubits with a quantum computer

Classical bit:
0 ou 1

Qubit: two level system
0 and 1

$$\frac{1}{\sqrt{2}}\left[a|0\rangle + b|1\rangle\right]$$

- Practically: extreeeeemely difficult to realize: a quantum computer manipulates huge Schrödinger cat states.
  - Anyway interesting for understanding

    the essence and the limits of quantum logic

CQED, brief history

- 1870-1920: Effect of boundary conditions on dipole radiation (Maxwell, Hertz, Sommerfeld).
- 1930: atom-metal surface interaction (London, Lennard Jones).
- 1946: Spins coupled with a tuned resonator (Purcel).
- Boundary effects on Synchrotron radiation (Schwinger).
- 1947: Vacuum fluctuations between two mirors, Casimir effect.
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow…)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- 1979⇒.. : Rydberg atoms in microwave cavities, Masers (ENS, Munich).
- 1983-87: Modification of spontaneous emission, experiments: ENS, MIT, Seattle, Yale, Rome…

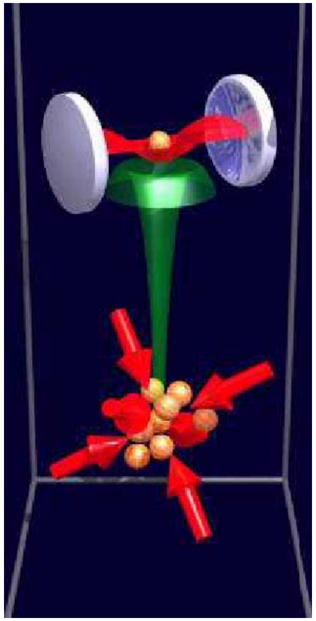
⇒CQED in the PERTUBATIVE regime: Low Q cavity or effect of a single mirror: coupling strength << dissipation rates

Perturbation of atomic radiative properties which remains qualitatively the same as in free space.

Cavity Quantum electrodynamics: strong coupling

The strong coupling: $\boxed{\Gamma_{at}, \Gamma_{cav} << \Omega_0}$ Ω_0 "vacuum Rabi frequency"

- CQED in optics: direct detection of the field



- Caltech: J. Kimble
- Munich: G. Rempe

- cold atoms "trapped" by vacuum forces
-single photon gun

- difficulties:

- control of atomic motion within λ_{opt}
- atomic lifetime: 10 *ns*

- also: Excitons and microcavities

- Microwave: detection of atoms

- Munich: H.Walther
- closed cavities
- Low l Rydberg atoms

micromaser, trapping states, number states

- Paris:
- open cavities
- circular Rydberg atoms

Main topic of this course

Outline of the course

Aim of this course: CQED with Rydberg atoms in the strong coupling regime.

- Lecture 1: the strong coupling regime
- Lecture 2: quantum gates and quantum logic
- Lecture 3: Quantum measurement and decoherence
- Lecture 4: perspectives

Outline of Course 1

1. One atom, one mode, the Jaynes-Cummings model

2. Rydberg atoms in a cavity:
the tools achieving the strong coupling regime

- ☐ The experimental setup
- ☐ Vacuum Rabi oscillations

3. Rabi oscillation in a small coherent field

- ☐ Direct observation of field graininess

4. Rabi oscillation Ramsey interferometry and complementarity

1. One atom, one mode, the Jaynes-Cummings model

Field quantization in a cavity

- Same procedure as in free space:

1- Find the classical eigenmodes of the resonator satisfying the boundary conditions.

Classical electric field: $\boxed{\vec{E}_\alpha(\vec{r},t) = E_\omega.\vec{f}_\alpha(\vec{r}).e^{i\omega t} + cc}$

2- Each mode is quantized as an harmonic oscillator.

Electric field operator: $\boxed{\hat{\vec{E}}_\alpha(\vec{r},t) = E_\omega.\left(\vec{f}_\alpha(\vec{r}).\hat{a}_\alpha + \vec{f}^{\,*}_\alpha(\vec{r}).\hat{a}^+_\alpha\right)}$ $\boxed{\left[a,a^+\right]=1}$

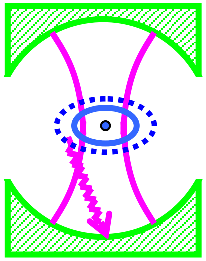
Where:

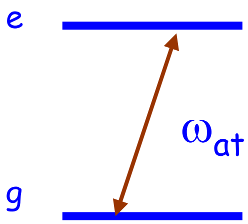
- $E_\omega = \sqrt{\dfrac{\hbar\omega}{2\varepsilon_0 V_{cav}}}$ "vacuum electric field".
- $V_{cav} = \int_{Cavity} \left|\vec{f}_\alpha(\vec{r})\right|^2 d^3\vec{r}$ volume of the mode. V_{cav} is really a physical volume.
- $\vec{f}_\alpha(\vec{r})$ complex function of Normalization: $Max\left|\vec{f}_\alpha(\vec{r})\right| = 1$

  (real functions will be enough for us)

- Here the quantized object is a collective excitation of the field and all the electric charges at the surface of the mirror.
- We now consider a single mode and drop the index α.

The Jaynes Cummings model:





+ a single two level atom, frequency ω_{at}
+a single field mode, frequency ω_c
+ dipole coupling
+ negligible damping

- Atom-field Hamiltonian: $H = H_{at} + H_{cav} + V_{at\text{-}cav}$

$$\boxed{\begin{aligned} H_{at} &= \frac{\hbar\omega_{at}}{2}\left[|e\rangle\langle e| - |g\rangle\langle g|\right] \\ H_{cav} &= \hbar\omega_c\left[a^+a + 1/2\right] \\ V_{at-cav} &= -\vec{d}.\hat{\vec{E}}(\vec{r}) \\ \vec{d} &= d_{eg}\left[|e\rangle\langle g| + |g\rangle\langle e|\right] \end{aligned}}$$

Condition of validity:

- ω_c close to a single atomic transition: $\boxed{|\delta| = |\omega_c - \omega_{at}| << \omega_c, \omega_{at}}$

- small cavity: $\boxed{FSR >> \delta}$

The Jaynes Cummings hamiltonian

- Rotating wave approximation (RWA):

$$V_{at-cav} = \hbar\Omega(\vec{r})/2\left[a|e\rangle\langle g| + a|g\rangle\langle e| + a^{+}|g\rangle\langle e| + a^{+}|e\rangle\langle g|\right]$$

Non-resonant terms are neglected

$$\boxed{V_{at-cav} \approx \hbar\Omega(\vec{r})/2\left[a|e\rangle\langle g| + a^{+}|g\rangle\langle e|\right]}$$

Vacuum Rabi frequency:

$$\boxed{\Omega(\vec{r}) = -2d_{eg}.\vec{f}(\vec{r}).E_{\omega} = \Omega_0.\left|\vec{f}(\vec{r})\right|}$$

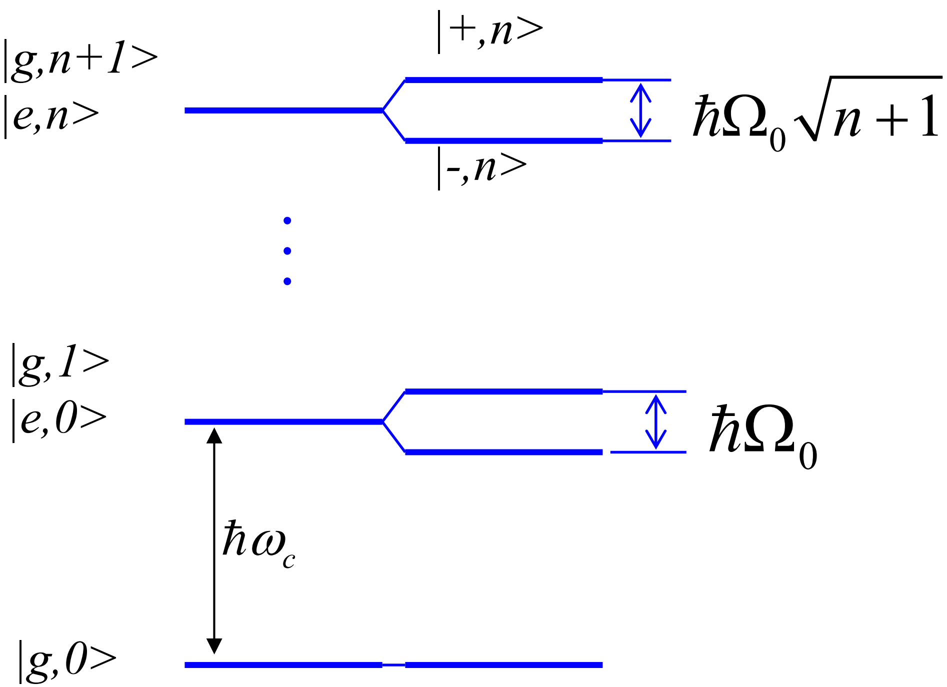
$$\boxed{\Omega_0 = 2d_{eg}.\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{cav}}}}$$

Validity of RWA:

$$\boxed{\Omega << \omega_{at}, \omega_c}$$

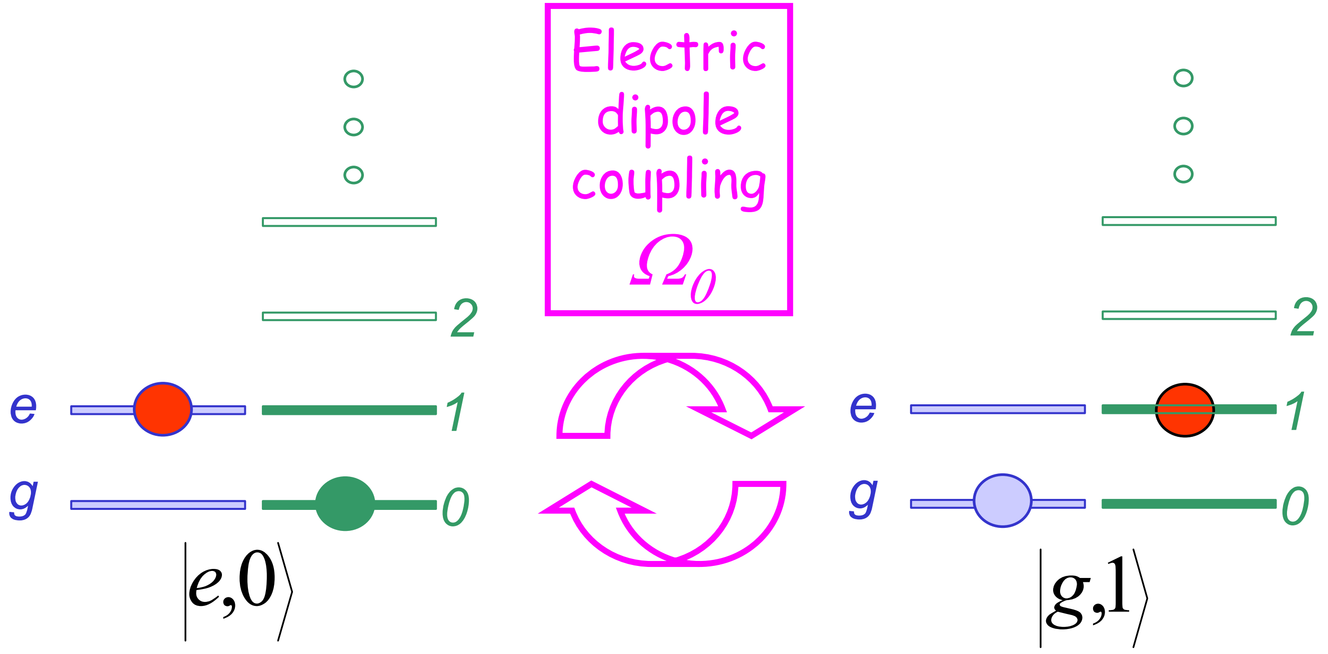
Dressed energy levels at resonance ($\omega_{at}=\omega_c$)

- Eigenvalues: $E_{\pm n} = \hbar\omega_c(n+1/2) + \hbar\omega_{at} \pm \hbar\Omega_0/2\sqrt{n+1}$
- Eigenstates: $|\pm n\rangle = 1/\sqrt{2}\left[|e,n\rangle \pm |g,n+1\rangle\right]$



- Levels just couple by pairs (except the ground state)
- level splitting scales as the Field amplitude

Resonant atom-field coupling: dynamic point of view



Electric dipole coupling Ω_0

$|e,0\rangle$

$|g,1\rangle$

$\Omega_0/2\pi = 50\ kHz$

$T_{rabi} = 20\ \mu s$

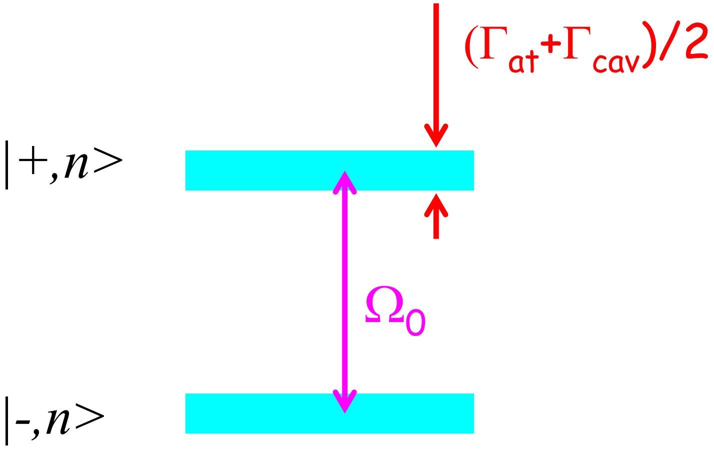
$$|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0 t}{2}\right)\cdot|e,0\rangle - i\sin\left(\frac{\Omega_0 t}{2}\right)\cdot|g,1\rangle$$

- Coherent Rabi oscillation

Condition of observation of vacuum Rabi splitting

- Cavity damping rate: $\boxed{\Gamma_{cav} \quad , \quad T_{cav} = \Gamma_{cav}^{-1}}$
- Atomic lifetime: $\boxed{\Gamma_{at} \quad , \quad T_{at} = \Gamma_{at}^{-1}}$
- The width of dressed levels must be smaller than the vacuum Rabi frequency:

$$\boxed{\Omega_0 \gg \Gamma_{cav}, \Gamma_{at}}$$

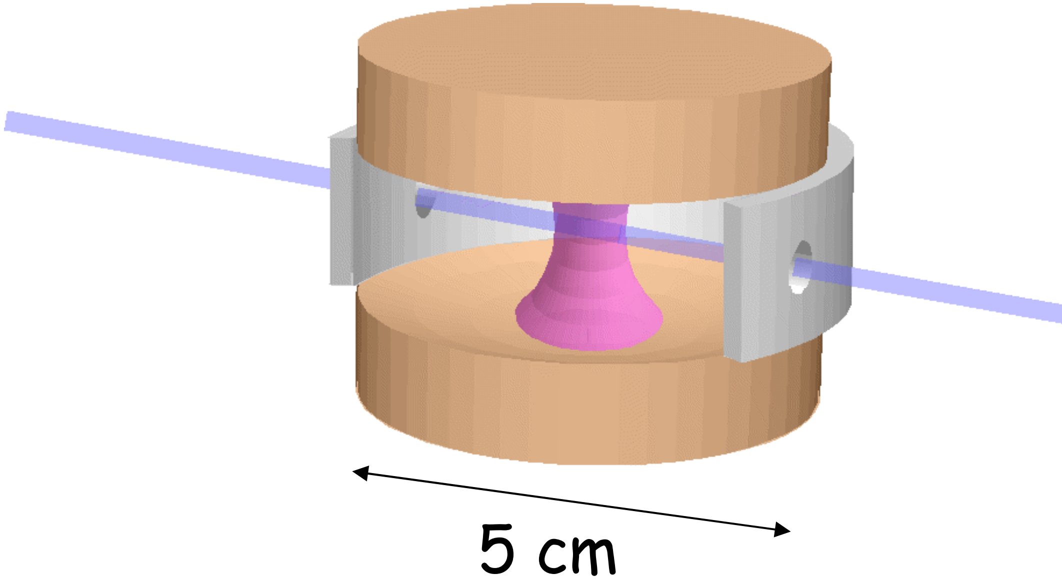


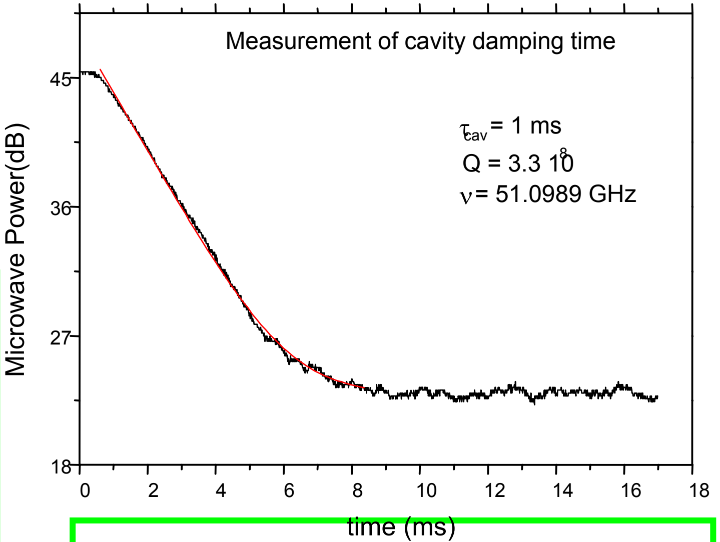
2. Rydberg atoms in a cavity: achieving the strong coupling regime

One photon and one atom in a box:

- ☐ Photon box: superconducting microwave cavity
- ☐ “circular" Rydberg atoms

The cavity





a "photon box":
- superconducting Niobium mirrors
- microwave photons:

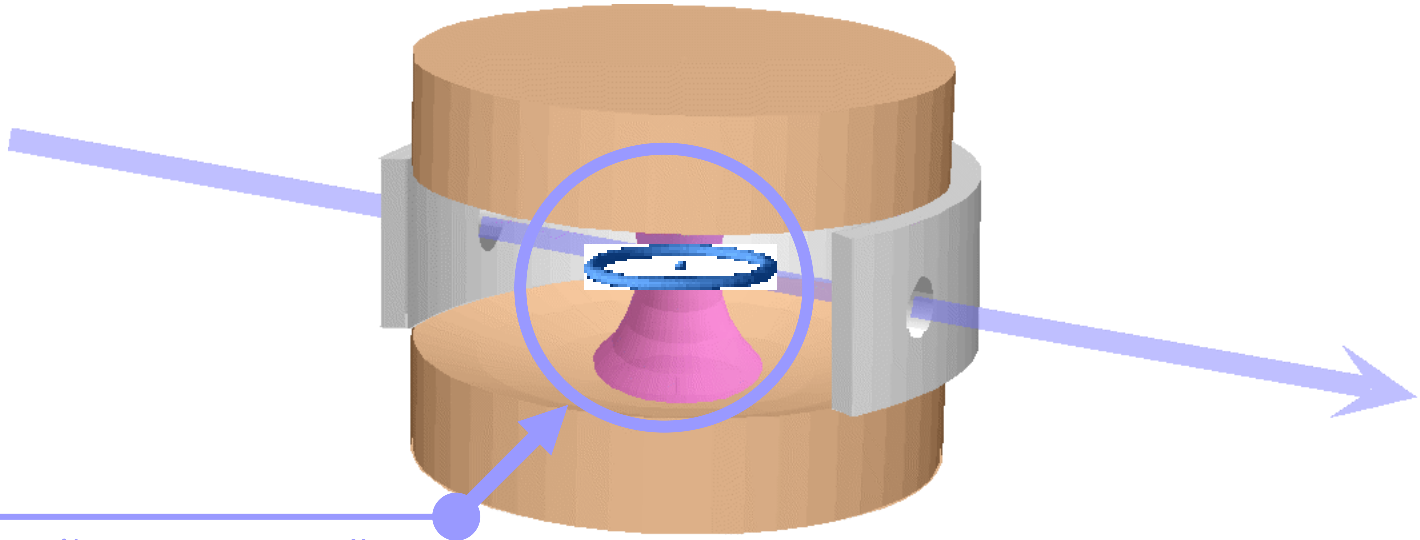
$\lambda = 6\ mm,\ \nu_{cav} = 51 GHz$

- photon lifetime:

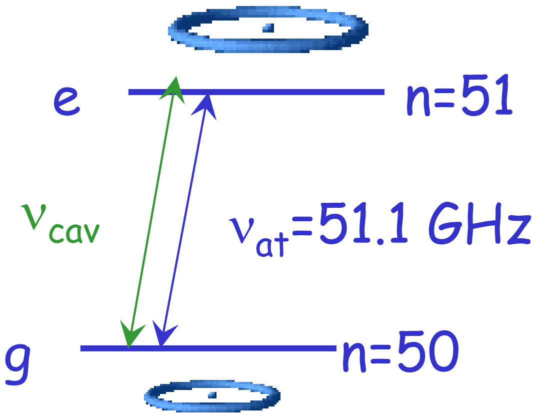
$$T_{cav} = 1 ms\ (\ Q = 3.10^8\ )$$

- one mode
- a few trapped photons

The "circular" Rydberg atoms



"Circular Rydberg atoms": $l=|m|=n-1$

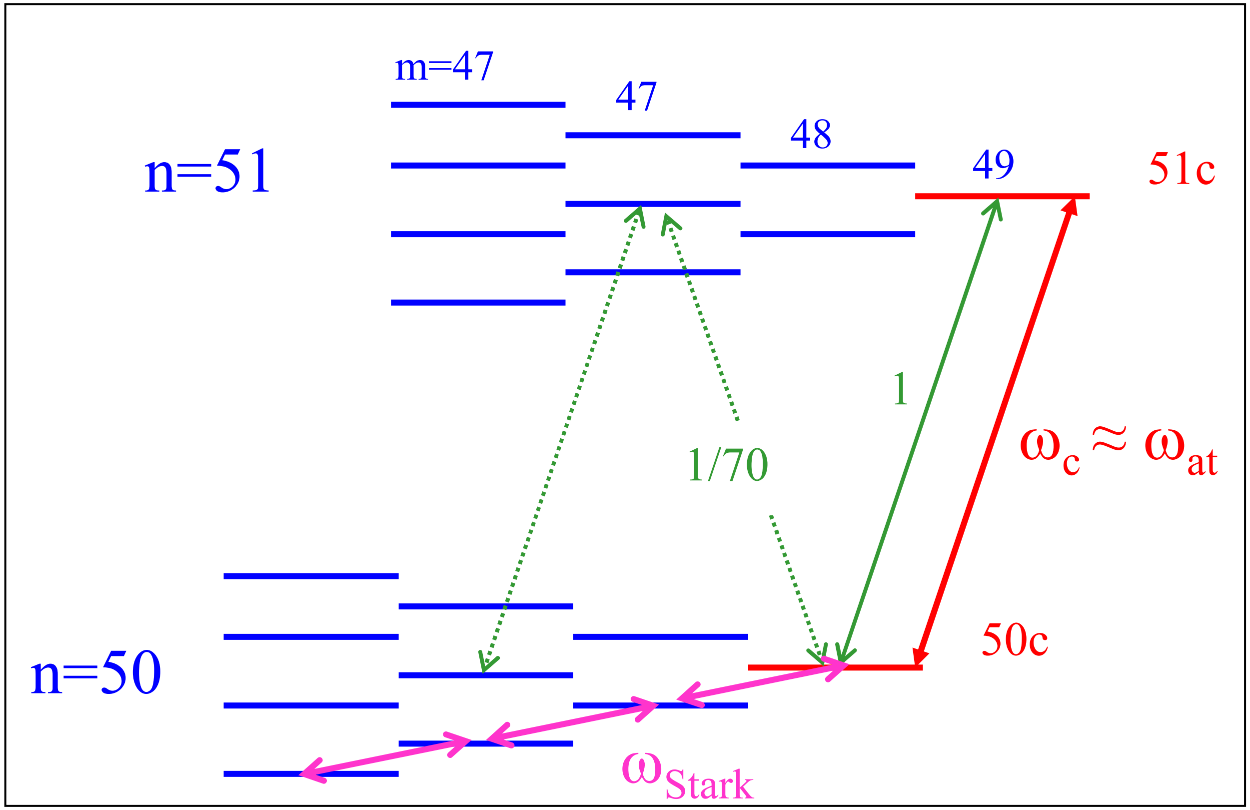


- radiative lifetime: 30 *ms*
- dipôle: *d*= 1500 *u.a.*
- ideal closed two level system

"Circular" atoms as two level atoms

- Stark diagram of Rydberg levels:

Good quantum number: m

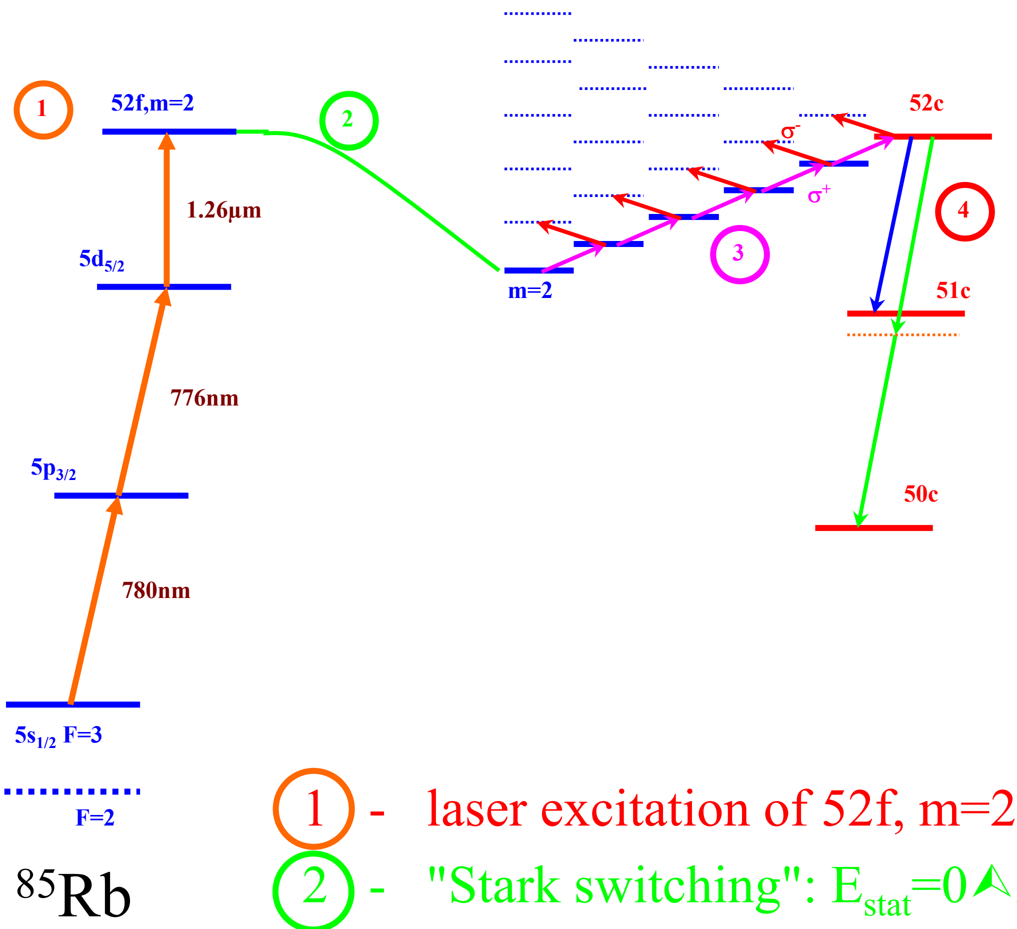


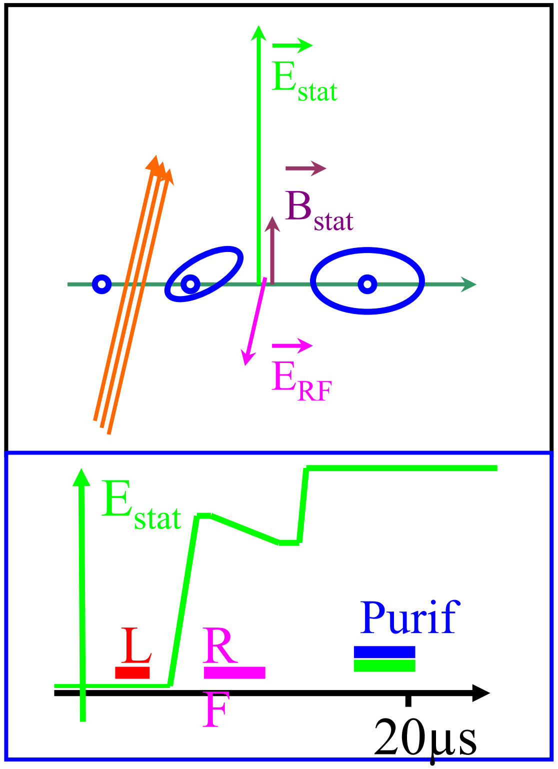
The electric field removes the degeneracy between transitions:

good isolation of the 51c to 50c transition

- Linear Stark effect: $\omega_{Stark}/2\pi = 100\ MHz/(V/cm)$
- Quadratic Stark shift of the 51c-50c transition: $255\ kHz/(V/cm)^2$ used for fast tuning of the atom in or out of resonance

Preparation of circular atoms





① - laser excitation of 52f, m=2.

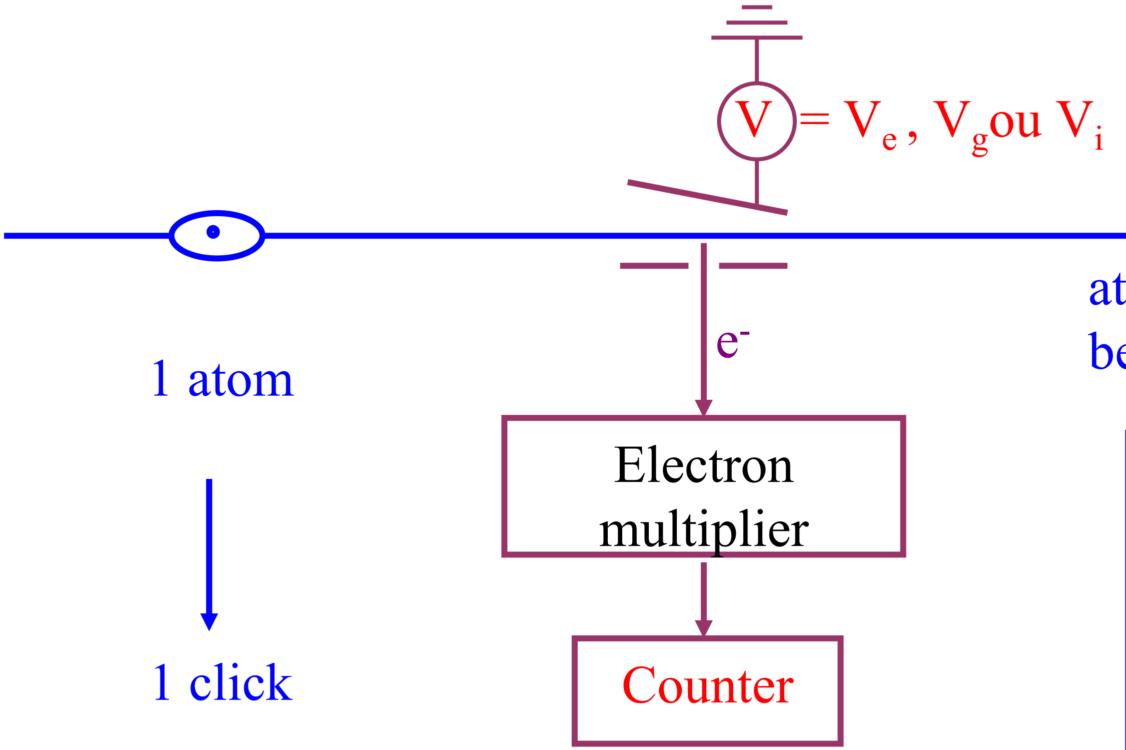
② - "Stark switching": E_{stat}=0→2.5V/cm.

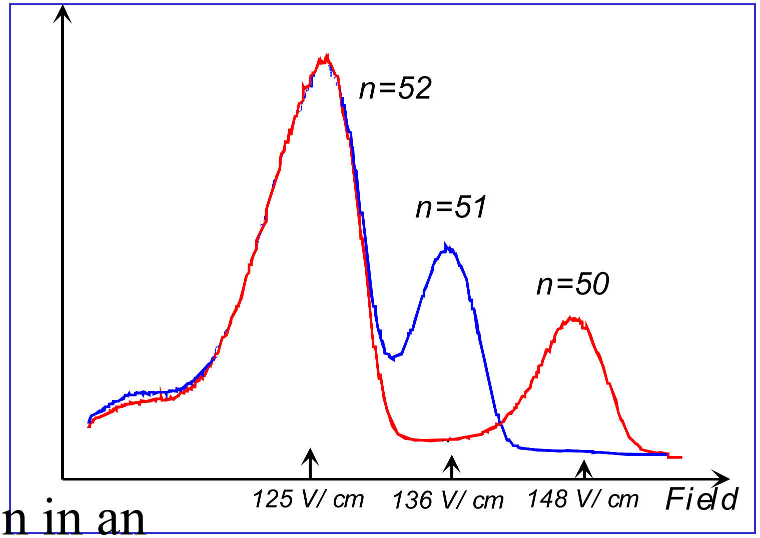
③ - 49 photons adiabatic transfer to 52c in duced by E_{RF}(ν=250MHz). B_{stat} removes degeneracy betwen σ^+ and σ^-.

④ - "Purification": selective transfer to 51c or 50c.

→ 300 circular atom/laser pulse. Purity > 99%

Detection of Rydberg atoms (1)





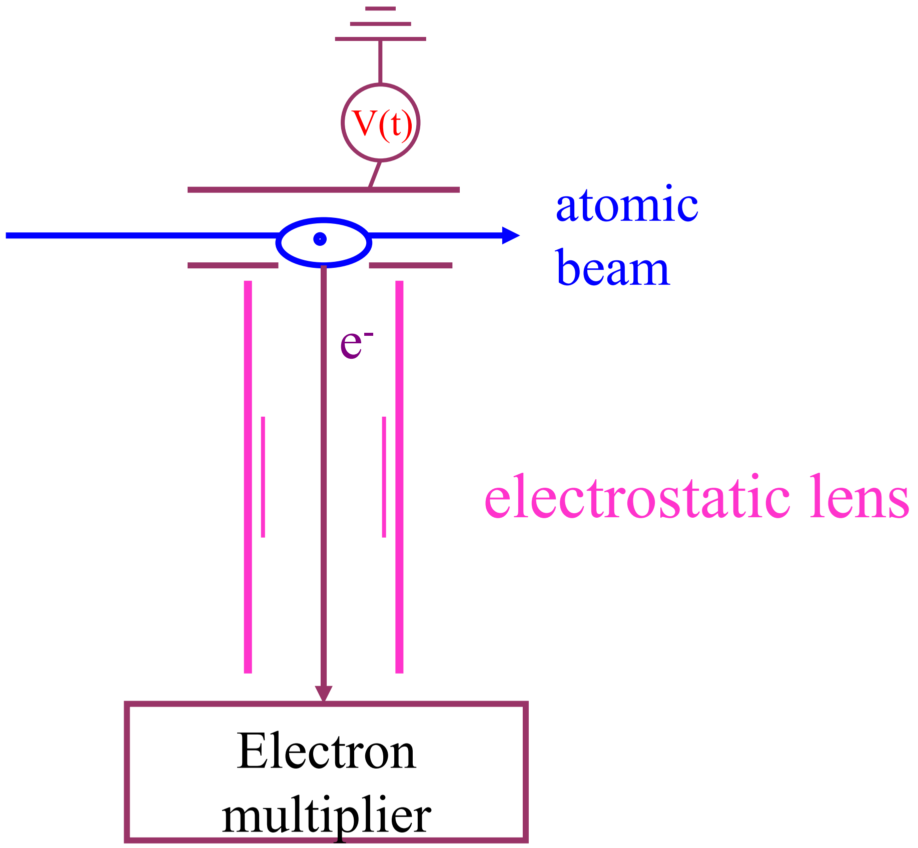
- atoms detected one by one by selective ionization in an electric field

➤ measurement of internal energy state of the atom after interaction with C

CW detection in a field gradient: efficiency 40%

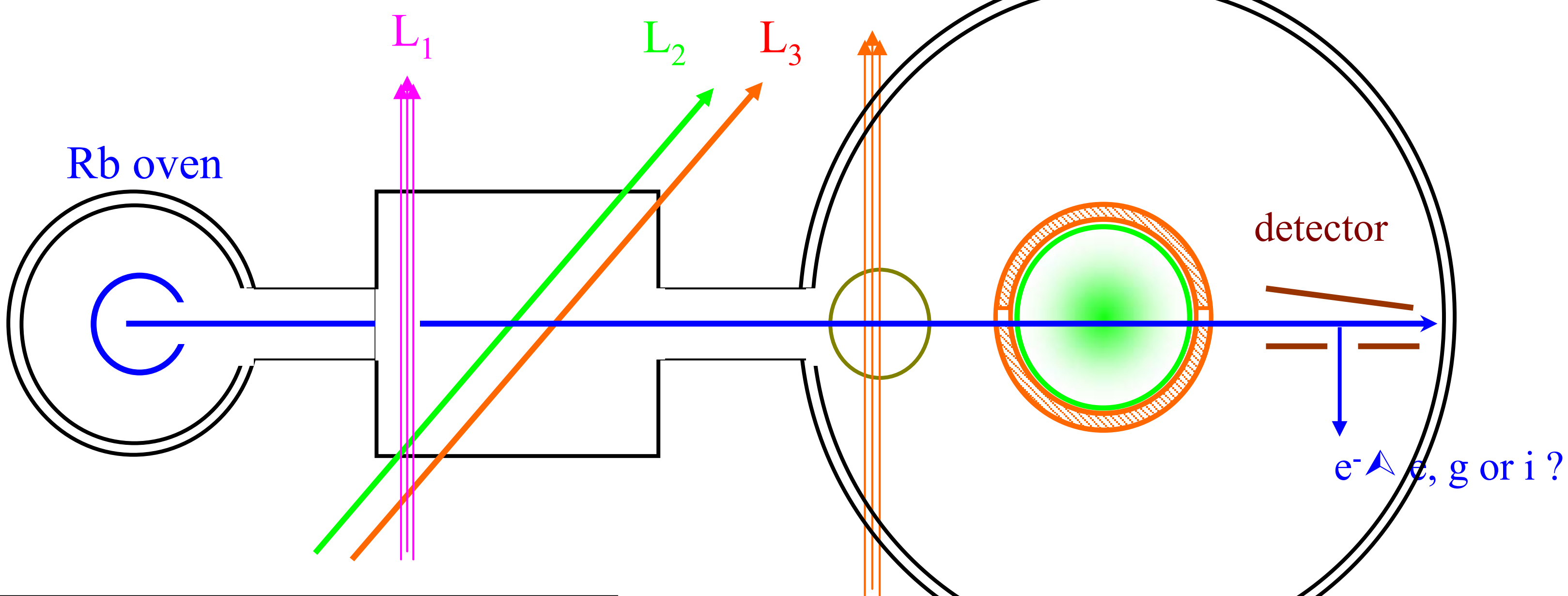
Detection of Rydberg atoms (2)

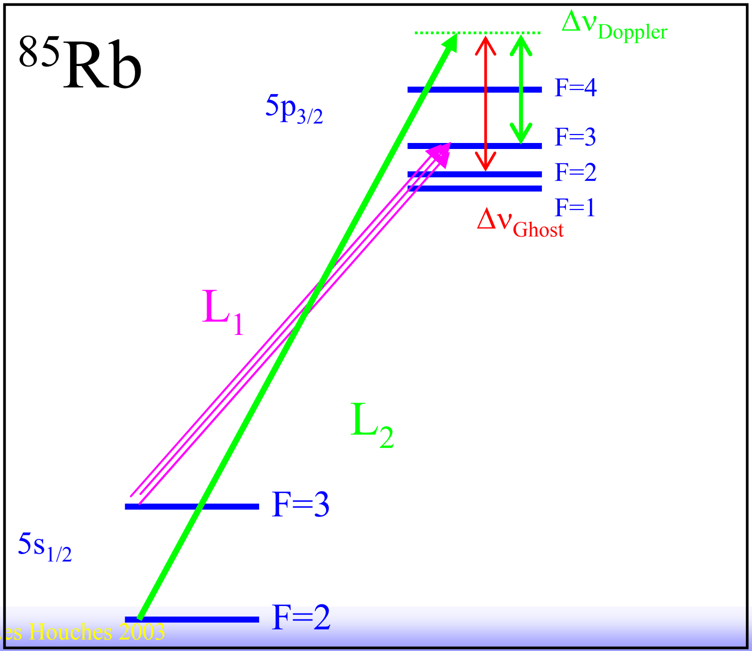
- New detector: atoms are ionized in a pulsed homogeneous electric field:



Improved detection efficiency: 70% +/- 10%

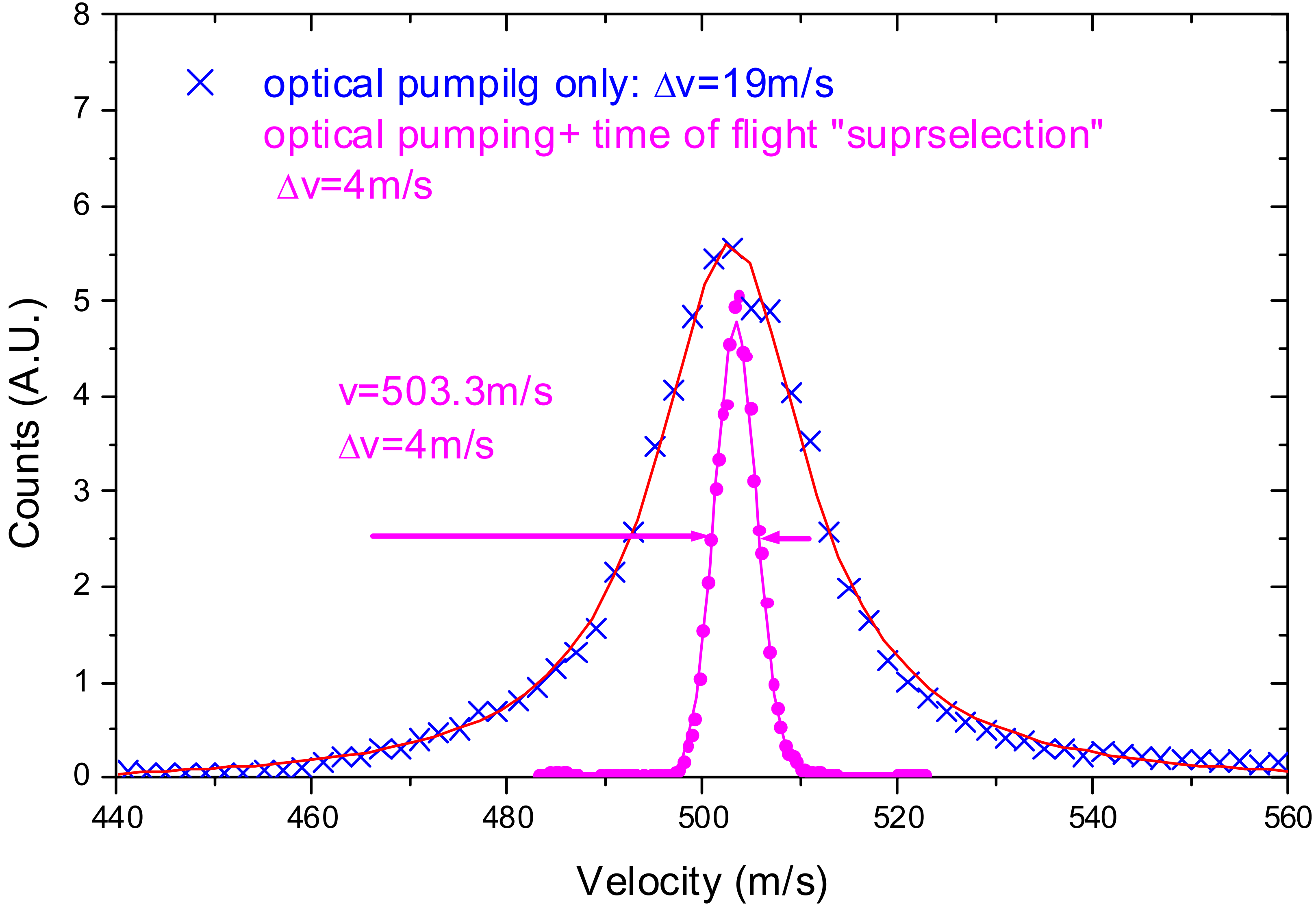
Velocity selection by optical pumping:



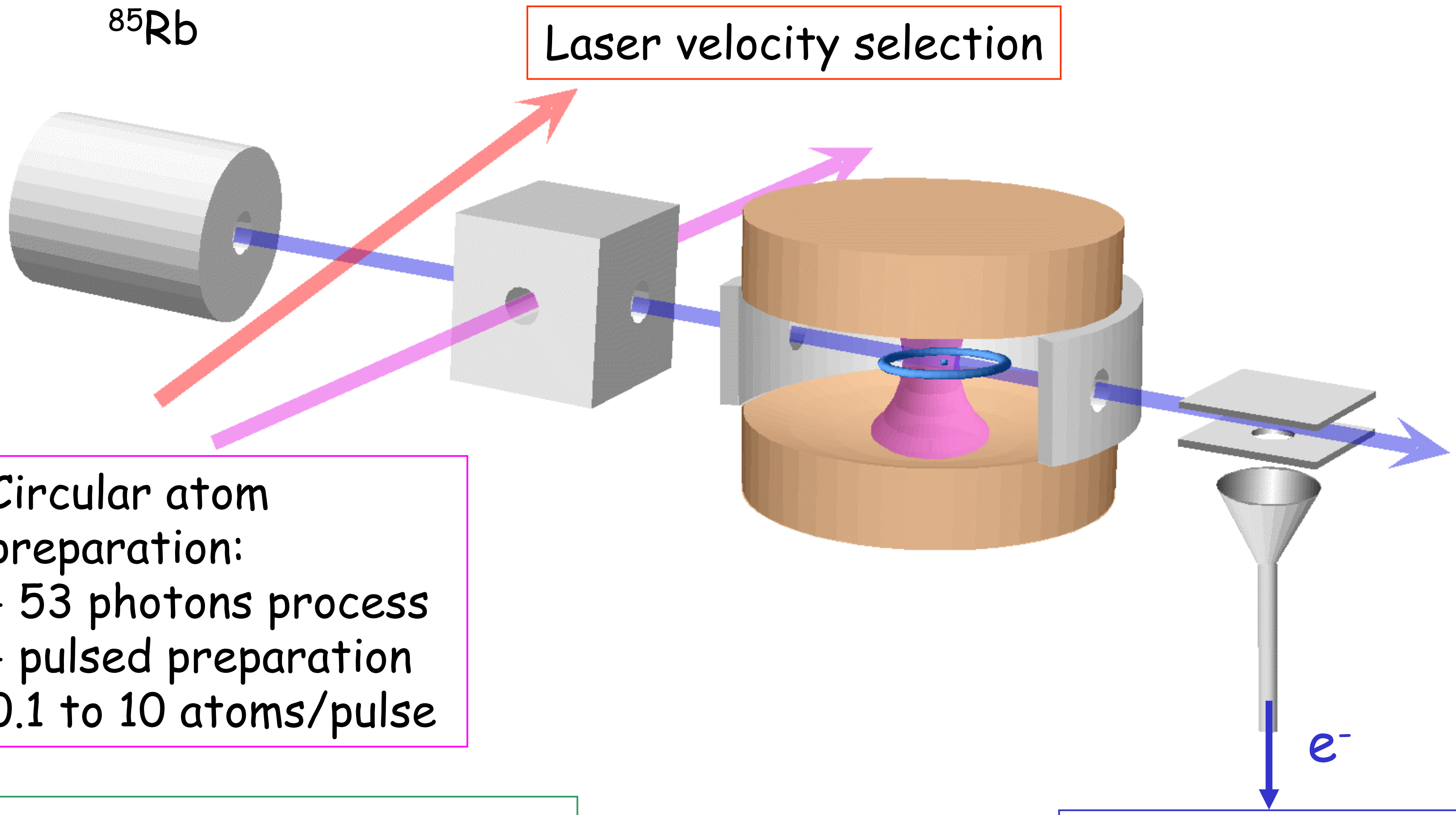


- L1: depumping beam
- L2: velocity selective repumping beam, tuned on $\nu_{F=2\text{-}F=3}+\Delta\nu_{Doppler}$
- L3: velocity selective depumping beam for removing "ghost"velocity pumped by L2 on the F=2-F=2 transition

Velocity "superselection"



Experimental set-up



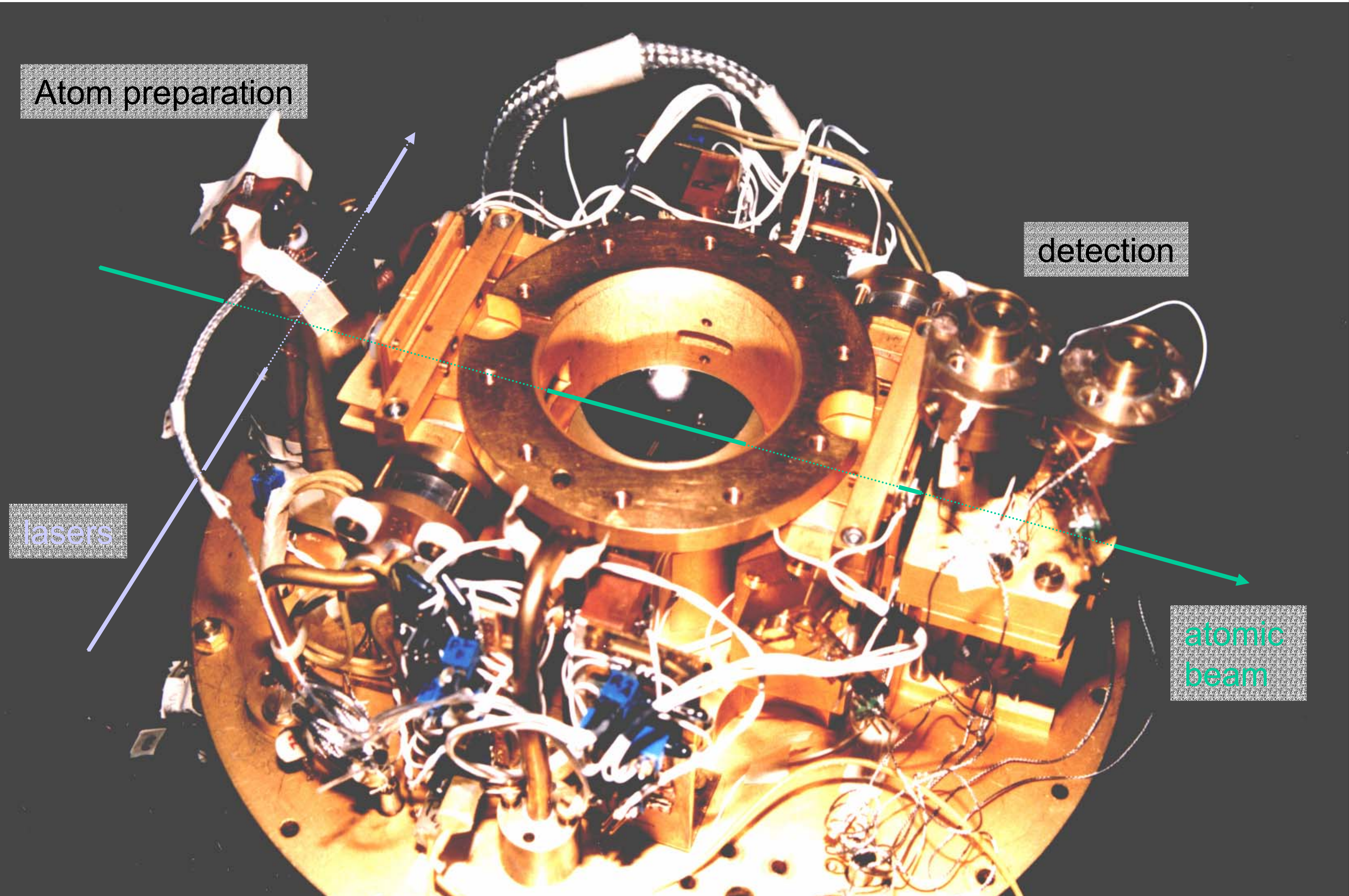
Laser velocity selection

Circular atom preparation:
- 53 photons process
- pulsed preparation
0.1 to 10 atoms/pulse

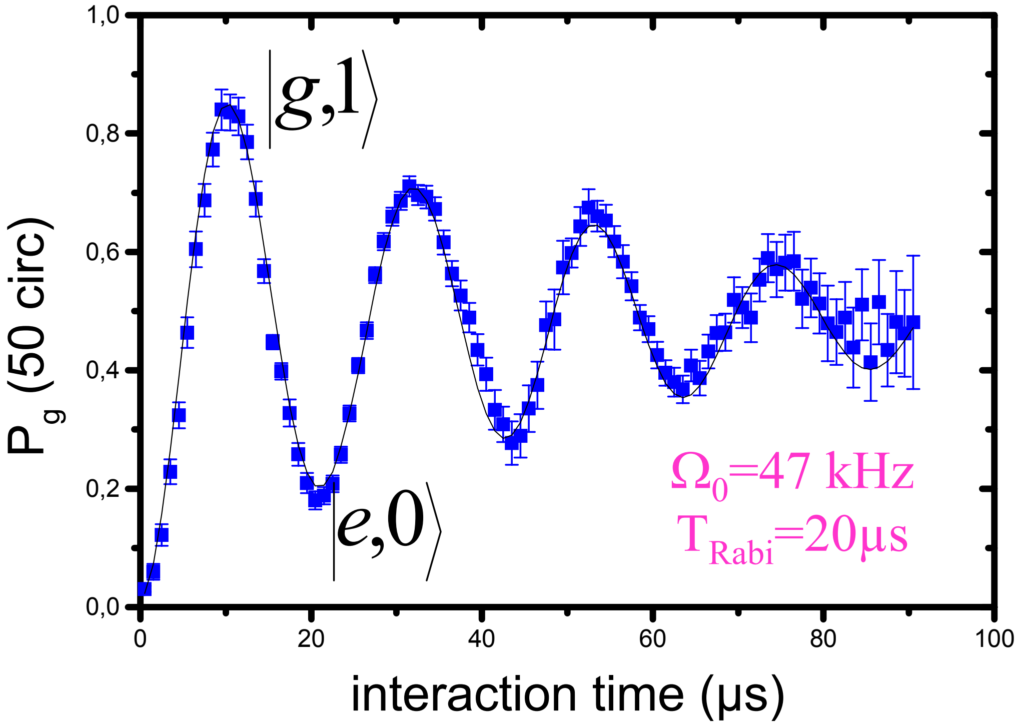
Cryogenic environment
T=0.6 to 1.3 K
⇒weak blackbody radiation

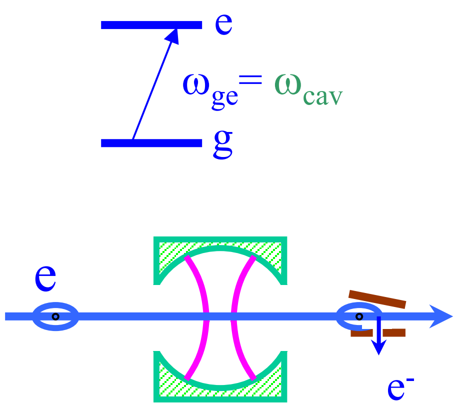
State selective detector
One atom = one click

Experimental setup



Single photon induced Rabi oscillation





Coherent Rabi oscillation
replaces irreversible damping
by spontaneous emission

3. Rabi oscillation in a small coherent field

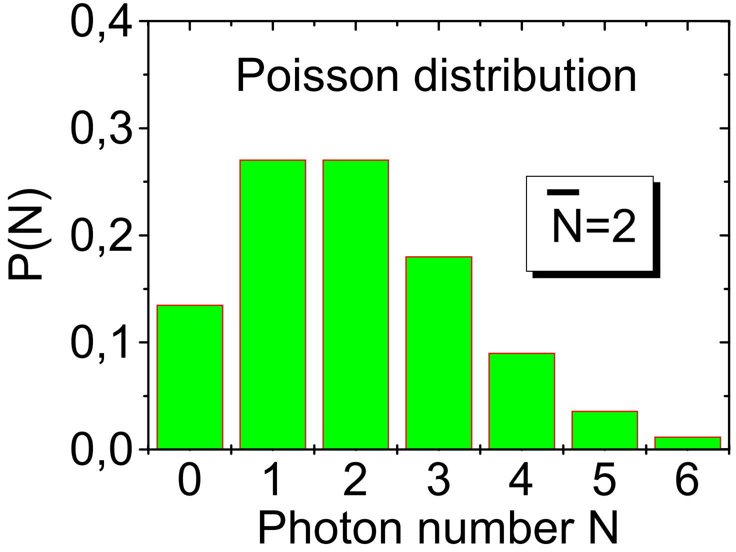
Direct observation
of discrete Rabi frequencies

Coherent field states

- Number state: $|N\rangle$
- Quasi-classical state: $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_N \frac{\alpha^N}{\sqrt{N!}}|N\rangle \quad ; \quad \alpha = |\alpha| e^{i\Phi}$

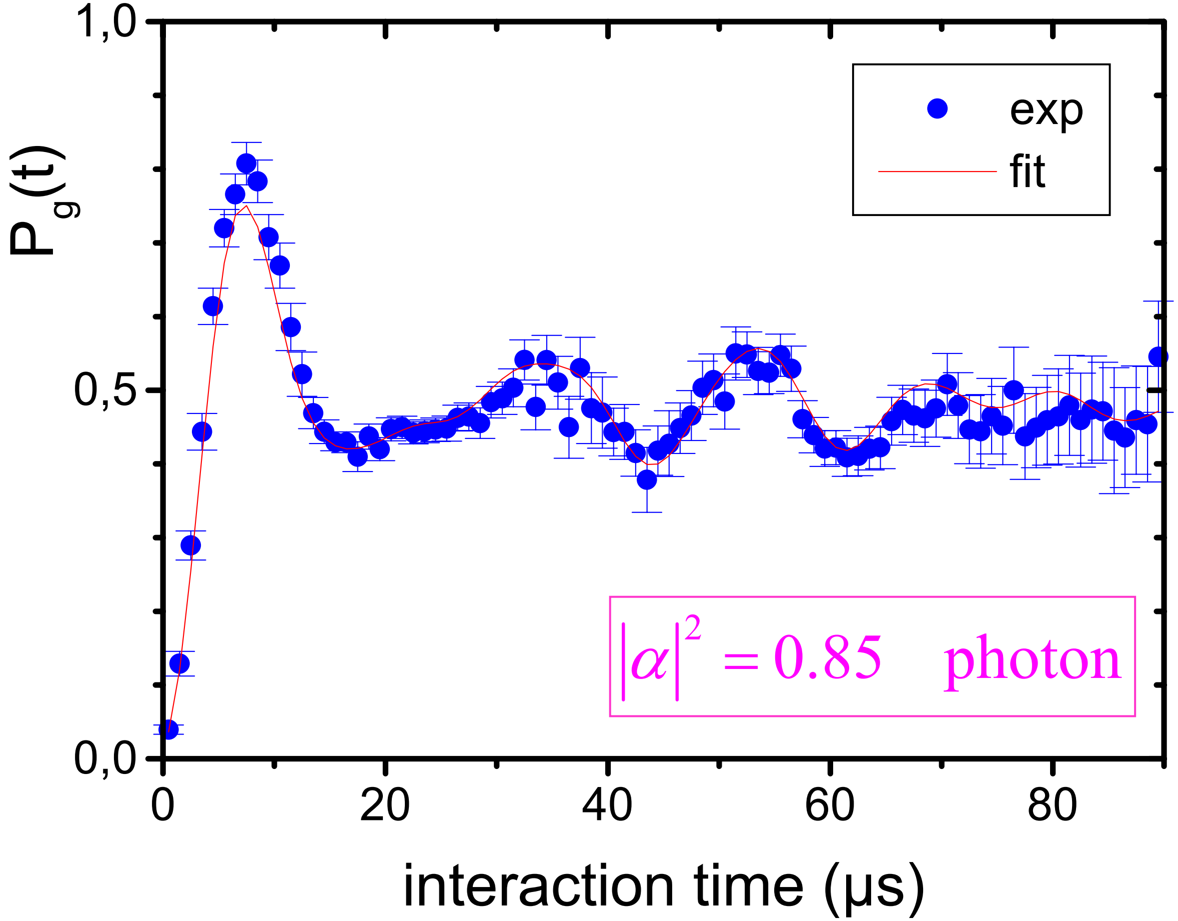
Photon number distribution:

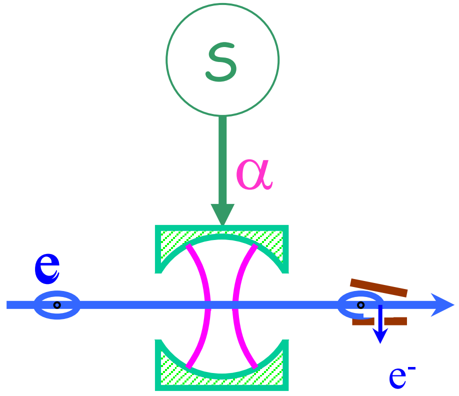
$$P(N) = e^{-\bar{N}}\frac{\bar{N}^N}{N!} \quad ; \quad \bar{N} = |\alpha|^2$$



$$P_g(t) = \sum_N P(N)\frac{1}{2}\left(1-\cos\left(\Omega_0 t\sqrt{N+1}\right)\right)$$

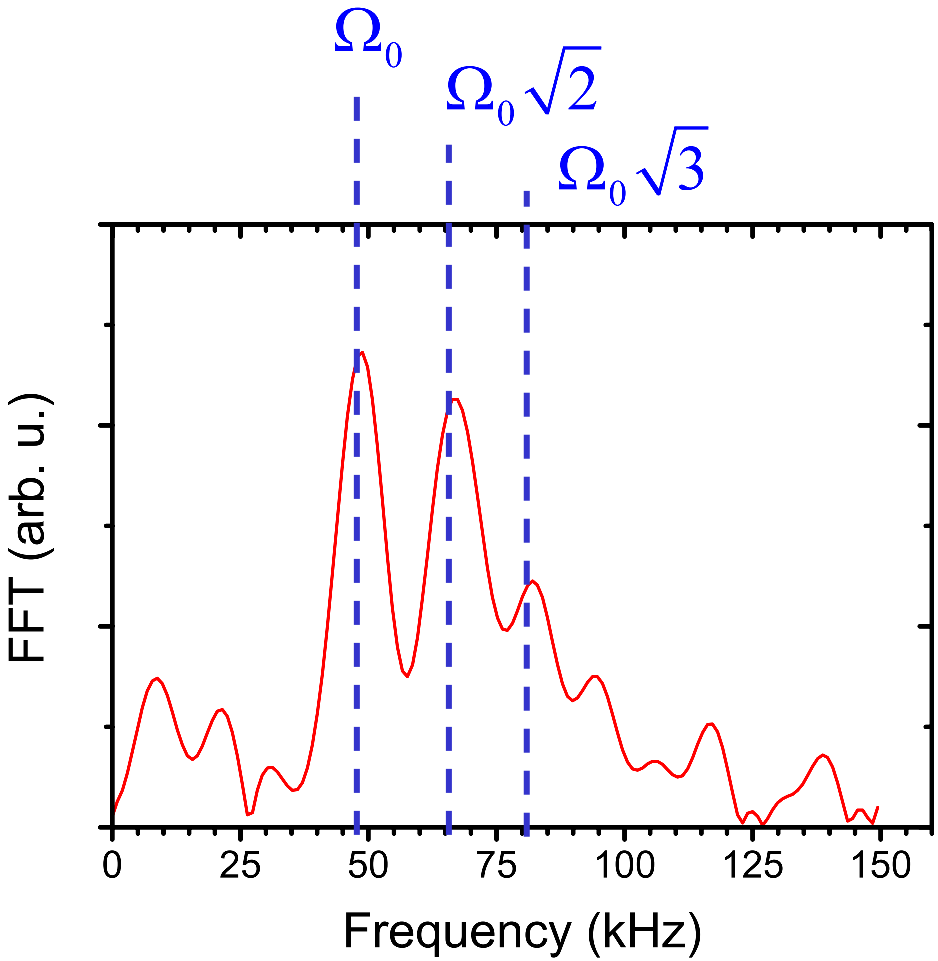
Rabi oscillation in a small coherent field





Rabi oscillation in a small coherent field: observing discrete Rabi frequencies

Fourier transform of the Rabi oscillation signal



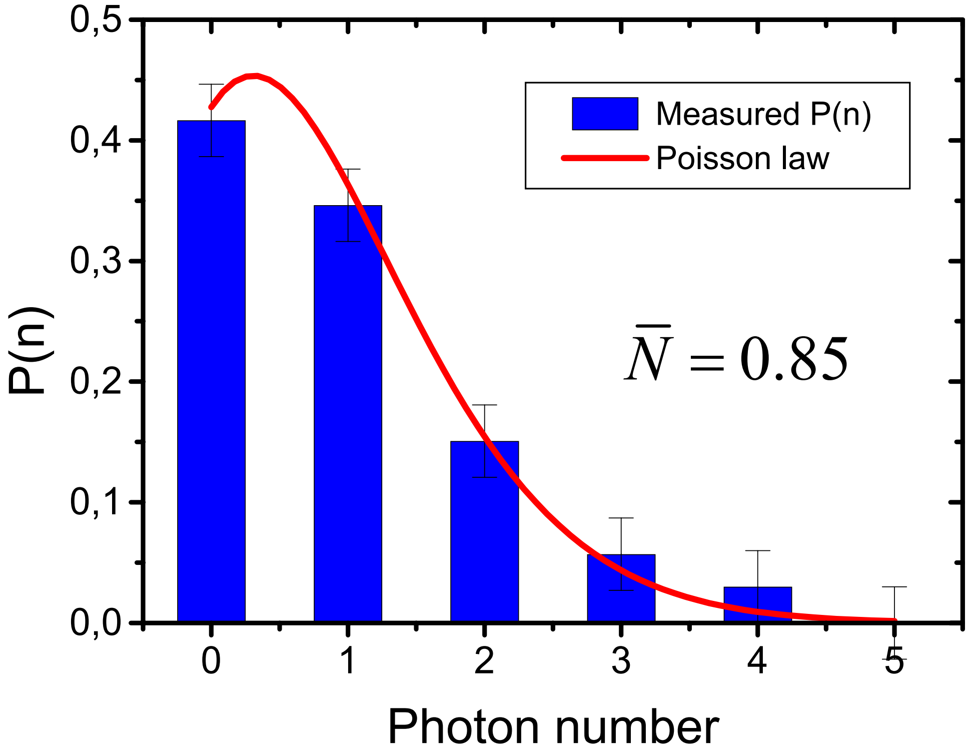
Discrete peaks corresponding to discrete photon numbers

➢ Direct observation of field quantization in a "box"

Rabi oscillation in a small coherent field: Measuring the photon number distribution

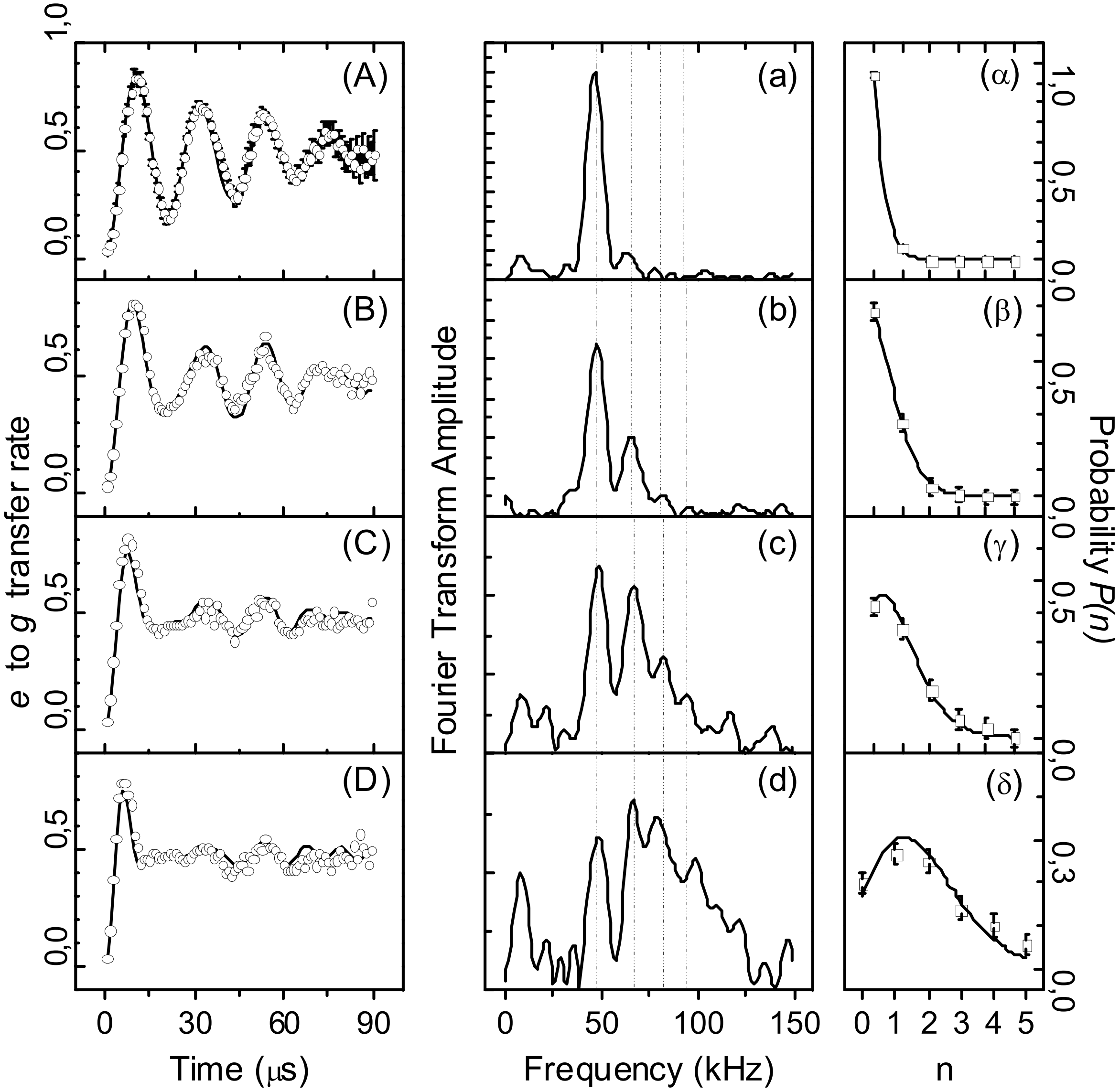
$$P_g(t) = \sum_N P(N) \frac{1}{2}\left(1 - \cos\left(\Omega_0 t \sqrt{N+1}\right).e^{-t/\tau}\right)$$

- Fit of P(n) on the Rabi oscillation signal:



- accurate field statistics measurement

Rabi oscillation in small coherent fields



<n>=0.06

<n>=0.4

<n>=0.85

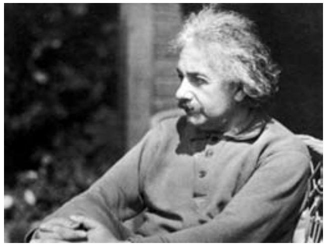
<n>=1.77

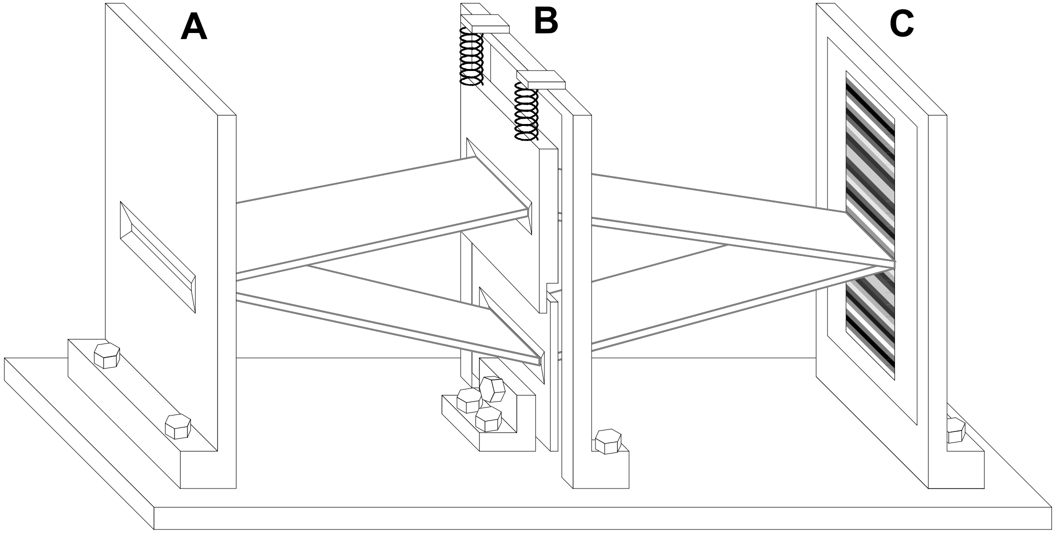
Phys. Rev. Lett. ***76****, 1800 (1996)*

4. Rabi oscillation Ramsey interferometry and complementarity

- Entanglement and complementarity
- quantum eraser

Probing the EPR atom-field entanglement





Illustrating Bohr-Einstein dialog

central concept: complementarity

Massive slits: insensitive to collisions with single particles

Interferences: mater behave as waves

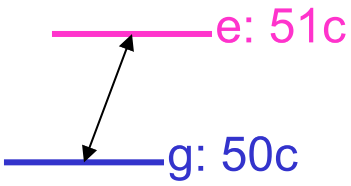
Experiment performed with photons, electrons, atoms, molecules.

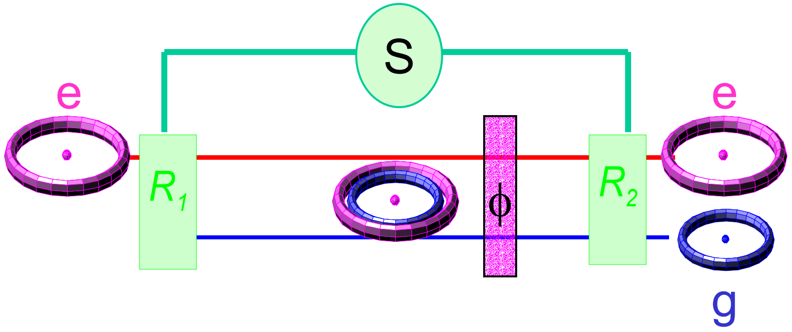
Light slits: recoil of the slit monitors which path information

No interferences: mater behave as particles

More practical interferometers

- Ramsey interferometer



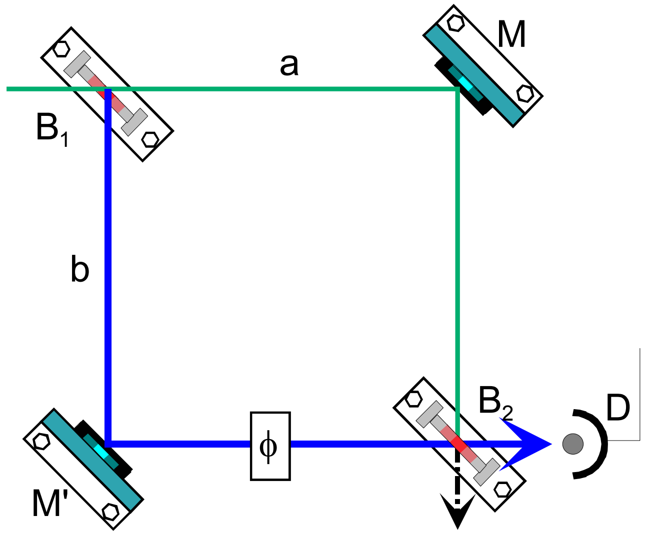


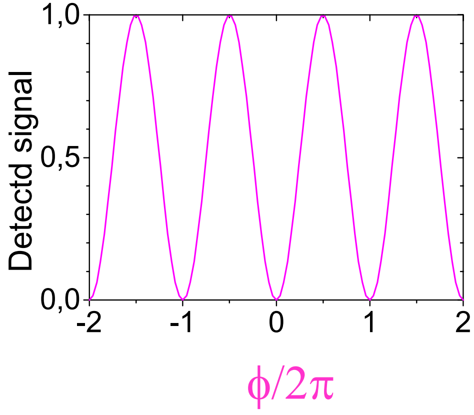
- R1 and R2: resonant $\pi/2$ pulses induced by CLASSICAL microwave fields.

acts as "beam splitter" for the internal atomic state

- The phase ϕ of the interferometer is scanned using a Stark pulse

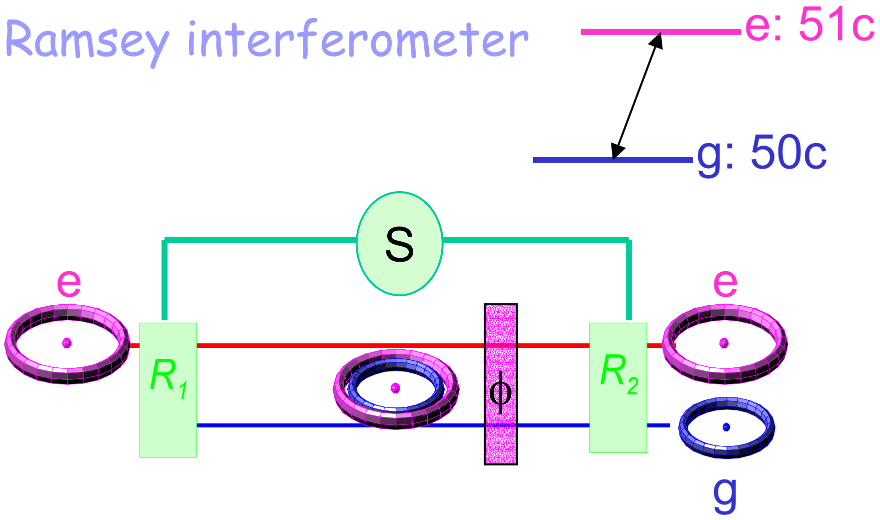
- Mach Zender Interferometer





More practical interferometers

- Ramsey interferometer

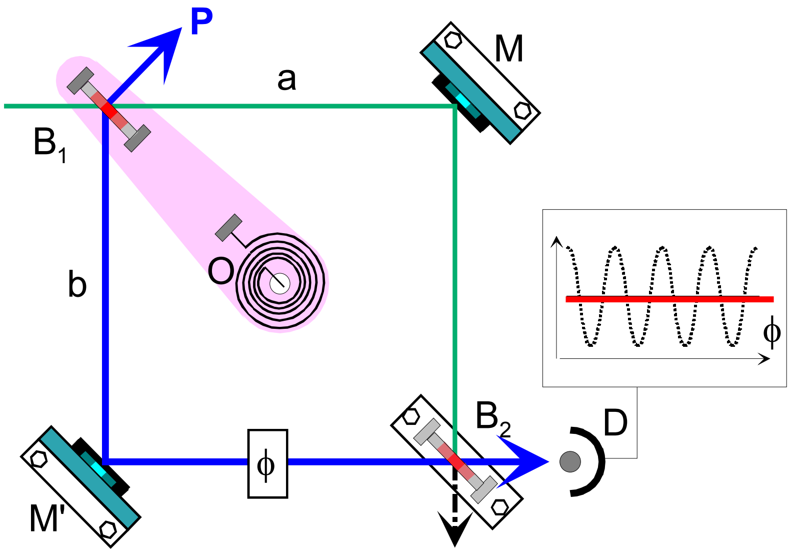


- R1 and R2: resonant π/2 pulses induced by CLASSICAL microwave fields.

acts as "beam splitter" for the internal atomic state

- The phase ϕ of the interferometer is scanned using a Stark pulse

- Mach Zender Interferometer



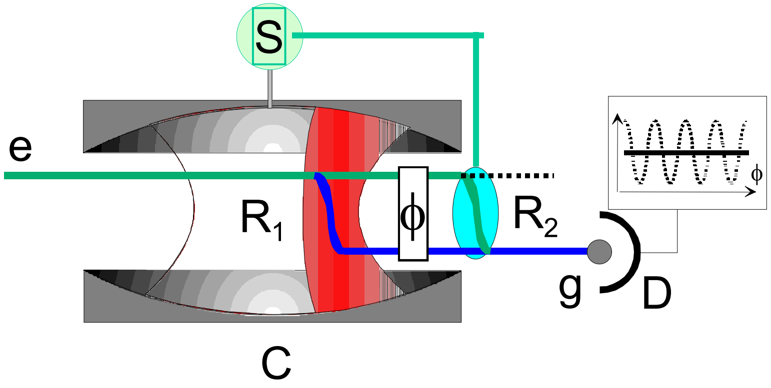
Small mass beam splitter: the recoil momentum P at reflexion of the particle on B1 keeps track of which path information

No fringes for a microscopic beam splitter

Ramsey interferometry with a "quantum beam splitter"

- $\pi/2$ pulse R_1: Rabi oscillation in a small coherent field injected in C.

  S injects in C a small coherent field $|\alpha\rangle$.



$$\boxed{|e\rangle \otimes |\alpha\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|e\rangle \otimes |\alpha_e\rangle + |g\rangle \otimes |\alpha_g\rangle\right)}$$

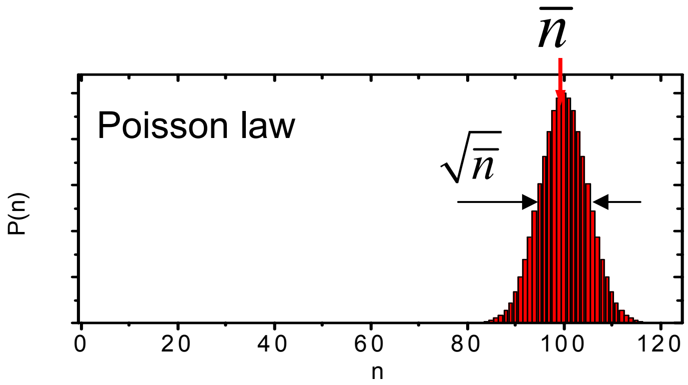
- $|\alpha_e\rangle$ and $|\alpha_g\rangle$ are not coherent states
- Generaly: $|\langle \alpha_e | \alpha_g \rangle| < 1$
  - partial atom-field entanglement
  - partial which path information is stored in the final field state

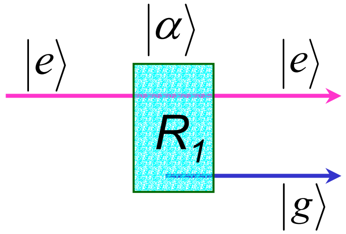
From classical to quantum beam splitters:

- $\pi/2$ pulse in a large coherent state: $\boxed{\bar{n} \gg \sqrt{\bar{n}}}$

$$\boxed{\left|\alpha_e\right\rangle \approx \left|\alpha_g\right\rangle \approx \left|\alpha\right\rangle}$$

$$\boxed{\left|e\right\rangle \otimes \left|\alpha\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|e\right\rangle + \left|g\right\rangle\right) \otimes \left|\alpha\right\rangle}$$





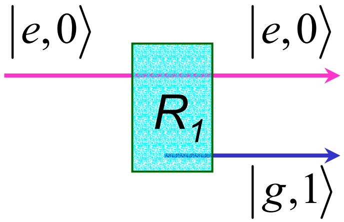
When α is large enough,
one more photon in the field does not
make any difference on the field state

→ NO which path information stored in the field: "classical beam splitter"

- $\pi/2$ pulse in vacuum: α=0

$$\boxed{\left|e\right\rangle \otimes \left|0\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|e,0\right\rangle + \left|g,1\right\rangle\right)}$$

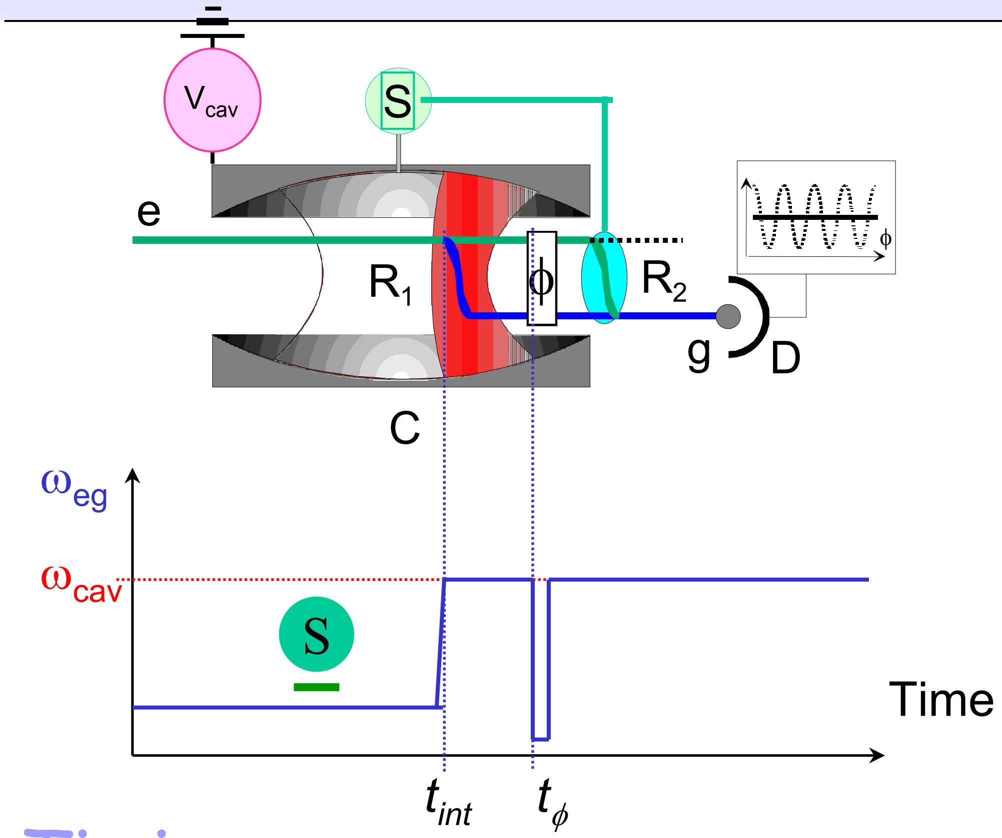
Atom-field EPR pair:
Hagley et al. PRL ***79****,1 (1997)*



The photon number is a perfect label of the atomic state

→ FULL which path information stored in the field: "quantum beam splitter"

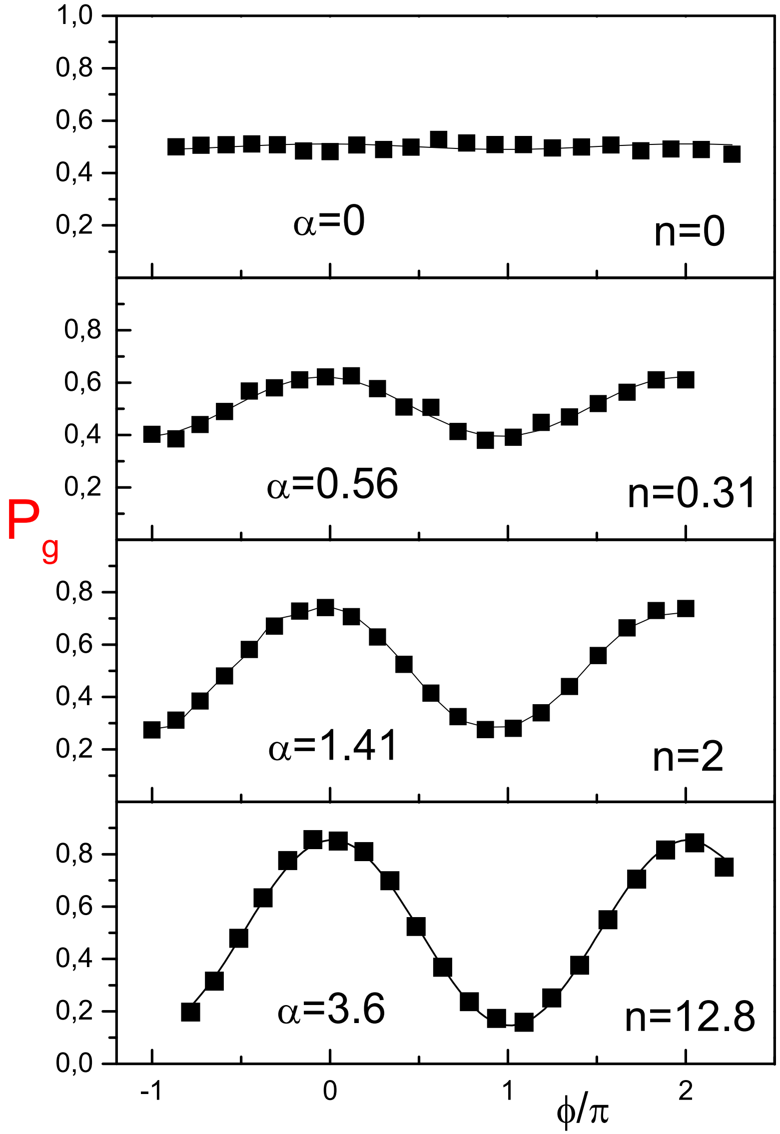
Practical realization



The atomic frequency is adjusted by Stark effect with V_{cav}

- Timing:
  - ❑ Inject a controlled coherent field. (average photon number <n>measured by measuring light shifts in an auxiliary experiment)
  - ❑ Put atom on resonance at time t_{int} . For each value of <n>, t_{int} is adjusted for performing a $\pi/2$ Rabi oscillation pulse
  - ❑ Vary ϕ by varying V_{cav} at time t_{ϕ}.

Fringes signal for various values of n:



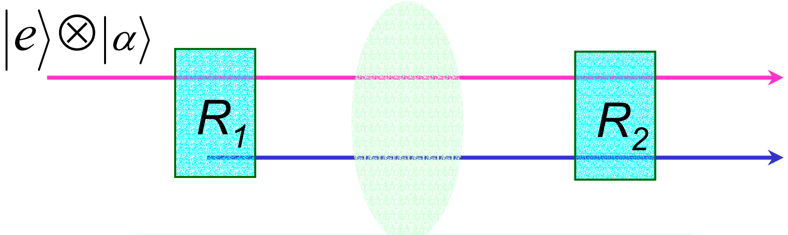
Perfect distingishability:
No fringes

Saturated contrast: η=0.73

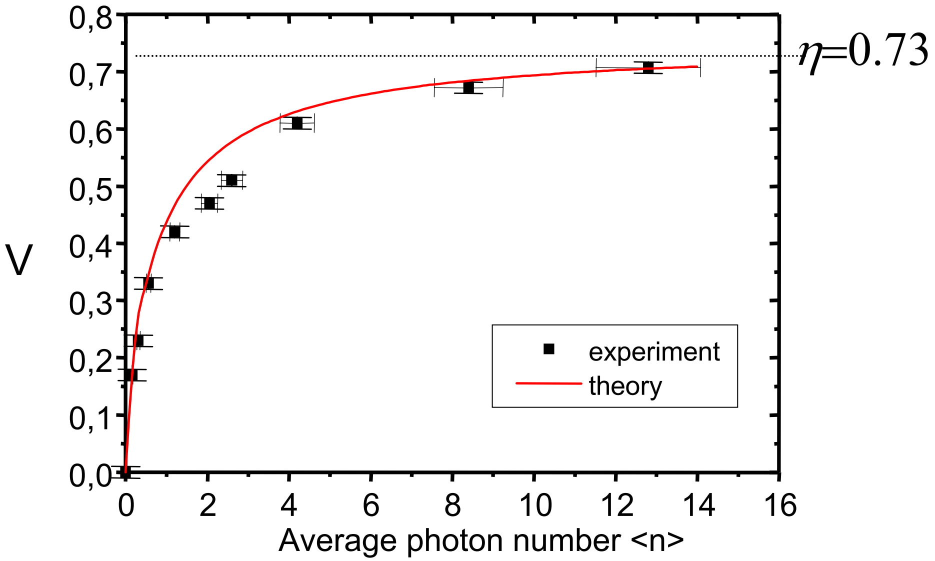
Bertet et al. Nature ***411****, 166 (2001)*

Quantitative interpretation in term of atom-cavity entanglement

- Variation of fringe Visibility V:



$$\boxed{\frac{1}{\sqrt{2}}\left(|e\rangle\otimes|\alpha_e\rangle+|g\rangle\otimes|\alpha_g\rangle\right)}$$



Reduced atom density matrix:

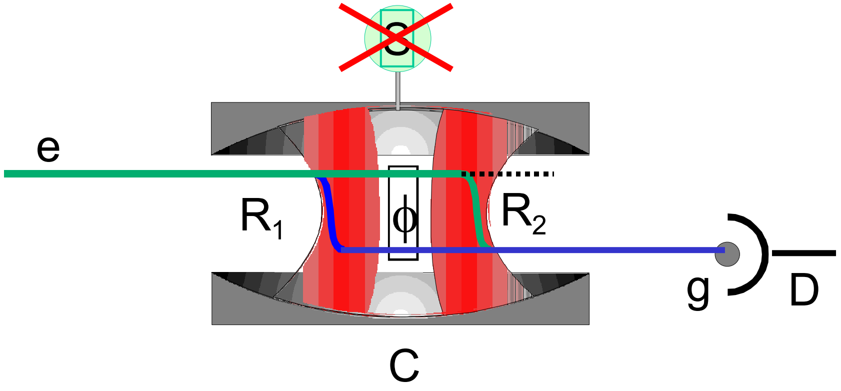
$$\rho_{at}=\frac{1}{2}\begin{pmatrix} 1 & \langle\alpha_e|\alpha_g\rangle^* \\ \langle\alpha_e|\alpha_g\rangle & 1 \end{pmatrix}$$

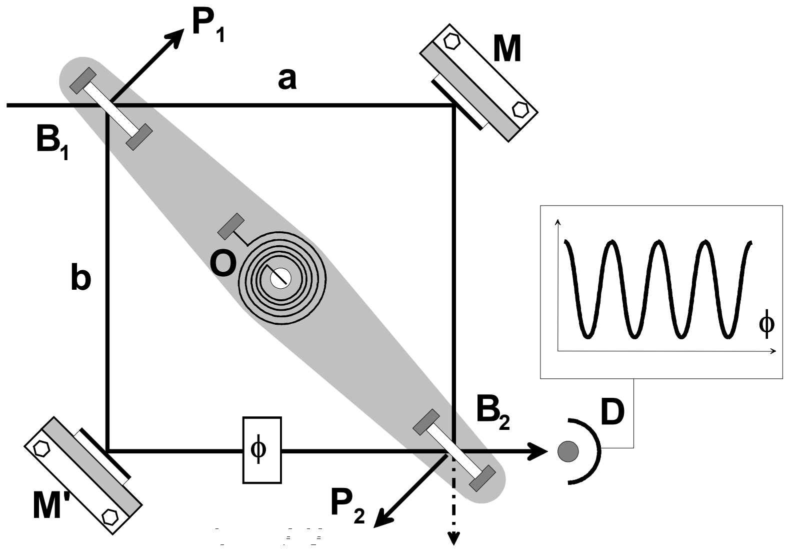
$$\boxed{V=\left|\langle\alpha_e|\alpha_g\rangle\right|.\eta}$$

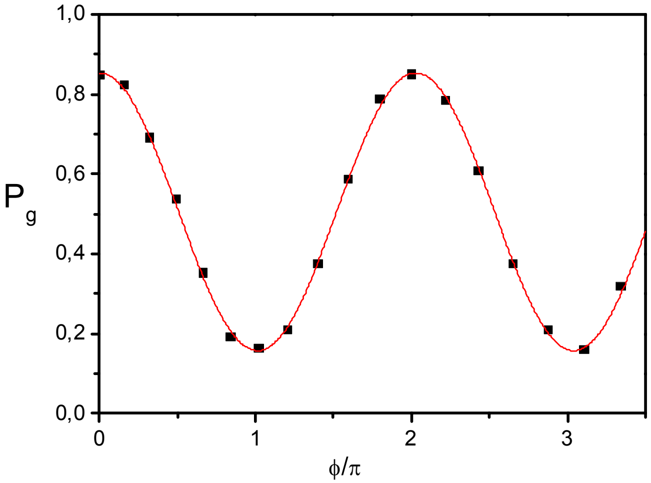
η: saturated contrast at large n

Checking the atom-"beam splitter"entanglement: a quantum eraser

- *C is initially empty:*





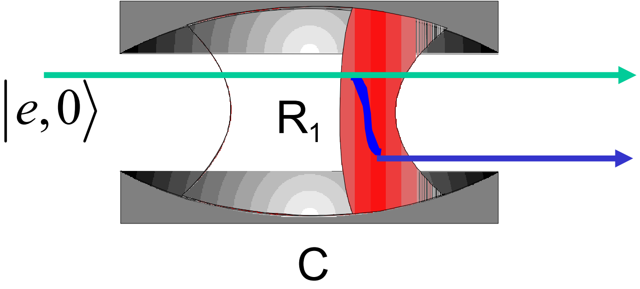


- The which path information is erased by the second interaction with C

variations of fringe contrast can not be interpreted as simply resulting from noise added by the interaction with the beam-splitter

A genuine quantum eraser:

- Which path information is erased by appropriate measurement of the "which path meter":



$$\boxed{\frac{1}{\sqrt{2}}\left(|e,0\rangle + |g,1\rangle\right)}$$

Measurement of the cavity field in the basis:

$$\frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

Projection of atomic state on:

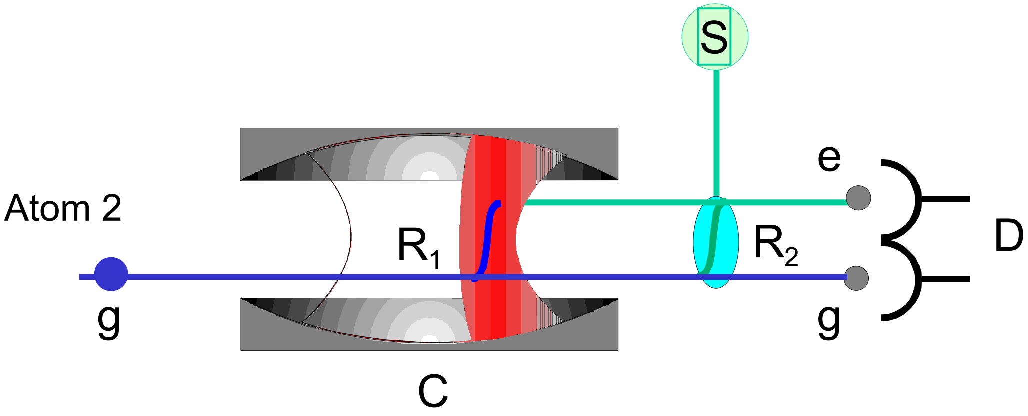
$$\frac{1}{\sqrt{2}}\left(|e\rangle \pm |g\rangle\right)$$

R_1 is then equivalent to a classical $\pi/2$ pulse

whose phase depends on the result of the measurement of the field.

When R_2 is activated, one expects conditional Ramsey fringes.

Quantum eraser: measuring the cavity field with a second atom



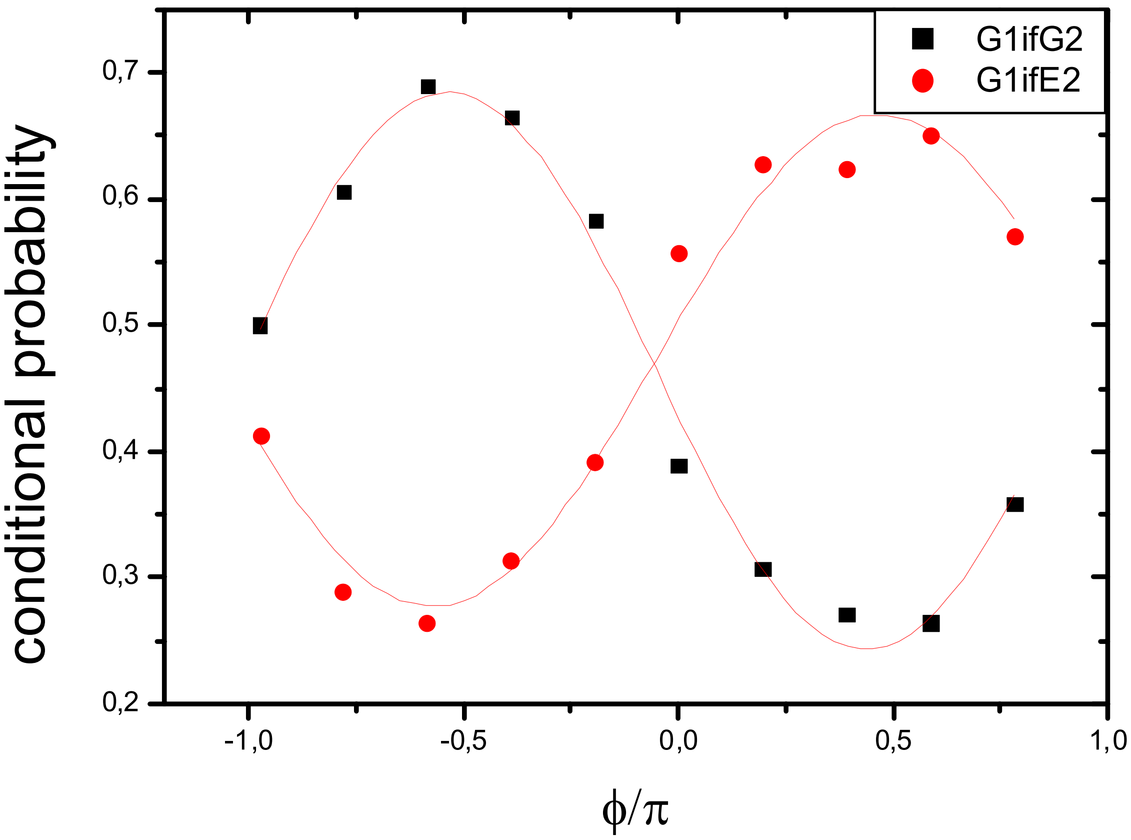
1. π pulse in C: swaps a 0 or 1 photon state into an atomic state:
Maître et al. PRL 79,769 (1997)

$$|g,0\rangle \Rightarrow |g,0\rangle$$
$$|g,1\rangle \Rightarrow |e,0\rangle$$

2. Detection of the atom after a classical $\pi/2$ pulse:
   equivalent to the measurement of the phase of the atomic dipole
   equivalent to the measurement of the phase on the initial field state.

This performs the appropriate measurement of the cavity field state in C

Quantum eraser: experimental results



High visibility is restored

Conclusion of lecture 1:

Cavity QED with microwave photons and circular Rydberg atoms:

…. a powerfull tool for:

- *Achieving* strong coupling between single atoms and single photons
- manipulating entanglement and complementarity

…. next lecture:

  - ☐ Rabi oscillation in vacuum and quantum gates

References

- Strong coupling regime in CQED experiments:

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  - J.M. Raimond, M. Brune and S. Haroche : "Manipulating quantum entanglement with atoms and photons in a cavity", Rev. Mod. Phys. vol.73, p.565-82 (2001).
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  - E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond and S. Haroche: "Generation of Einstein-Podolsky-Rosen pairs of atoms", PRL 79,1 (1997).