Classical and Quantum impurities in superconductors

100 years old and still dirty

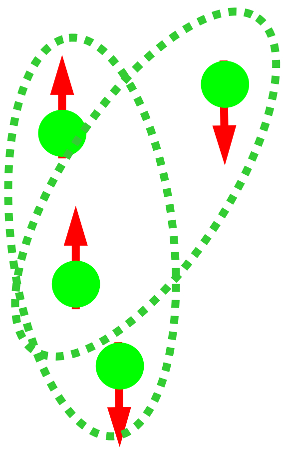
Ilya Vekhter

Louisiana State University, USA

Superconductivity review I

Simplest (well understood) correlated system:

often even when emerges from a strange normal state



pairing of electrons
near the Fermi surfc

\+

Bose-condensation
of Cooper pairs

Superconductivity

Nontrivial object: ***pairing amplitude***

$$\Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2,\beta) \propto \langle \psi_\alpha(\mathbf{r}_1)\psi_\beta(\mathbf{r}_2)\rangle$$

Simplest case: $\alpha,\beta = \uparrow,\downarrow$

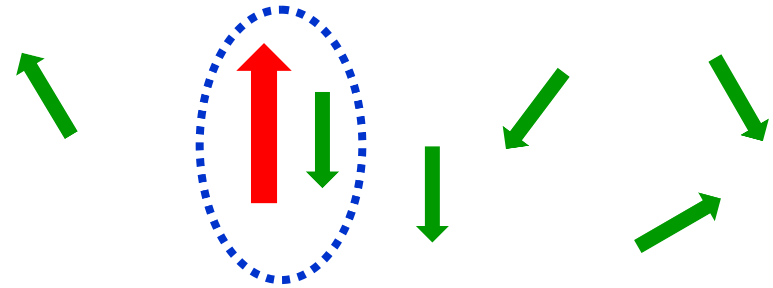
$$\Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2\beta) = \chi_{\alpha\beta}\varphi(\mathbf{r}_1-\mathbf{r}_2)$$

Cooper pairs have well-defined spin (singlet or triplet pairs)

Why impurities?

Kondo impurity:

local singlet + electrons



Simplest superconductor: no spin-orbit

$$\Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2\beta) = \chi_{\alpha\beta}\varphi(\mathbf{r}_1-\mathbf{r}_2)$$

singlet ***S=0***

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$$\chi_{s,\alpha\beta} = (i\sigma^y)_{\alpha\beta}$$

$$\chi_{s,\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

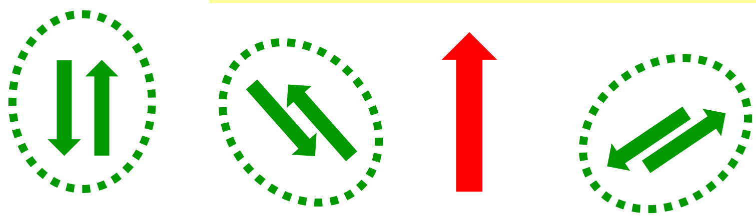
triplet ***S=1***

$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$$

$$\chi_{t,\alpha\beta} = [(i\sigma^y)(\mathbf{d}\cdot\boldsymbol{\sigma})]_{\alpha\beta}$$

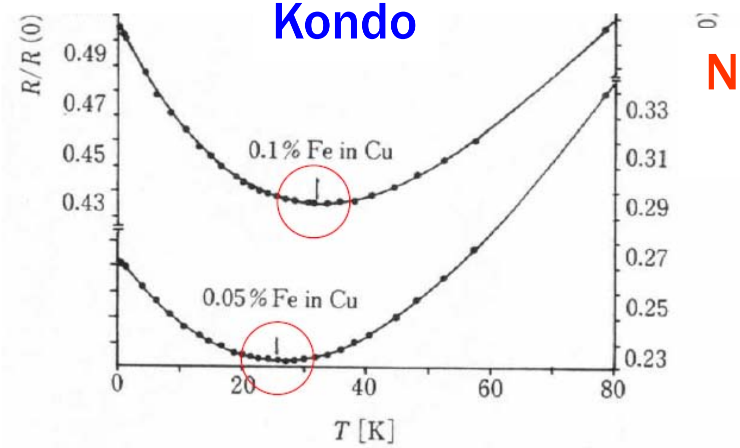
$$\chi_t = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

Competition of energy scales:
impurities vs pairing



Superconductors vs Kondo metals

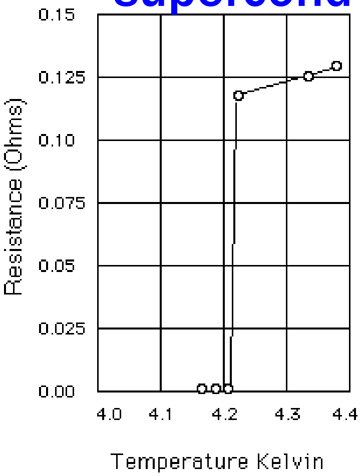
Kondo



From D. MacDonald et al. 1962

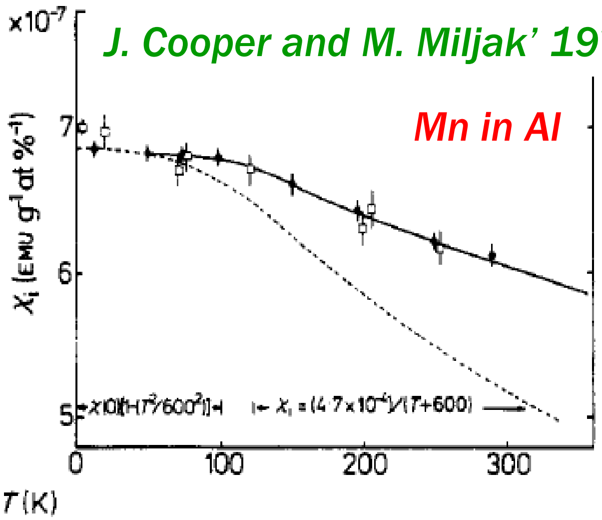
No resistance minimum: superconductivity

supercond



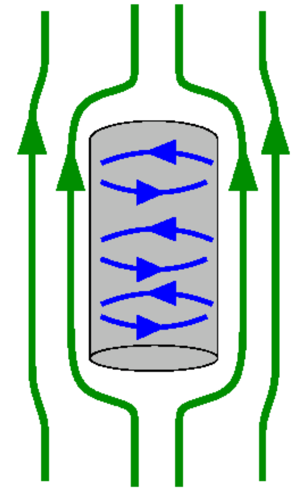
H. Kamerlingh-Onnes 1911

J. Cooper and M. Miljak' 1976



No susceptibility: Meissner effect

NB: sometimes NMR



Superconductivity review II

BCS Hamiltonian: $$H_{BCS} = \sum \xi_{\mathbf{k}} c^{+}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} - \Delta_{\alpha\beta}(\mathbf{k}) c^{+}_{\mathbf{k}\alpha} c^{+}_{-\mathbf{k}\beta}$$

band — pairing, "anomalous"

Order parameter: $$\Delta_{\alpha\beta}(\mathbf{k}) = \sum V_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}') \left\langle c_{-\mathbf{k}'\delta} c_{\mathbf{k}'\gamma} \right\rangle$$

singlet/triplet;
isotropic/anisotropic;
unitary or not…

Superconductivity review II

BCS Hamiltonian:

$$H_{BCS} = \sum \xi_{\mathbf{k}} c^{+}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} - \Delta_{\alpha\beta}(\mathbf{k}) c^{+}_{\mathbf{k}\alpha} c^{+}_{-\mathbf{k}\beta}$$

band — pairing, “anomalous”

Order parameter:

$$\Delta_{\alpha\beta}(\mathbf{k}) = \sum V_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}') \left\langle c_{-\mathbf{k}'\delta} c_{\mathbf{k}'\gamma} \right\rangle$$

singlet/triplet;
isotropic/anisotropic;
unitary or not…

Matrix form:

$$H_{BCS} = \begin{pmatrix} c^{+}_{\mathbf{k}\alpha} & c_{-\mathbf{k}\bar{\alpha}} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} & \Delta(\mathbf{k}) \\ \Delta^{*}(\mathbf{k}) & -\xi_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\alpha} \\ c^{+}_{-\mathbf{k}\bar{\alpha}} \end{pmatrix}$$

singlet

Superconductivity review II

BCS Hamiltonian:

$$H_{BCS} = \sum \xi_{\mathbf{k}} c^{+}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} - \Delta_{\alpha\beta}(\mathbf{k}) c^{+}_{\mathbf{k}\alpha} c^{+}_{-\mathbf{k}\beta}$$

band — pairing, "anomalous"

Order parameter:

$$\Delta_{\alpha\beta}(\mathbf{k}) = \sum V_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}') \langle c_{-\mathbf{k}'\delta} c_{\mathbf{k}'\gamma} \rangle$$

singlet/triplet;
isotropic/anisotropic;
unitary or not…

Matrix form:

$$H_{BCS} = \begin{pmatrix} c^{+}_{\mathbf{k}\alpha} & c_{-\mathbf{k}\bar{\alpha}} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} & \Delta(\mathbf{k}) \\ \Delta^{*}(\mathbf{k}) & -\xi_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\alpha} \\ c^{+}_{-\mathbf{k}\bar{\alpha}} \end{pmatrix}$$

singlet

$$H_{BCS} = \sum E(\mathbf{k}) \gamma^{+}_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}$$

Bogoliubov transformation

Excitation energies

$$E(\mathbf{k}) = \sqrt{\xi^2_{\mathbf{k}} + |\Delta(\mathbf{k})|^2}$$

energy gap

Eigenstates

$$\gamma_{\mathbf{k}\sigma} = u_{\mathbf{k}} c_{\mathbf{k}\sigma} - \sigma v^{*}_{\mathbf{k}} c^{\dagger}_{-\mathbf{k}\bar{\sigma}}$$

$|u_{\mathbf{k}}|^2$ electron

$|v_{\mathbf{k}}|^2$ hole

Isotropic vs anisotropic superconductors

Connection to pair wave function $\Delta_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2) \propto \Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2\beta) = \chi_{\alpha\beta}\varphi(\mathbf{r}_1-\mathbf{r}_2)$

Fermion exchange $\Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2\beta) = -\Psi(\mathbf{r}_2\beta;\mathbf{r}_1,\alpha)$ $\Psi(\mathbf{r}_1,\alpha;\mathbf{r}_2\beta) = \chi_{\alpha\beta}\varphi(\mathbf{r}_1-\mathbf{r}_2)$

Spin part: 2x2 matrix

singlet $S=0$

$$\chi_{s,\alpha\beta} = (i\sigma^y)_{\alpha\beta} = -\chi_{s,\beta\alpha}$$

triplet $S=1$

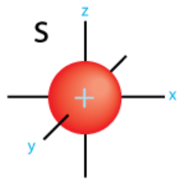
$$\chi_{t,\alpha\beta} = [(i\sigma^y)(\mathbf{d}\cdot\boldsymbol{\sigma})]_{\alpha\beta} = \chi_{t,\beta\alpha}$$

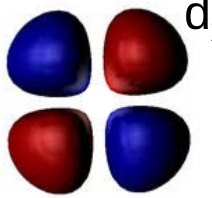
Spatial part: angular momentum l

$$l = 0,2,4...$$

s, d… wave

s



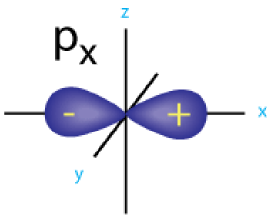


d_{xy}

$$l = 1,3,5...$$

p, f… wave

p_x

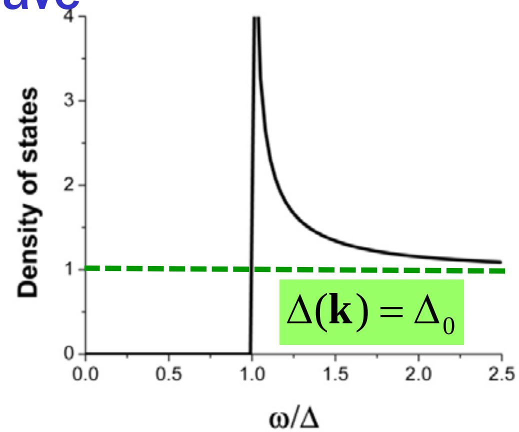


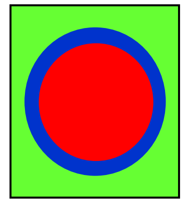
non-s-wave (anisotropic) states favored by strong Coulomb repulsion

Anisotropic superconductors

density of states

***s*-wave**

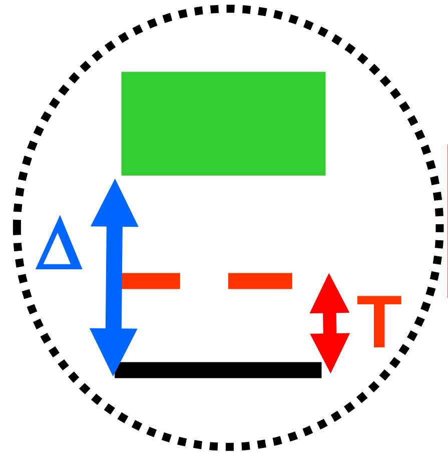




Isotropic gap

Al, Be, Nb_3Sn

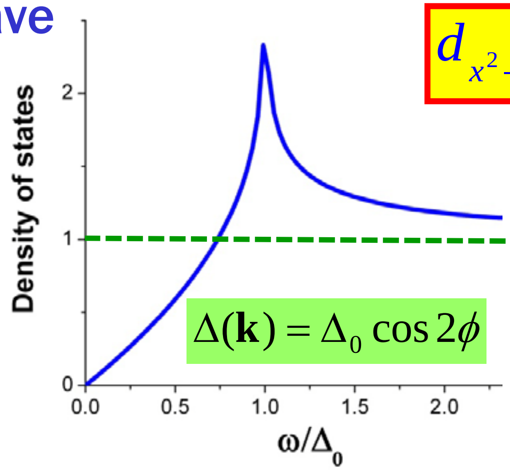
< 1978



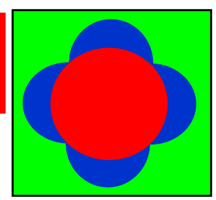
No excitations at low T

Activated behavior $e^{-\Delta/T}$

***d*-wave**



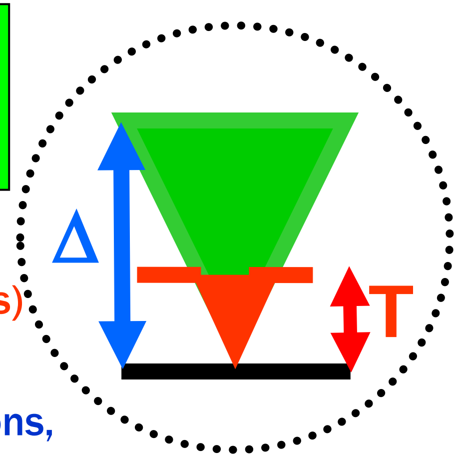
$d_{x^2-y^2}$



Gap with zeroes (nodes)

Cuprates, heavy fermions,

>1979

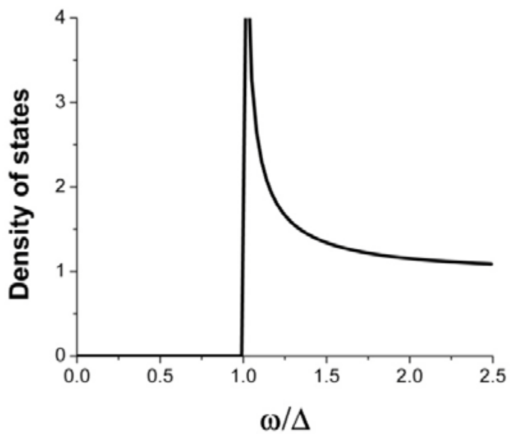


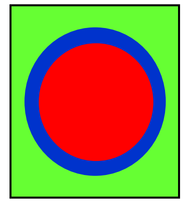
Density of qp	$\propto T$
Specific heat	$C(T) \propto T^2$
NMR	$T_1^{-1} \propto T^3$
universal	κ/T

Power laws

Pure and impure superconductors

Pure superconductor: density of states

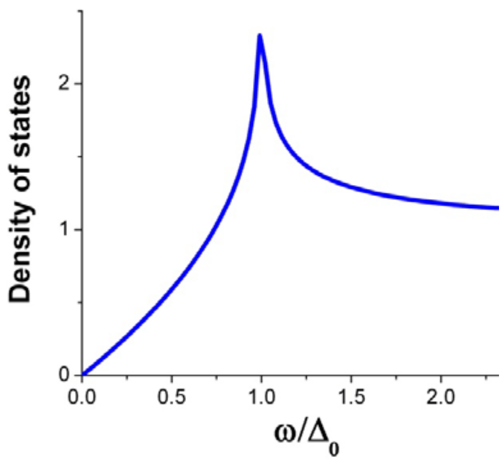


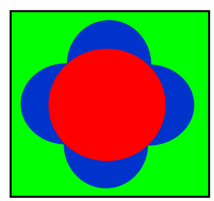


Isotropic gap

Al, Be, Nb_3Sn

< 1978





Gap with zeroes (nodes)

Cuprates,
heavy fermions,
>1979

What is the effect of:

1) an isolated impurity
(STM spectra)

2) ensemble of impurities
(T_c, planar junctions)

How is this picture modified by impurities:

1) locally

2) globally

Classical and quantum impurities

1. Potential scatterers

$$H_{\mathrm{imp}} = \sum_{\alpha} \int d\mathbf{r}\, \psi_{\alpha}^{\dagger}(\mathbf{r}) U_{\mathrm{pot}}(\mathbf{r}) \psi_{\alpha}(\mathbf{r})$$

2. Spin scattering

$$H_{\mathrm{imp}} = \sum_{\alpha\beta} \int d\mathbf{r} J(\mathbf{r}) \psi_{\alpha}^{\dagger}(\mathbf{r}) \mathbf{S} \cdot \boldsymbol{\sigma}_{\alpha\beta} \psi_{\beta}(\mathbf{r})$$

2a. Classical spin

$$[S_i, S_j] = 0$$

2b. Quantum spin

$$[S_i, S_j] \neq 0$$

3. Anderson impurity: interpolate between the two regimes

$$H_{imp} = E_0(n_{i\uparrow} + n_{i\downarrow}) + U n_{i\uparrow} n_{i\downarrow} + \sum_k V d_{\sigma}^{+} c_{k\sigma} + h.c.$$

4. Single Impurity vs. many impurities

5. Conventional vs unconventional superconductors

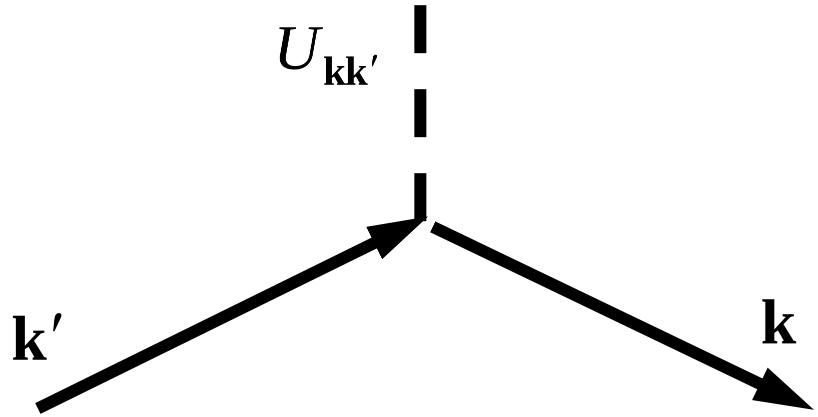
Single Impurities

Single Impurity Problem

We are solving a scattering problem (drop spin indices)

$$H_{imp} = \int U(\mathbf{r})\rho(\mathbf{r})d\mathbf{r} = \int d\mathbf{r}U(\mathbf{r})\Psi_\sigma^+(\mathbf{r})\Psi_\sigma(\mathbf{r})$$

$$= \int d\mathbf{r}U(\mathbf{r})\sum_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}'\sigma} e^{i(\mathbf{k}-\mathbf{k}')\,\mathbf{r}} = \sum_{\mathbf{k}\mathbf{k}'} U_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}'\sigma}$$



For classical impurities (*U* is a function) this can be solved exactly

Reminder: Green's functions

Prescription:

$$G_{\alpha\beta}(\mathbf{r}_1,\tau;\mathbf{r}_2,\tau') = -\left\langle T_\tau \psi_\alpha(\mathbf{r}_1,\tau)\psi^+_\beta(\mathbf{r}_2,\tau')\right\rangle \Rightarrow G_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\omega_n) \quad \omega_n = 2\pi T(n+1/2)$$

Matsubara

- obtain retarded Green's function $\quad i\omega_n \to \omega + i\delta \quad G^R(\mathbf{r}_1,\mathbf{r}_2;\omega)$
- poles=excitation energies
- density of states=Im part

$$N(\mathbf{r},\omega) = -\pi^{-1}\,\mathrm{Im}\,G^R(\mathbf{r},\mathbf{r};\omega) = -\pi^{-1}\int d\mathbf{k}\,\mathrm{Im}\,G^R(\mathbf{k};\omega)$$

Example: normal metal

$$G(\mathbf{k},\omega_n) = \left[i\omega_n - \xi_\mathbf{k}\right]^{-1} \to \left[\omega - \xi_\mathbf{k} + i\delta\right]^{-1}$$

$$N(\omega) = \int d\mathbf{k}\,\delta(\omega - \xi_\mathbf{k})$$

Nambu formalism and matrices I

- Mix particles/holes, spin up/down

$$\Psi^\dagger(\mathbf{r}) = (\psi^\dagger_\uparrow, \psi^\dagger_\downarrow, \psi_\uparrow, \psi_\downarrow)$$

4x4 matrix

$$\hat{G}(x,x') = -\langle T_\tau \Psi(x)\Psi^\dagger(x')\rangle$$

Nambu-Gor'kov

- BCS hamiltonian

$$\mathcal{H}_{\rm BCS} = \int d\mathbf{r}\Psi^\dagger(\mathbf{r})[\xi(-i\nabla)\tau_3 + \Delta\tau_1\sigma_2]\Psi(\mathbf{r})$$

- Matrices σ_i, τ_i in spin and particle-hole space respectively

- Matrix structure of the impurity scattering:

Potential: $\hat{U}(\mathbf{r}) \Rightarrow U(\mathbf{r})\tau_3$ e.g attracts electrons/repels holes

Magnetic: $\mathbf{S}\cdot\boldsymbol{\sigma} \Rightarrow \mathbf{S}\cdot\boldsymbol{\alpha}$ $\boldsymbol{\alpha} = [(1+\tau_3)\boldsymbol{\sigma} + (1-\tau_3)\sigma_3\boldsymbol{\sigma}\sigma_3]/2$

- Pure BCS

$$\hat{G}_0^{-1}(\mathbf{k},\omega) = i\omega_n - \xi(\mathbf{k})\tau_3 - \Delta(\mathbf{k})\sigma_2\tau_1$$

Nambu formalism and matrices II

Green's function of a superconductor

$$\hat{G}_0(\mathbf{k};\omega) = \begin{pmatrix} i\omega_n - \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & i\omega_n + \xi_{\mathbf{k}} \end{pmatrix}^{-1}$$

"anomalous" Green's function, ODLRO

"normal" particle & hole propagators

Nambu formalism and matrices II

Green's function of a superconductor

$$\hat{G}_0(\mathbf{k};\omega) = \begin{pmatrix} i\omega_n - \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & i\omega_n + \xi_{\mathbf{k}} \end{pmatrix}^{-1}$$

"anomalous" Green's function, ODLRO

"normal" particle & hole propagators

Density of states

$$N(\mathbf{r},\omega) = -\pi^{-1}[\operatorname{Im} G^R]_{11}(\mathbf{r},\mathbf{r};\omega)$$

"normal" part

poles: energies → energy gap

Self-consistency condition on the order parameter

$$\Delta_{\mathbf{k}} = T\sum_{\omega_n}\int d\mathbf{k}' V(\mathbf{k},\mathbf{k}')G_{12}(\mathbf{k}',\omega_n)$$

"anomalous" part

highest T with sol'n → transition temperature

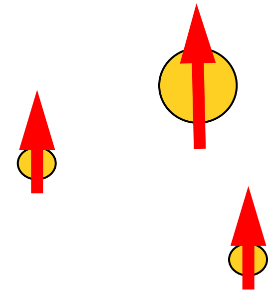
Not important for single impurity

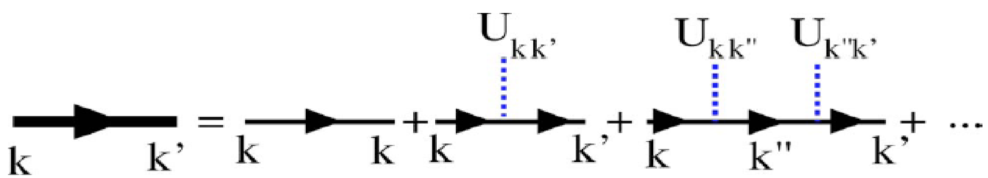
Crucial for multiple impurities

Single impurity

- **Key: multiple scattering** $H_{imp} = \sum_{\mathbf{kk}'} U_{\mathbf{kk}'} c^{+}_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma} + h.c.$

change of momentum/spin at each scattering event



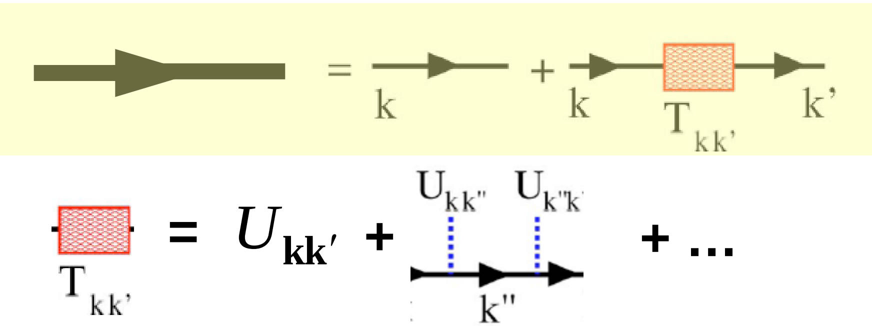


$$\hat{G}(\mathbf{k},\mathbf{k}') = \hat{G}_0(\mathbf{k}) + \hat{G}_0(\mathbf{k})\hat{U}_{\mathbf{k},\mathbf{k}'}\hat{G}_0(\mathbf{k}')$$

$$+ \sum_{\mathbf{k}''} \hat{G}_0(\mathbf{k})\hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{U}_{\mathbf{k}'',\mathbf{k}'}\hat{G}_0(\mathbf{k}') + \cdots$$

can include all the scattering events ... in principle

T-matrix solution

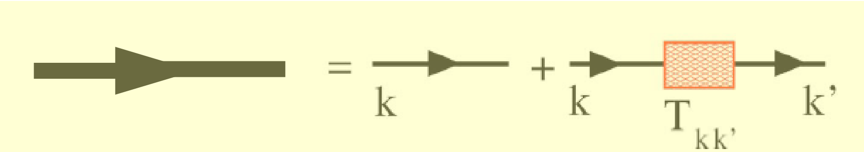


$$\hat{G}(\mathbf{k},\mathbf{k}') = \hat{G}_0(\mathbf{k}) + \hat{G}_0(\mathbf{k})\hat{T}_{\mathbf{k},\mathbf{k}'}\hat{G}_0(\mathbf{k}')$$

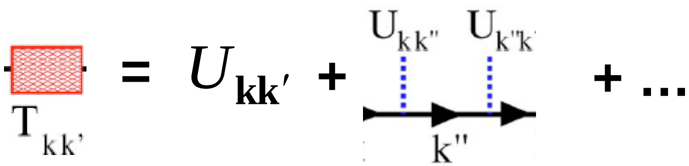
$$\hat{T}_{\mathbf{k},\mathbf{k}'} = \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{U}_{\mathbf{k}'',\mathbf{k}'} + \cdots$$

$$= \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{T}_{\mathbf{k}'',\mathbf{k}'}.$$

T-matrix solution



$$\hat{G}(\mathbf{k},\mathbf{k}') = \hat{G}_0(\mathbf{k}) + \hat{G}_0(\mathbf{k})\hat{T}_{\mathbf{k},\mathbf{k}'}\hat{G}_0(\mathbf{k}')$$



$$\hat{T}_{\mathbf{k},\mathbf{k}'} = \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{U}_{\mathbf{k}'',\mathbf{k}'} + \cdots$$

$$= \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{T}_{\mathbf{k}'',\mathbf{k}'}.$$

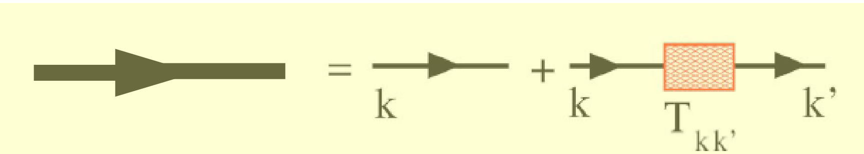
structure is especially simple for isotropic scatterers, $U_{\mathbf{k},\mathbf{k}'} = U$ T-matrix depends on ω only. $T_{\mathbf{k},\mathbf{k}'} = T(\omega)$

$$\hat{T}(\omega) = \hat{U} + \hat{U}\sum_{\mathbf{k}}\hat{G}_0(\mathbf{k},\omega)\hat{T}(\omega)$$

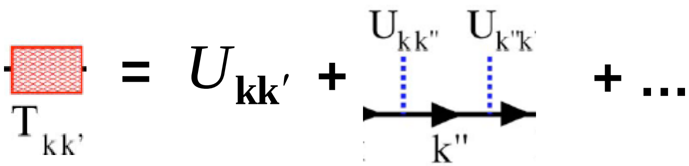
$$\hat{T}(\omega) = \left[1 - \hat{U}\sum_{\mathbf{k}}\hat{G}_0(\mathbf{k},\omega)\right]^{-1} U$$

T-matrix includes all the effects of multiple scattering on a single impurity

T-matrix solution



$$\hat{G}(\mathbf{k},\mathbf{k}') = \hat{G}_0(\mathbf{k}) + \hat{G}_0(\mathbf{k})\hat{T}_{\mathbf{k},\mathbf{k}'}\hat{G}_0(\mathbf{k}')$$



$$\hat{T}_{\mathbf{k},\mathbf{k}'} = \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{U}_{\mathbf{k}'',\mathbf{k}'} + \cdots$$

$$= \hat{U}_{\mathbf{k},\mathbf{k}'} + \sum_{\mathbf{k}''} \hat{U}_{\mathbf{k},\mathbf{k}''}\hat{G}_0(\mathbf{k}'')\hat{T}_{\mathbf{k}'',\mathbf{k}'}.$$

structure is especially simple for isotropic scatterers, $U_{\mathbf{k},\mathbf{k}'} = U$ T-matrix depends on ω only. $T_{\mathbf{k},\mathbf{k}'} = T(\omega)$

Local G at impurity site

$$\hat{T}(\omega) = \hat{U} + \hat{U}\sum_{\mathbf{k}}\hat{G}_0(\mathbf{k},\omega)\hat{T}(\omega)$$

$$\hat{T}(\omega) = \left[1 - \hat{U}\sum_{\mathbf{k}}\hat{G}_0(\mathbf{k},\omega)\right]^{-1} U$$

T-matrix includes all the effects of multiple scattering on a single impurity

Scattering strength

$$\hat{T}(\omega) = \left[1 - \hat{U}\sum_{\mathbf{k}} \hat{G}_0(\mathbf{k},\omega)\right]^{-1} \hat{U} = \left[(UN_0)^{-1} - \tau_3 \int_{FS} d\hat{\mathbf{k}}\, \hat{g}\;(\hat{\mathbf{k}},\omega)\right]^{-1} \tau_3$$

phase shift of scattering

integral over Fermi surface

$$\hat{g}(\hat{\mathbf{k}},\omega) = \int_{FS} d\xi\, \hat{G}_0(\hat{\mathbf{k}},\xi,\omega) = g_i \tau_i$$

$(UN_0)^{-1} = \cot^{-1}\delta_0$ generally depends on band structure

strong scatterers $(UN_0)^{-1} << 1$ $\delta_0 \approx \pi/2$ unitarity

weak scatterers $(UN_0)^{-1} >> 1$ $\delta_0 \approx 0$ Born

Classical impurities: fixed phase shift

Quantum impurities: phase shift depends on energy scale

DOS and T-matrix

- Density of states $$N(\omega,\mathbf{r}) = -\pi^{-1}\,\mathrm{Im}\,\hat{G}_{11}(\mathbf{r},\mathbf{r};\omega)$$
- T-matrix in real space $$G(\mathbf{r},\mathbf{r};\omega) = G_0(\mathbf{r},\mathbf{r};\omega) + G_0(\mathbf{r},\mathbf{r}_0;\omega)T(\omega)G_0(\mathbf{r}_0,\mathbf{r};\omega)$$
- With impurity $$N(\omega,\mathbf{r}) = N_0(\omega) - \pi^{-1}\,\mathrm{Im}\left[G_0(\mathbf{r},\mathbf{r}_0;\omega)T(\omega)G_0(\mathbf{r}_0,\mathbf{r};\omega)\right]$$
- Impurity-induced $$\delta N(\omega,\mathbf{r}) \propto \mathrm{Im}\,T(\omega)\left|G_0(\omega,\mathbf{r})\right|^2$$

Impurity-induced new states appear at energies where T-matrix has imaginary part: poles of $T(\omega)$

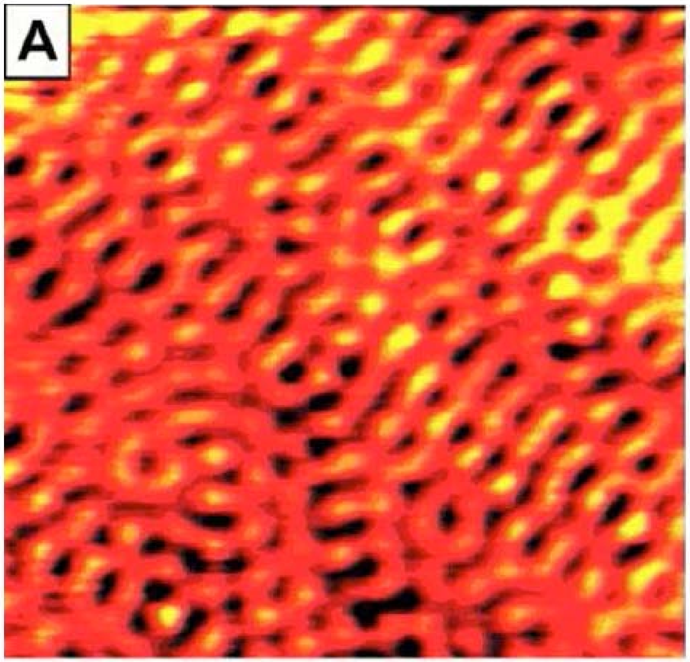
DOS and T-matrix

- Density of states $$N(\omega,\mathbf{r}) = -\pi^{-1}\,\mathrm{Im}\,\hat{G}_{11}(\mathbf{r},\mathbf{r};\omega)$$
- T-matrix in real space $$G(\mathbf{r},\mathbf{r};\omega) = G_0(\mathbf{r},\mathbf{r};\omega) + G_0(\mathbf{r},\mathbf{r}_0;\omega)T(\omega)G_0(\mathbf{r}_0,\mathbf{r};\omega)$$
- With impurity $$N(\omega,\mathbf{r}) = N_0(\omega) - \pi^{-1}\,\mathrm{Im}\left[G_0(\mathbf{r},\mathbf{r}_0;\omega)T(\omega)G_0(\mathbf{r}_0,\mathbf{r};\omega)\right]$$
- If for some w $$N_0(\omega) = -\pi^{-1}\,\mathrm{Im}\,G_0(\mathbf{r},\mathbf{r};\omega) = 0$$
- New states $$\delta N(\omega,\mathbf{r}) \propto \mathrm{Im}\,T(\omega)\left|G_0(\omega,\mathbf{r})\right|^2$$

Friedel oscillations

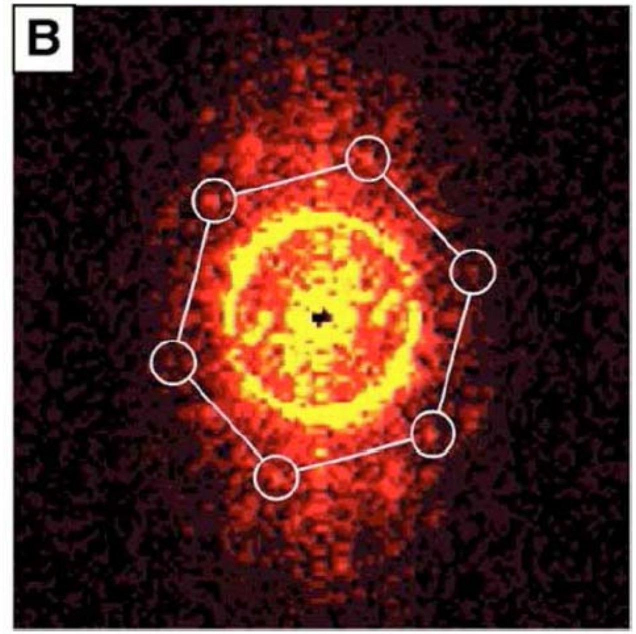
Impurity-induced new states appear at energies where T-matrix has imaginary part: poles of $T(\omega)$

2D Metal: experiment



real space

Be



Fourier transform

P. Sprunger et al. 1997

Spatial oscillations with $k_F r$: Fourier transform gives image of the Fermi surface

2D superconductors

$$E(\mathbf{k}) = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta(\mathbf{k})|^2}$$

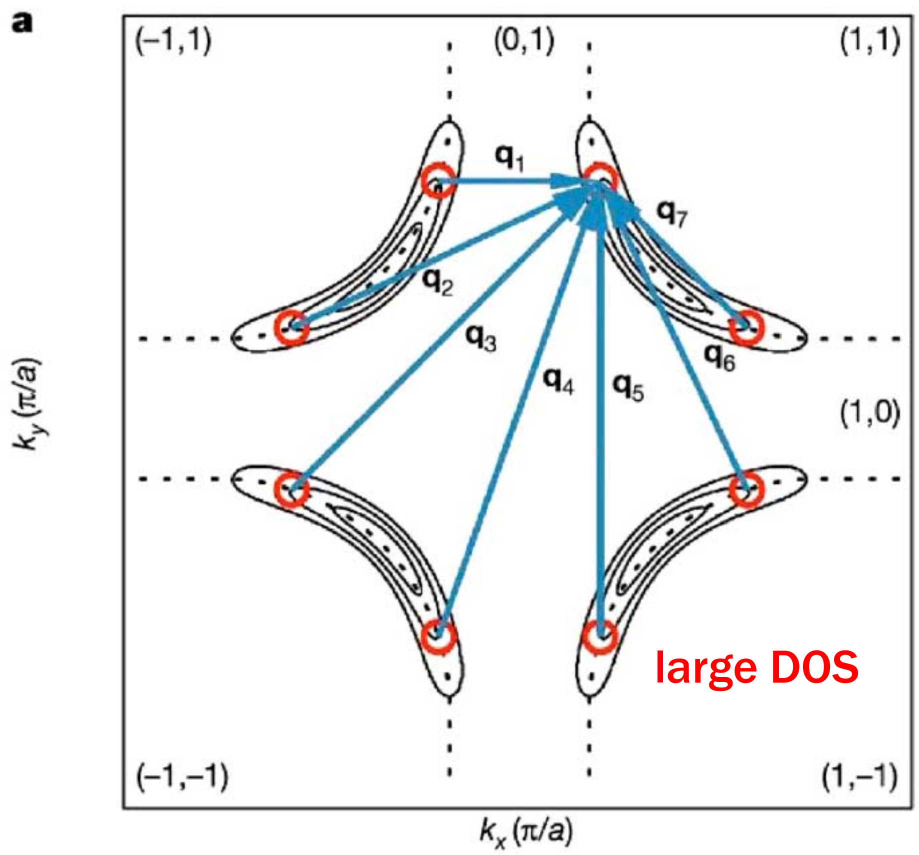
Tight binding dispersion

$$\xi_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu$$

d-wave gap

$$\Delta(\mathbf{k}) = \Delta_0(\cos k_x - \cos k_y)$$

Follow dominant wave vectors as a function of energy



K. McElroy et al. 2003

Simple example: potential scattering

4x4 $\rightarrow$ 2x2

spin is not "active"

$$\hat{T}(\omega) = \frac{g_0\tau_0 - g_1\tau_1 - (UN_0)^{-1}\tau_3}{(UN_0)^{-2} - g_0^2 - g_1^2}$$

$$g_0(\omega) = -\int_{FS} d\hat{\mathbf{k}} \frac{i\omega}{\sqrt{\omega^2 - \left|\Delta(\hat{\mathbf{k}})\right|^2}}$$

$$g_1(\omega) = -\int_{FS} d\hat{\mathbf{k}} \frac{\Delta(\hat{\mathbf{k}})}{\sqrt{\omega^2 - \left|\Delta(\hat{\mathbf{k}})\right|^2}}$$

$$\sum_{\mathbf{k}} \Delta(\mathbf{k}) = 0 \Rightarrow g_1 \text{ vanishes}$$

Different structure of T-matrix for conventional and nodal superconductors: check for new poles

Potential scatterer: s-wave

$$H_{imp} = \sum_{\mathbf{k}\mathbf{k}'} U\, c^{+}_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma}$$

$$\Delta(\mathbf{k}) = const$$

$$\hat{T}(\omega) = \frac{\sum_i a_i \tau_i}{(UN_0)^{-2} + 1}$$

no new poles

Physics: we are pairing time-reversed states: potential impurity makes states not simply |k>, but does not violate time-reversal.

$$\left|\mathbf{k}\uparrow\right\rangle, \left|-\mathbf{k}\downarrow\right\rangle$$

$$\left|n\uparrow\right\rangle, T\left|n\uparrow\right\rangle$$

The only situation where impurities are not harmful to superconductivity at all: no impurity states

P. W. Anderson, 1957

Resonant impurity: s-wave

Non-magnetic

“resonant scattering”

$$H_{imp} = E_0(n_{i\uparrow} + n_{i\downarrow}) + \sum_k V d_\sigma^+ c_{k\sigma} + h.c.$$

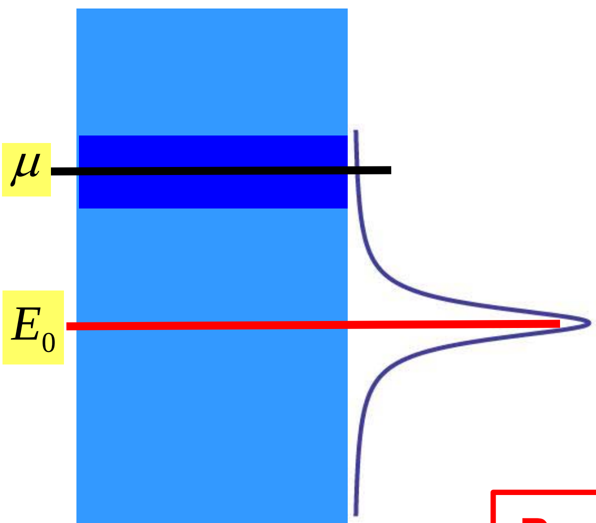
Machida & Shibata 1972, H. Shiba 1973

Hybridization with

the conduction band

$$\Gamma = \pi |V|^2 N_0 \qquad \Gamma >> \Delta$$

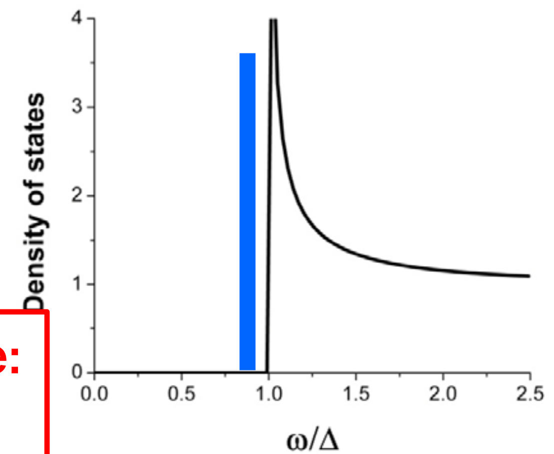
$$\hat{T}(\omega) = |V|^2 \tau_3 \left[\omega - E_0\tau_3 - |V|^2 \tau_3 \sum_{\mathbf{k}} \hat{G}_0(\mathbf{k},\omega)\tau_3 \right]^{-1} \tau_3$$



$$N_d(0) = \pi^{-1}\Gamma/(\Gamma^2 + E_0^2)$$

$$\omega_0 = \pm\Delta\{1 - 2\pi^2[\Delta N_d(0)]^2\}$$

$\underbrace{2\pi^2[\Delta N_d(0)]^2}_{\sim 10^{-3}}$



Bound state pinned to the gap edge: largely irrelevant

Potential scatterer: d-wave

$$H_{imp} = \sum_{\mathbf{kk}'} U\, c^{+}_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma}$$

$$\Delta(\phi) = \Delta_0 \cos 2\phi$$

$$\int_{FS} d\hat{\mathbf{k}}\, \Delta(\hat{\mathbf{k}}_F) = 0$$

Poles of T-matrix ? Yes

Wait, no gap!

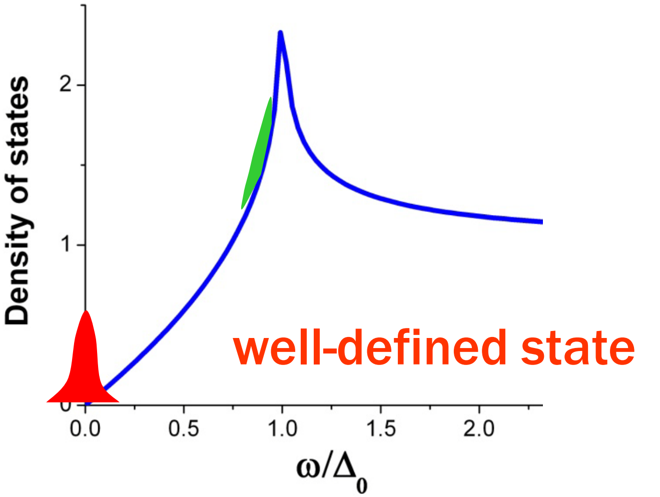
$$\Omega_0 = \Omega_1 + i\Omega_2 = -\Delta_0 \frac{\pi c/2}{\ln 8/\pi c}\left[1 + \frac{i\pi/2}{\ln 8/\pi c}\right]$$

$$c = (UN_0)^{-1}$$

resonance | energy | lifetime

strong scatterers $\Omega_2 << \Omega_1$ sharp

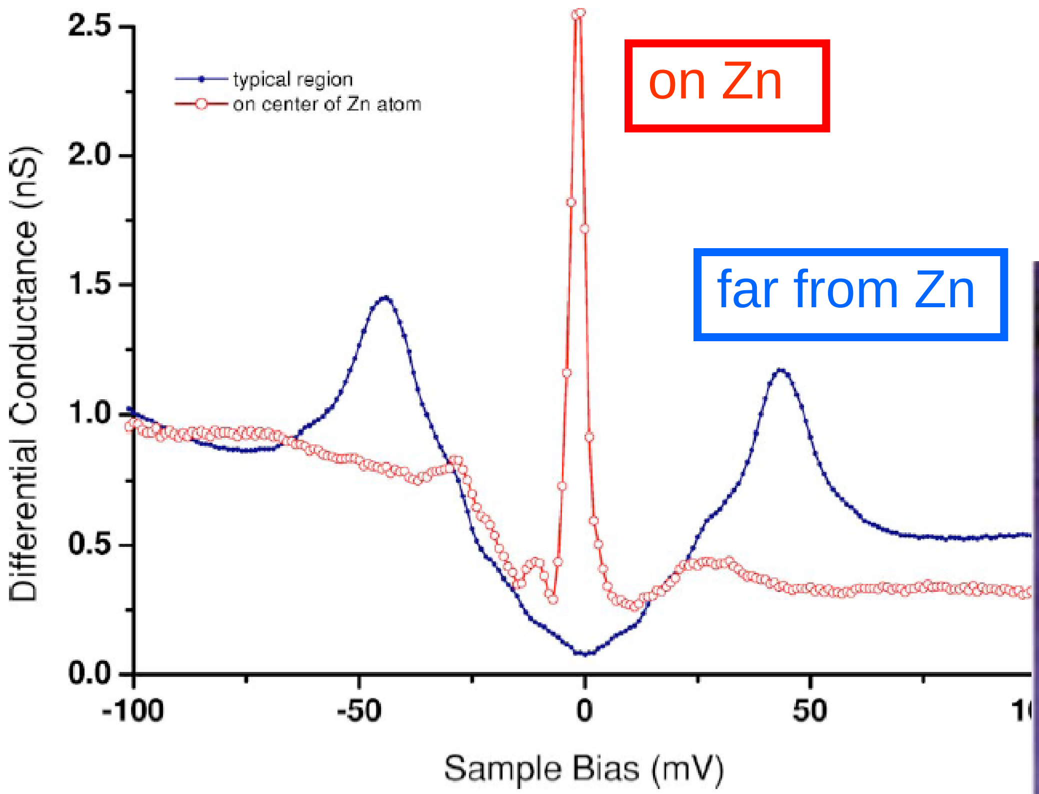
weak scatterers $\Omega_2 \geq \Omega_1$ smeared



P. Stamp 1987, J. Byers and D. Scalapino 1993, A. V. Balatsky et al. 1995

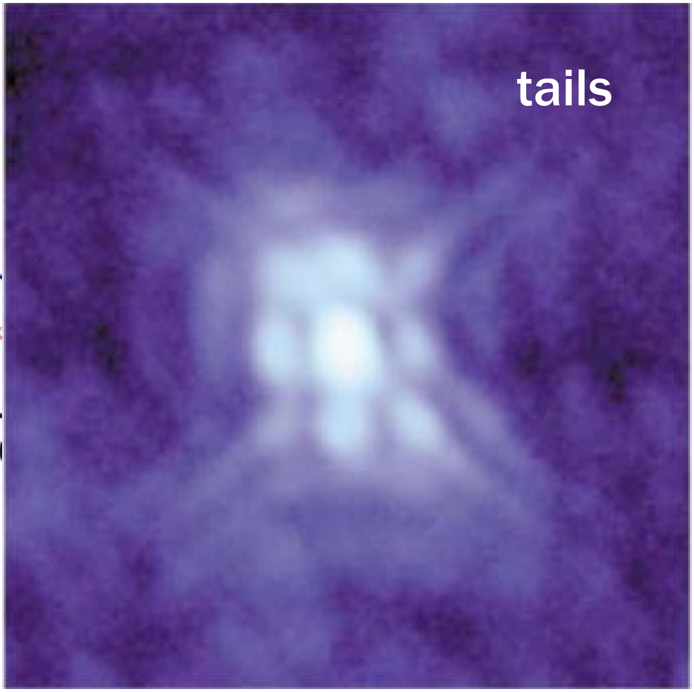
Experiment: d-wave

Zn impurity in BSCCO



S. Pan, E. Hudson et al., 2000

Spatial dependence is poorly understood:



Message: part I

Potential (electrostatic) scattering

- Isotropic s-wave gap: normally no bound state, never states deep in the gap
- Anisotropic states with sign changing order parameter: all scattering produced bound states, these states are deep in the gap for strong scattering

Now on to spin-dependent scattering starting with classical spin

Classical spin: isotropic gap

$$H_{imp} = J\sum_{\mathbf{kk}'} c^{+}_{\mathbf{k}\alpha}\mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\beta}c_{\mathbf{k}'\beta}$$

classical spin S $\Delta(\mathbf{k}) = \Delta$ s-wave

$$\hat{T}(\omega) \propto \frac{(JS/2)^2\,\hat{g}_0^2(\omega)}{1-\left[JS\hat{g}_0(\omega)/2\right]^2}$$

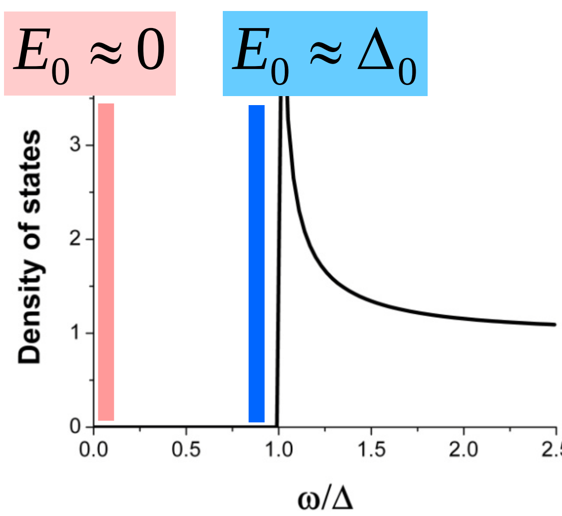
new poles

$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1-(JS\pi N_0/2)^2}{1+(JS\pi N_0/2)^2}$$

Time-reversal violated: new states below the gap edge

$JSN_0 \approx 1$ state in midgap

$JSN_0 << 1$ state near gap edge



A. Rusinov, 1968; H. Shiba, 1968, L. Yu 1965

第21卷 第1期
1965年1月

物理学报
ACTA PHYSICA SINICA

Vol. 21, No. 1
January, 1965

含順磁杂質超导体中的束縛态*

于 渌

提 要

本文利用广义正則变换和自洽場方法，討論了单个杂質对超导体的影响。証明在磁性杂質附近，可能形成一个束縛态的元激发，其能量位于能隙之中。求出了能級和波函数的解析表达式，并計算了束縛能級所引起的附加电磁吸收。討論了与此有关的隧道和高頻吸收实驗。此外，还討論了非磁性杂質对連續譜元激发的影响及杂質附近能隙的变化。

一、引 言

BOUND STATE IN SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

YU LUH

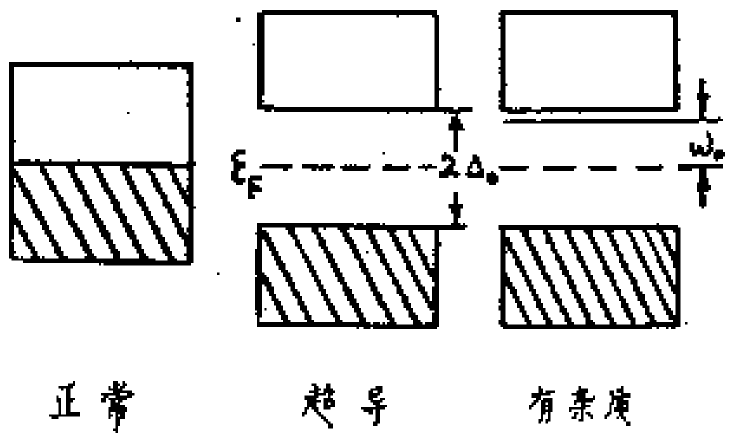


图 1

Quantum phase transition

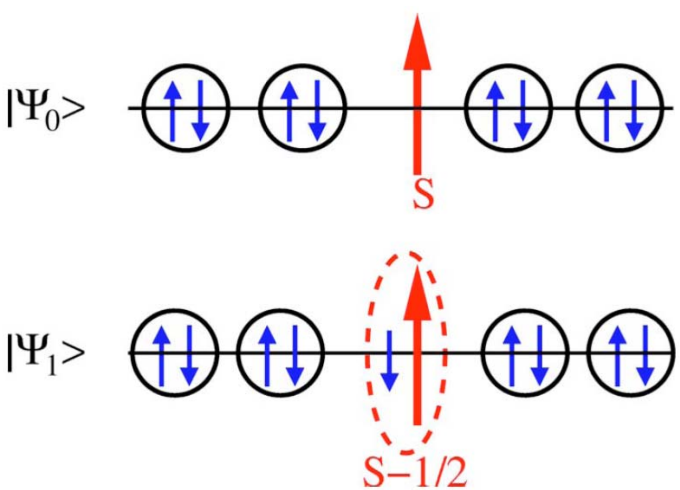
Bound state energy

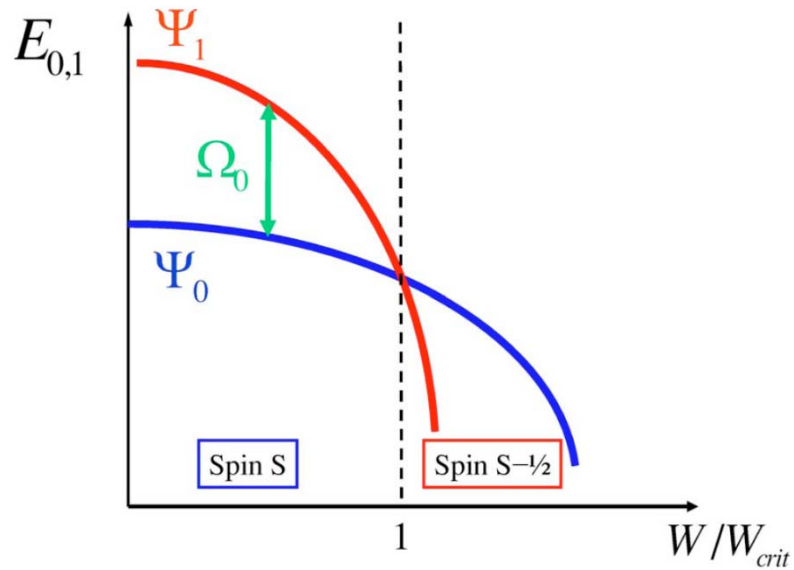
$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1-(JS\pi N_0/2)^2}{1+(JS\pi N_0/2)^2}$$

Critical value

$$J_c = \left[\pi N_0 S/2\right]^{-1}$$

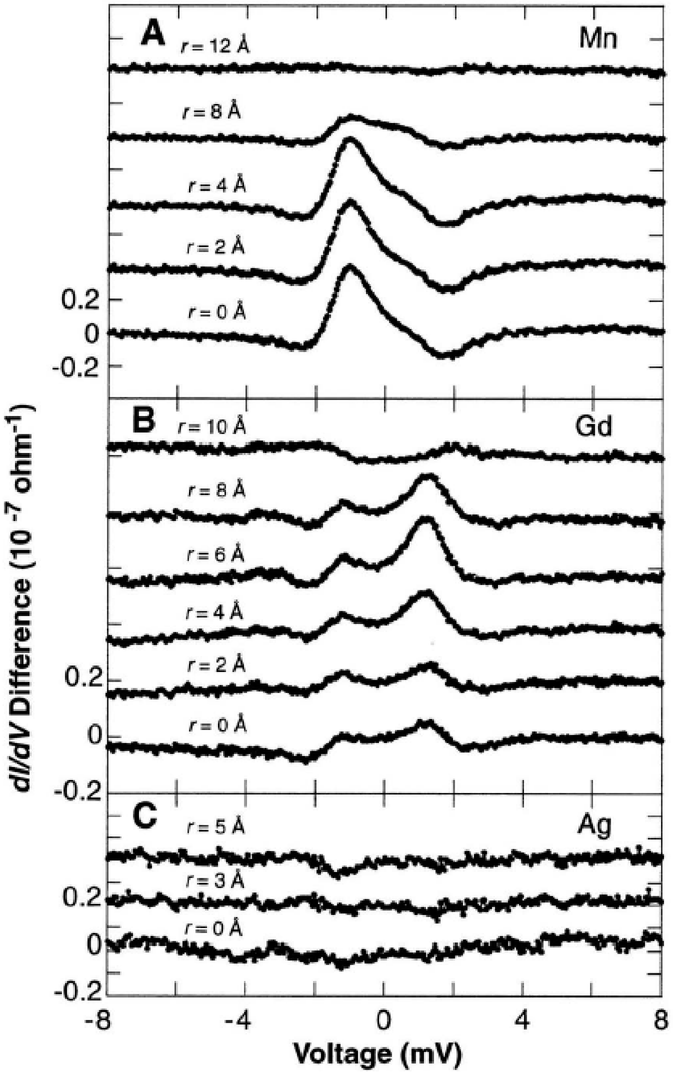
Occupied to unoccupied transition





In both cases similar bound state spectra (extra Cooper pair does not count)

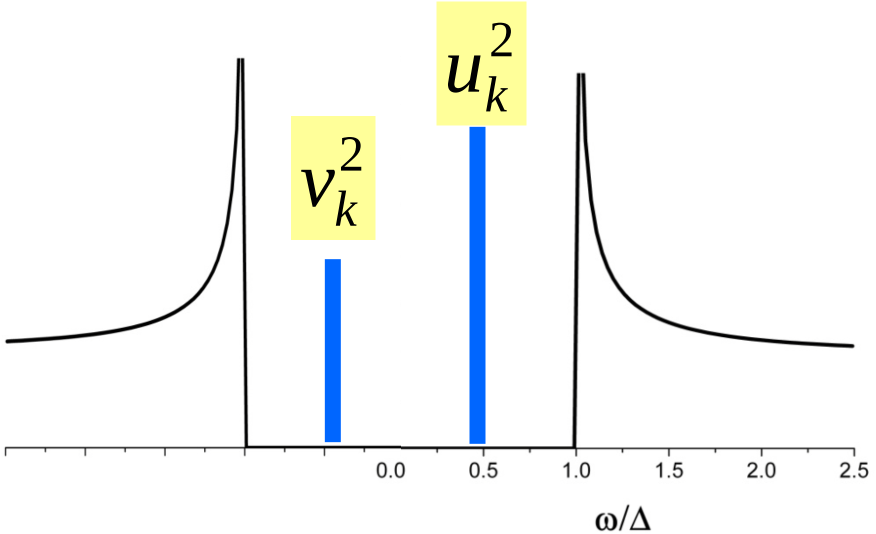
Experiment: s-wave



A. Yazdani et al, 1997

Mn & Gd magnetic, Ag non-magnetic

Asymmetric spectra: extract/inject e



Decay of the state on the scale:

$$|\psi|^2 \propto \exp(-r/r_0)$$

$$r_0 \approx \xi_0 / \sqrt{1-(E_0/\Delta_0)^2}$$

Quantum impurities

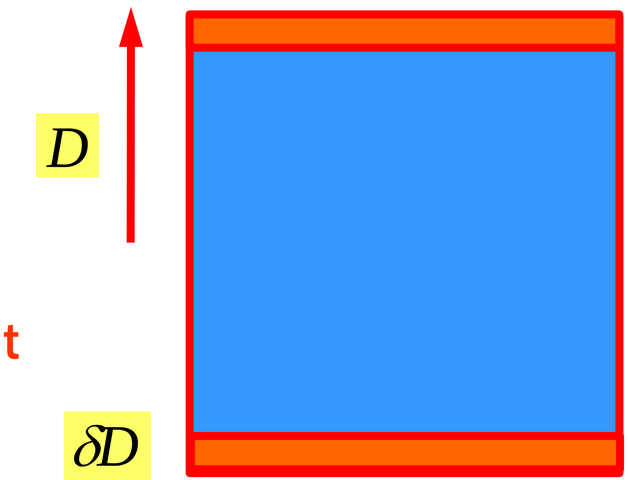
why can't we do the same for quantum impurities?

Recall: single ion Kondo model perturbative RG

$$H_{imp} = J\mathbf{S}\cdot\boldsymbol{\sigma}(\mathbf{r}_0) = J\sum_{\mathbf{k}\mathbf{k}'}\mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\beta}c^{+}_{\mathbf{k}\alpha}c_{\mathbf{k}'\beta}$$

$$\frac{dJ}{d\ln D} = -\rho(D)J^2$$

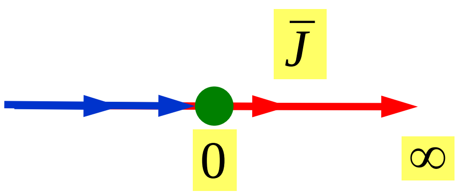
value of coupling depends on what energy we are looking at



constant density of states: $\rho(E) = N_0$

$$\bar{J} = \frac{J}{1 - JN_0 \ln D/T}$$

AFM $J > 0$ $\bar{J} \to \infty$ $T_K \approx D\exp(-1/JN_0)$ impurity screened



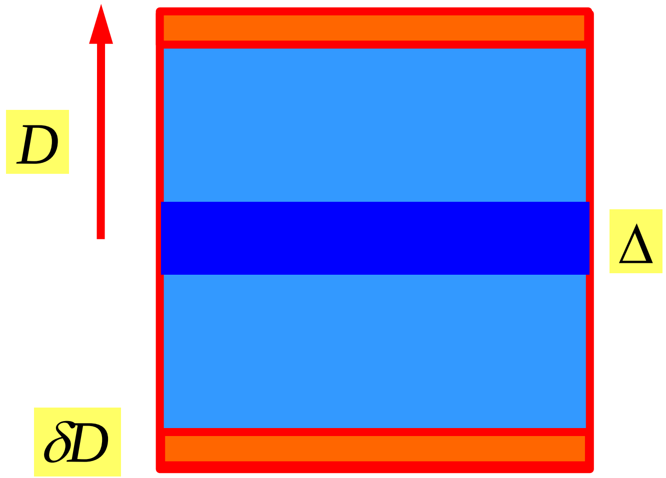
FM $J < 0$ $\bar{J} \to 0$ → Impurity decouples

Quantum impurities

why can't we do the same for quantum impurities?

Recall: single ion Kondo model perturbative RG

$$H_{imp} = J\mathbf{S}\cdot\boldsymbol{\sigma}(\mathbf{r}_0) = J\sum_{\mathbf{kk}'}\mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\beta}c^{+}_{\mathbf{k}\alpha}c_{\mathbf{k}'\beta}$$



$$\frac{dJ}{d\ln D} = -\rho(D)J^2$$

value of coupling depends on what energy we are looking at

constant density of states: $\rho(E) = N_0$

$$\bar{J} = \frac{J}{1 - JN_0 \ln D/T}$$

$$T_K \approx D\exp(-1/JN_0)$$

superconductor: $\rho(E < \Delta) << N_0$

$T_K >> \Delta$ impurity screened

$T_K << \Delta$ impurity not screened

Quantum character of spin

Classical spin:
T-matrix result

$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1-(JS\pi N_0/2)^2}{1+(JS\pi N_0/2)^2}$$

Does not depend on sign of the exchange interaction

Expect: difference between AFM *(J>0)* and FM *(J<0)* exchange

AFM *(J>0)*:
- Renormalizes to large *J*
- Competition with pairing
- May be Kondo screened

FM *(J<0)*:
- Renormalizes to small *J*
- Always unscreened

Need new approaches: numerical RG etc. *K. Satori et al. 1992, O. Sakai et al 1993*

Kondo S=1/2 spin: NRG

Classical spin:

T-matrix result

$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1 - (JS\pi N_0/2)^2}{1 + (JS\pi N_0/2)^2}$$

Ferromagnetic:

$$\frac{E_0}{\Delta} \approx 1 - \frac{\pi^2}{8}\left[\frac{J/D}{1+(J/D)\ln[D/\Delta]}\right]^2$$

RG flow stops at Δ

bound state close to gap edge

Kondo S=1/2 spin: NRG

Classical spin:
T-matrix result

$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1-(JS\pi N_0/2)^2}{1+(JS\pi N_0/2)^2}$$

Ferromagnetic:

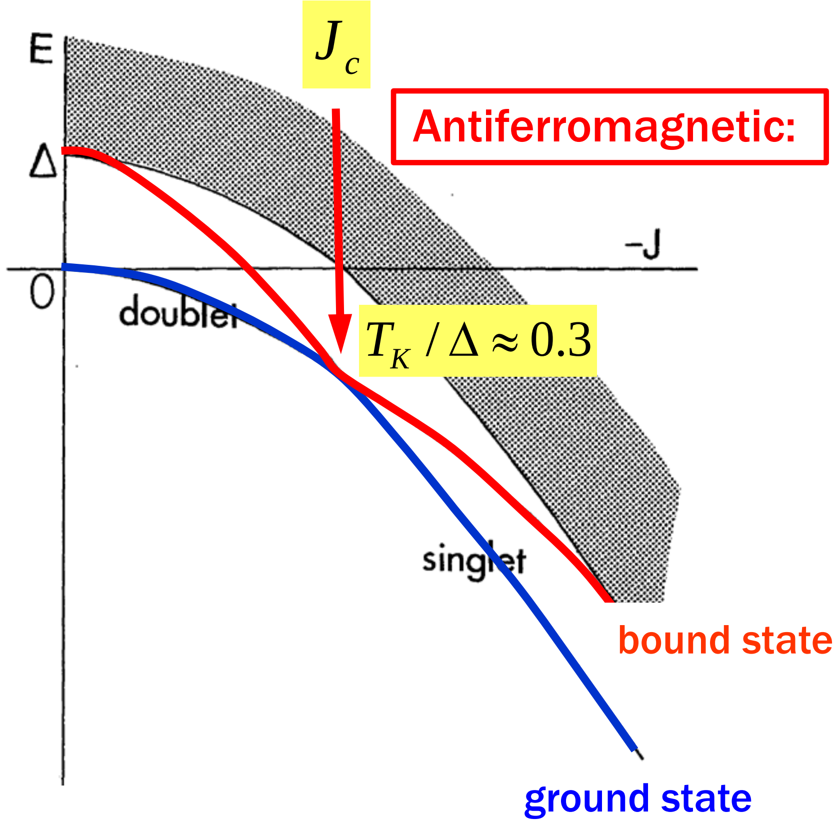
$$\frac{E_0}{\Delta} \approx 1 - \frac{\pi^2}{8}\left[\frac{J/D}{1+(J/D)\ln[D/\Delta]}\right]^2$$

RG flow stops at Δ

bound state close to gap edge

Antiferromagnetic:

critical value of coupling



K. Satori et al. 1992, O. Sakai et al 1993

Kondo S=1/2 spin: NRG

Classical spin:
T-matrix result

$$\epsilon_0 = \frac{E_0}{\Delta_0} = \frac{1-(JS\pi N_0/2)^2}{1+(JS\pi N_0/2)^2}$$

Ferromagnetic:

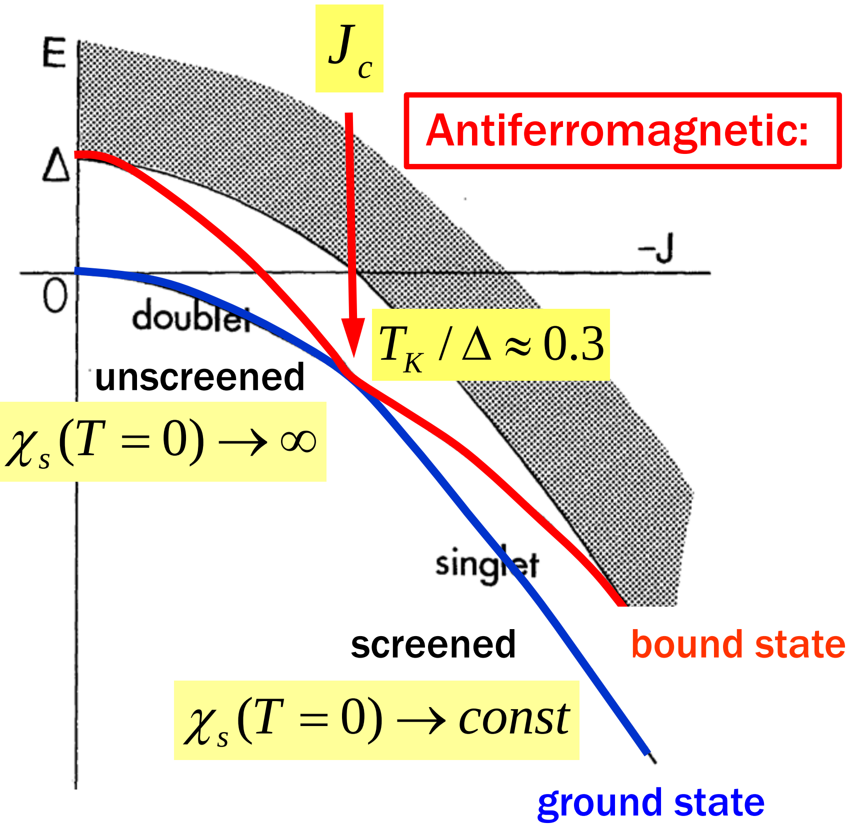
$$\frac{E_0}{\Delta} \approx 1 - \frac{\pi^2}{8}\left[\frac{J/D}{1+(J/D)\ln[D/\Delta]}\right]^2$$

RG flow stops at Δ

bound state close to gap edge

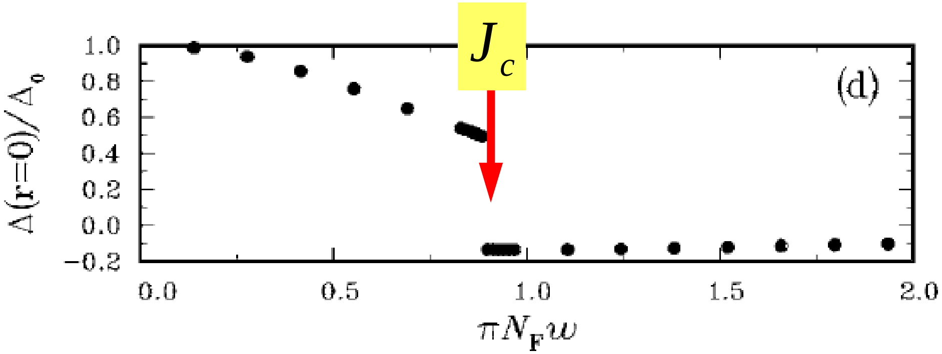
Antiferromagnetic:

critical value of coupling

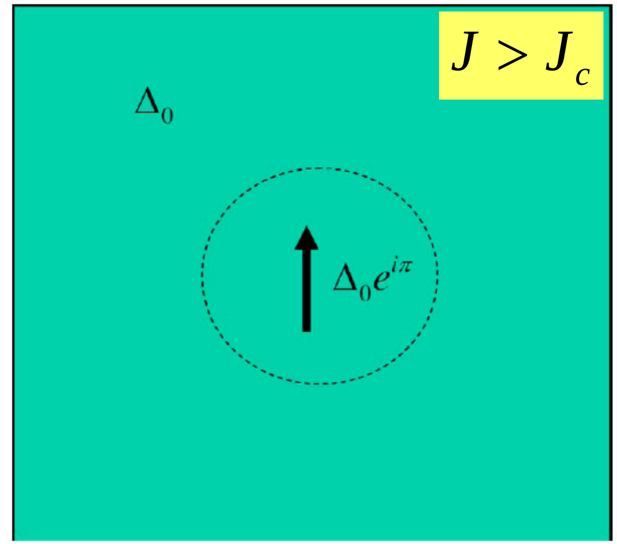


K. Satori et al. 1992, O. Sakai et al 1993

π-phase shifts

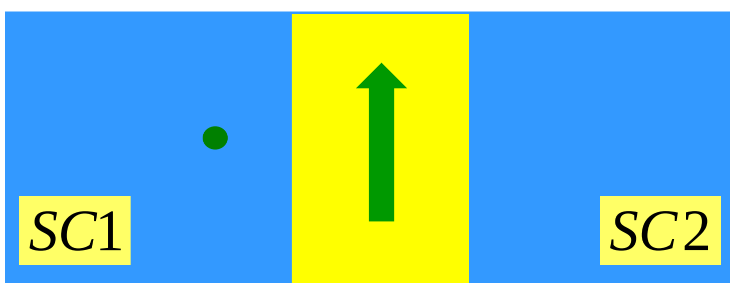


M. Salkola, A. Balatsky, J. R. Schrieffer 1997



Self-consistent calculation including OP suppression

Cf. π-Josephson junction *L. Bulaevskii et al. 1983*



Ground state: order parameters have opposite signs on both sides of the junction

Cooper pair tunneling via the spin

Gapless superconductors I

Classical spin: potential part of scattering needed

$$U_0 \geq J$$

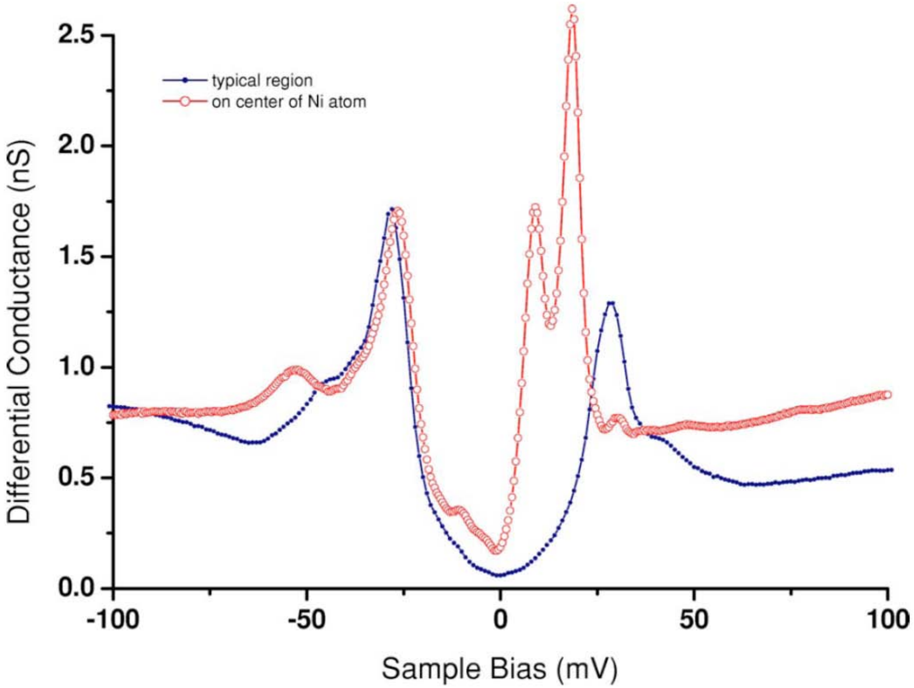
T-matrix result:

$$\Omega_{1,2} = -\frac{\Delta_0}{2N_0(U_0 \pm J)\ln|8N_0(U_0 \pm J)|}$$

M. Salkola, et al. 1997

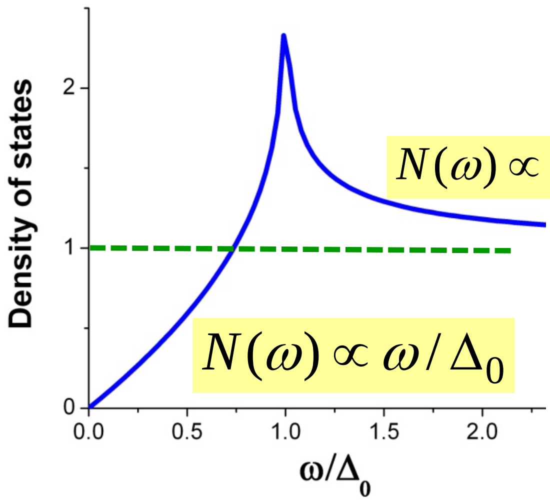
Splitting of the resonances (one for each spin species)

Ni impurity in BSCCO



E. Hudson ,et al. 2001

Kondo effect in gapless superconductors



Density of states suppressed, does not vanish: Kondo or not?

RG until Δ

$$\bar{J} \approx \frac{J}{1 - JN_0 \ln D/\Delta}$$

After that?

Pseudogap Kondo models $N(\omega) \propto \omega^r$

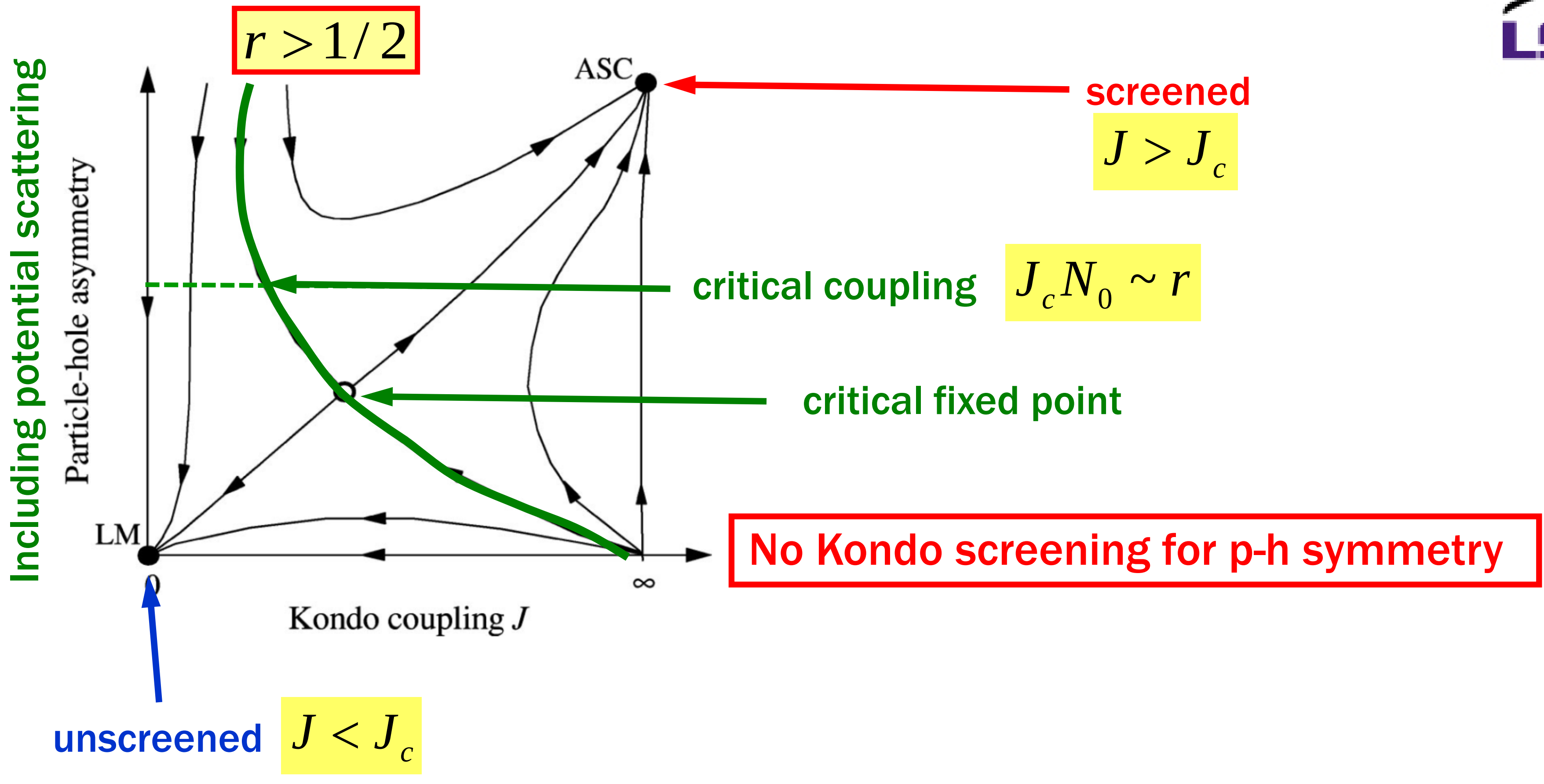
D. Withoff and E. Fradkin 1990
L. Borkowski and P. Hirschfeld 1992
K. Ingersent 1996
C. Gonzales-Buxton and K. Ingersent 1998
R. Bulla and M. Vojta 2001
L. Fritz and M. Vojta 2004
......

$r = \infty$ Hard gap

$r = 0$ Normal metal

$r = 1$ Semimetals, d-wave superconductors… **Inaccessible from r<<1**

Pseudogap Kondo model

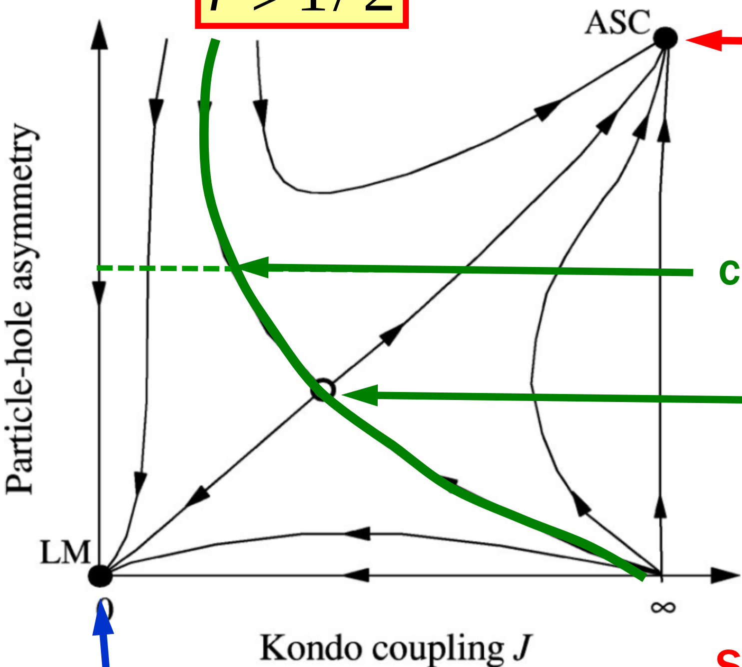


C. Gonzales-Buxton and K. Ingersent 1998
R. Bulla and M. Vojta 2001

Pseudogap Kondo model

$r > 1/2$

Including potential scattering



screened $J > J_c$

critical coupling $J_c N_0 \sim r$

critical fixed point

No Kondo screening for p-h symmetry

unscreened $J < J_c$

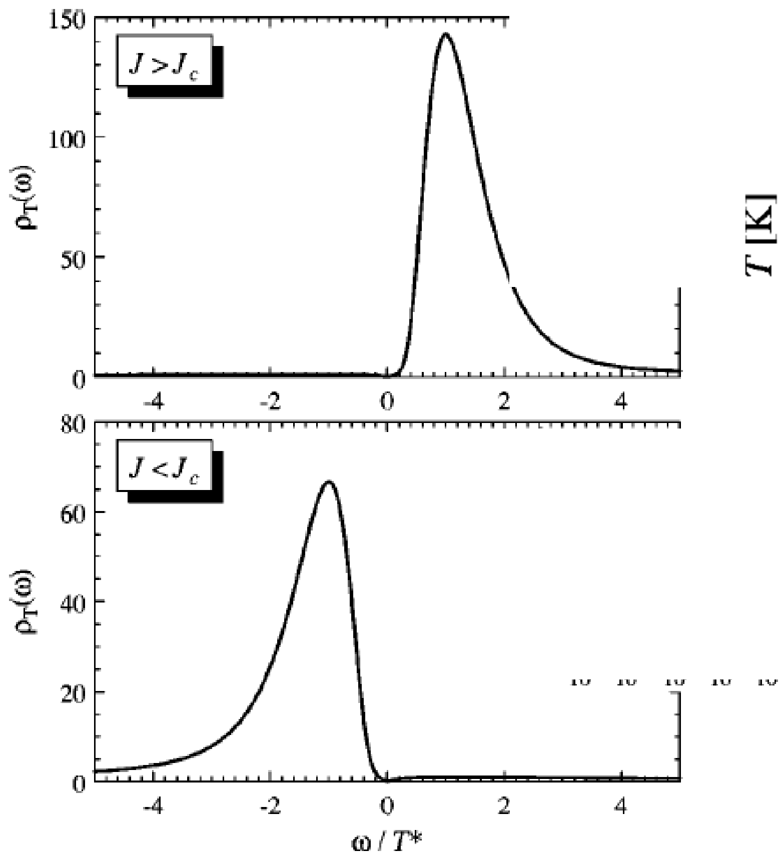
Screened if $\bar{J} \approx \dfrac{J}{1 - JN_0 \ln D/\Delta} \geq J_c \sim r/N_0$

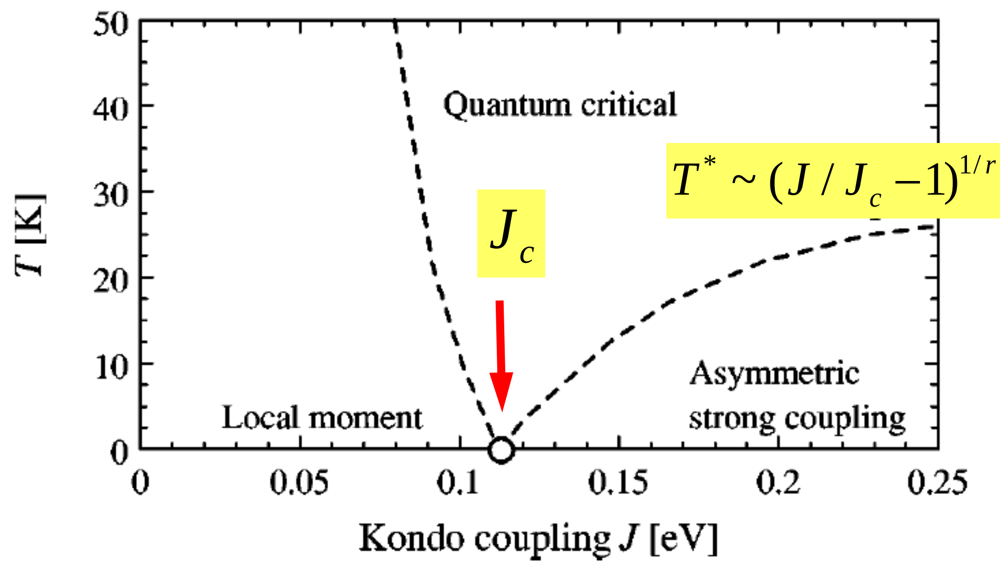
$\Delta < e^{1/r} T_K \sim T_K$

Similar to the gapped case

C. Gonzales-Buxton and K. Ingersent 1998
R. Bulla and M. Vojta 2001
M. Vojta and L. Fritz 2004

Impurity density of states





M. Vojta and R. Bulla 2001

Localized states on both sides

Message: part II

Spin-dependent scattering

- Isotropic s-wave gap:
  - FM coupling: bound state near the gap edge
  - AFM coupling: screening requires critical Kondo coupling, bound state deep into the gap if $T_K / \Delta \approx 0.3$
- Gap with nodes: need potential scattering, form bound states, screening requires critical Kondo coupling.

What about anomalous propagators?

$$\hat{T}(\omega) = \hat{U} + \hat{U}\sum_{\mathbf{k}} \hat{G}_0(\mathbf{k},\omega)\hat{T}(\omega)$$

Recall: if interaction is local, T-matrix depends on local Green's function

$$\hat{G}_0(\mathbf{r},\mathbf{r};\omega) = \sum_k \begin{pmatrix} i\omega_n - \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & i\omega_n + \xi_{\mathbf{k}} \end{pmatrix}^{-1}$$

Off-diagonal part vanishes if $\sum_{\mathbf{k}} \Delta_{\mathbf{k}} = 0$

For local coupling only density of states matters

For non-local coupling anomalous propagators are relevant

$$H_{imp} = \sum_{\mathbf{k}\mathbf{k}'} J(\mathbf{k},\mathbf{k}')\mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\beta} c^+_{\mathbf{k}\alpha} c_{\mathbf{k}'\beta}$$

M. Vojta and R. Bulla, 1998-2001
M. Vojta & L. Fritz 2004

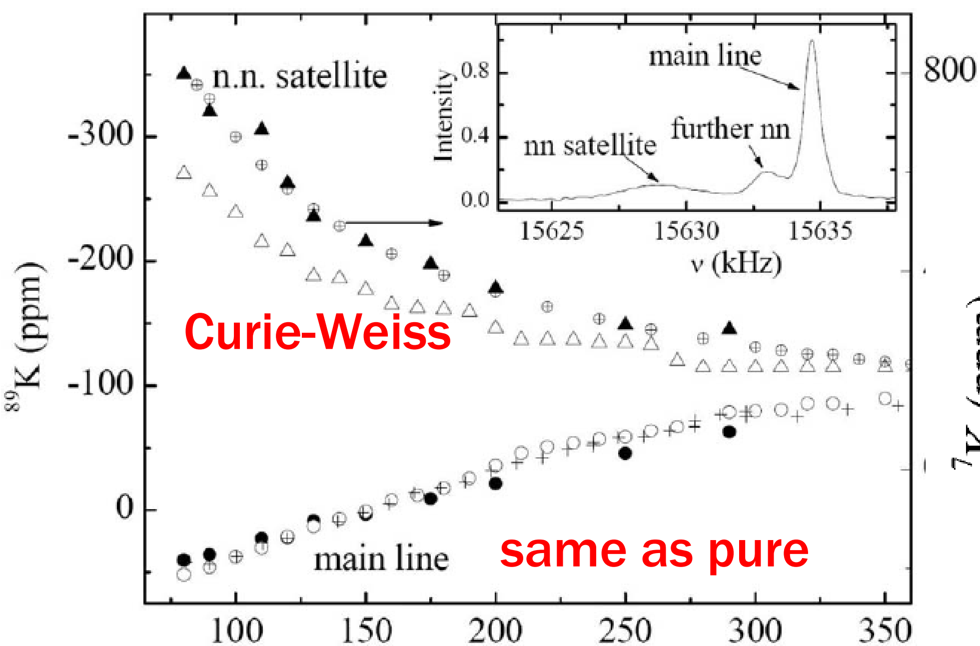
Multichannel Kondo etc.

Why may this be relevant?

Question: can one get Kondo behavior from a non-magnetic impurity?

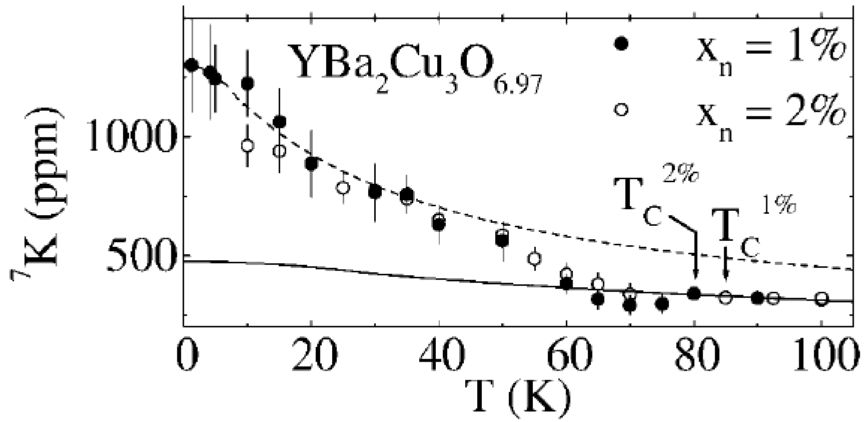
Answer: non-magnetic impurity in a correlated host can generate a magnetic moment distributed around it

Example: Li or Zn in high-temperature superconductor



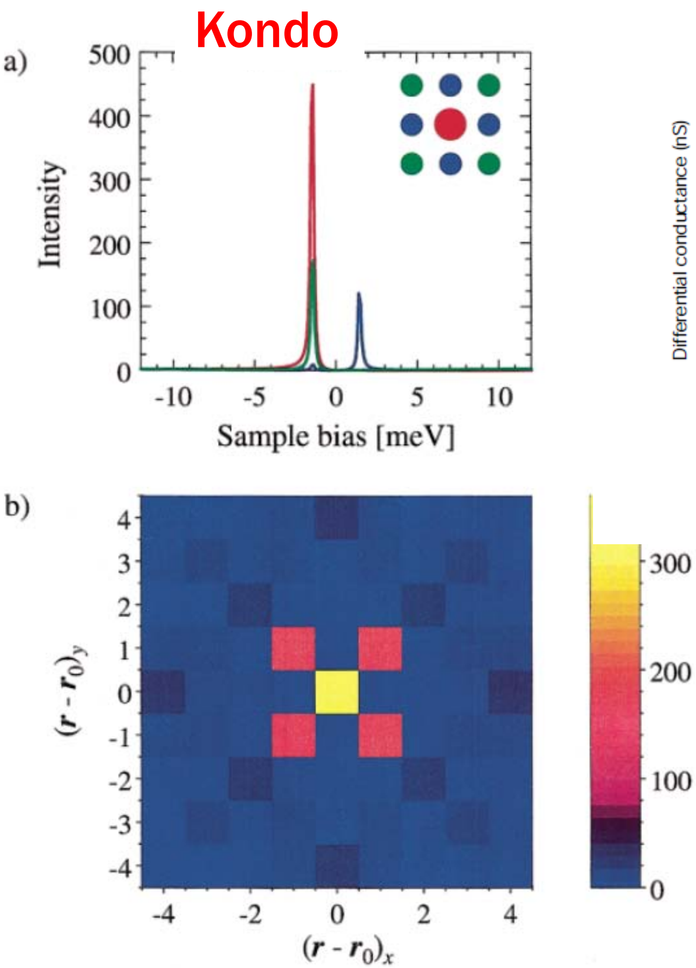
J. Bobroff et al. 1999-2001

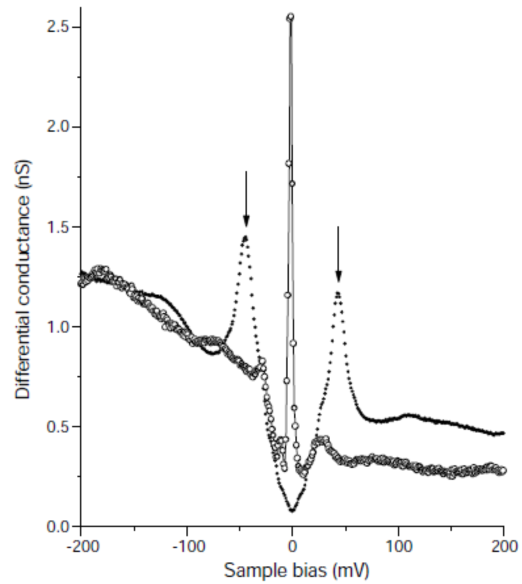
Moment distributed over nearest neighbors



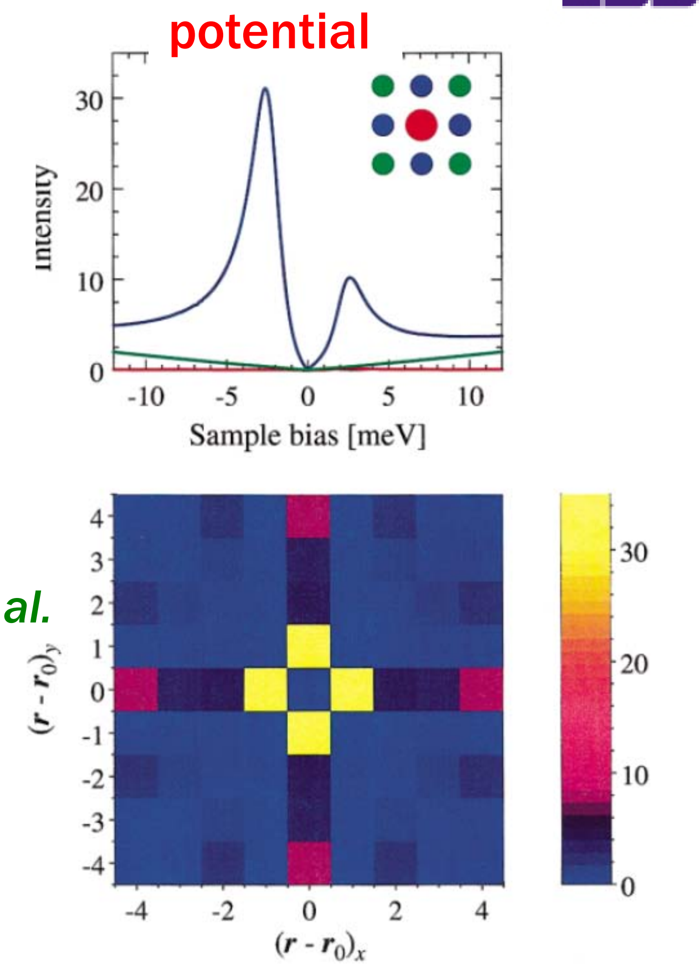
lines: scaled from Cu:Fe alloys

Kondo vs potential scattering





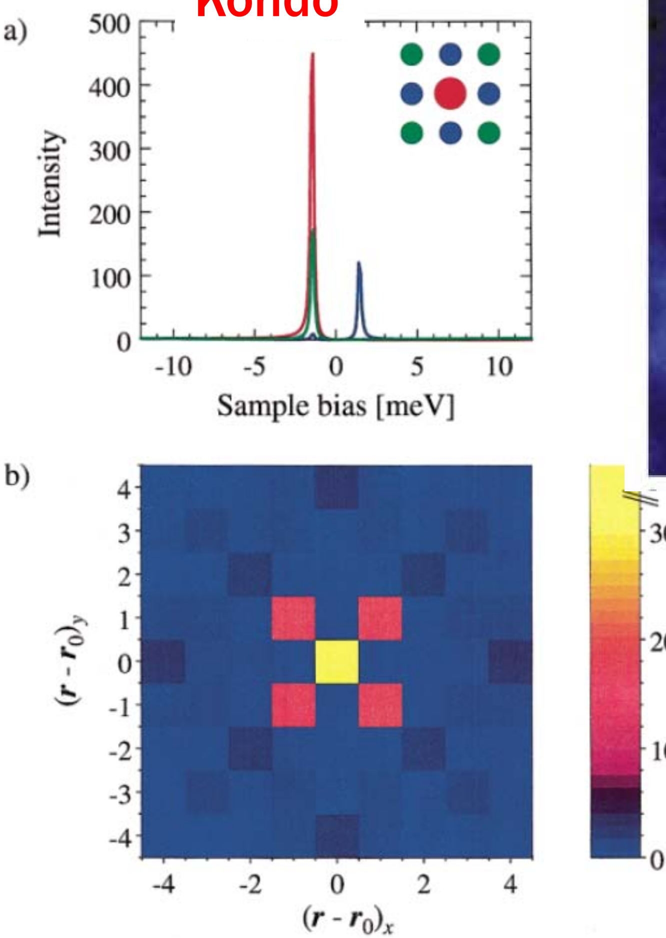
S.-H. Pan et al. 2000

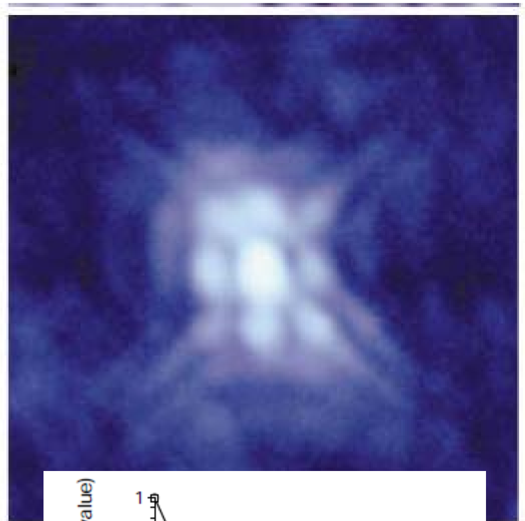


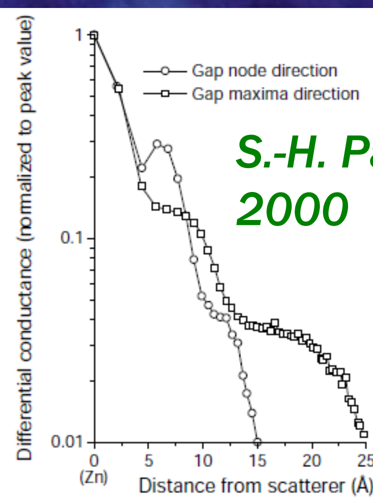
A. Polkovnikov, M. Vojta, S. Sachdev 2001

Kondo vs potential scattering

Kondo

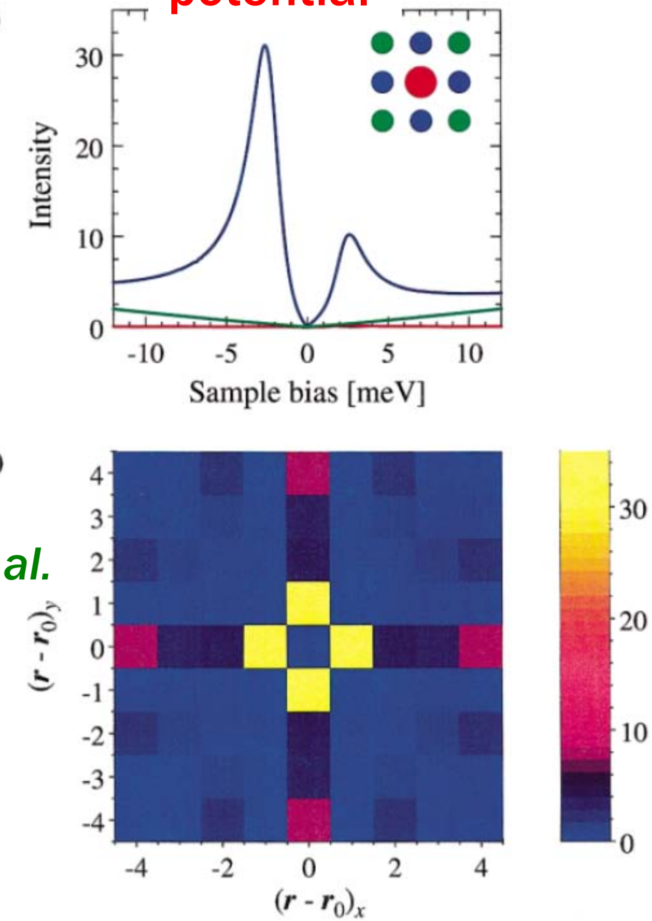






S.-H. Pan et al. 2000

potential



A. Polkovnikov, M. Vojta, S. Sachdev 2001

Neither fits experiment

Message: part III

Sometimes moments appear unexpectedly in correlated systems with magnetic tendencies

But that does not mean that Kondo can explain everything

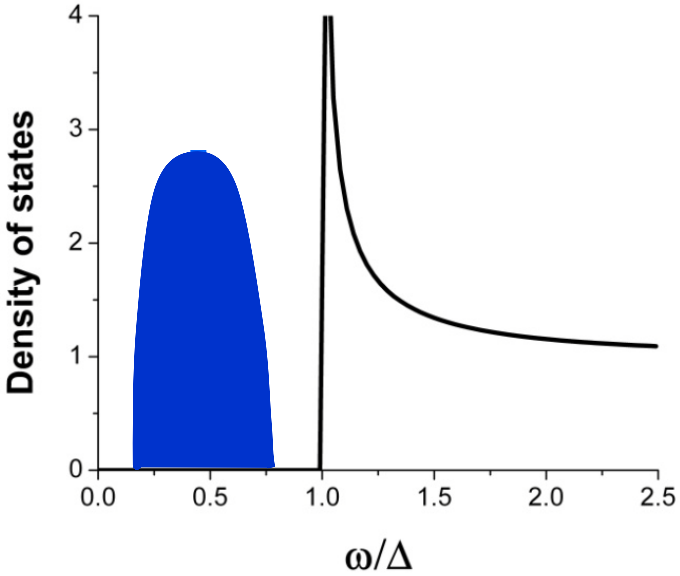
Corollary: draw conclusions about cuprates at your own risk

Many Impurities

From single to many impurities

1. Individual bound states around the impurities broaden into a band

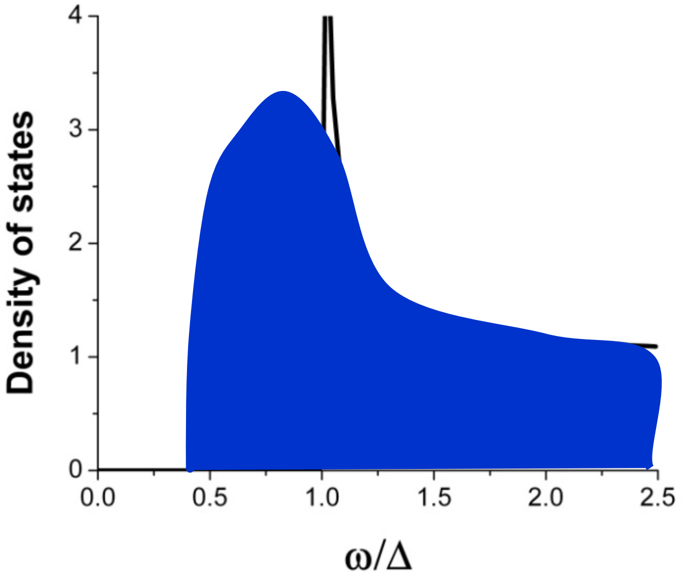
2. The bandwidth grows with the impurity concentration. Depending on the location of the single imp state:
   - either touches Fermi energy first (gapless superconductivity)
   - or mixes with continuum first



From single to many impurities

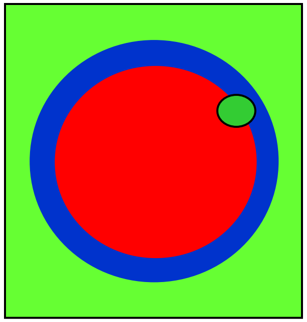
1. Individual bound states around the impurities broaden into a band

2. The bandwidth grows with the impurity concentration. Depending on the location of the single imp state:
   – either touches Fermi energy first (gapless superconductivity)
   – or mixes with continuum first

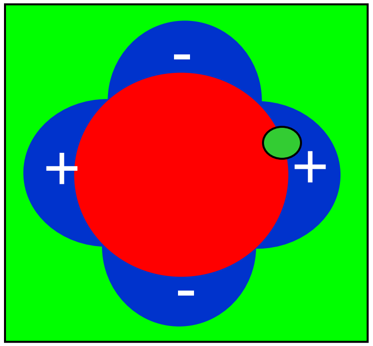


At the same time impurities affect superconductivity

Impurities and superconductivity

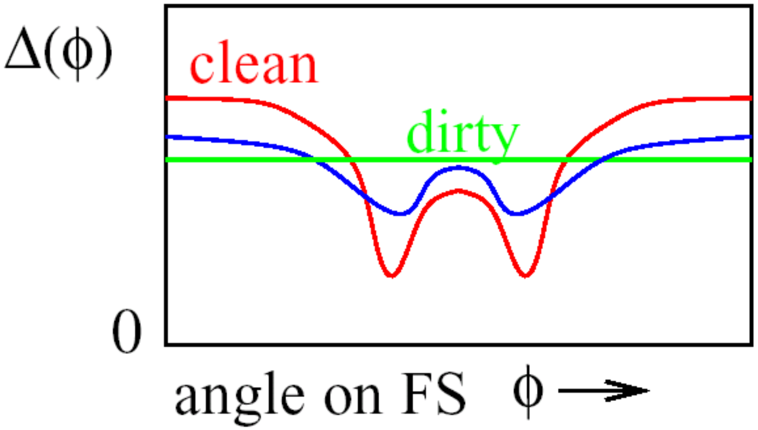


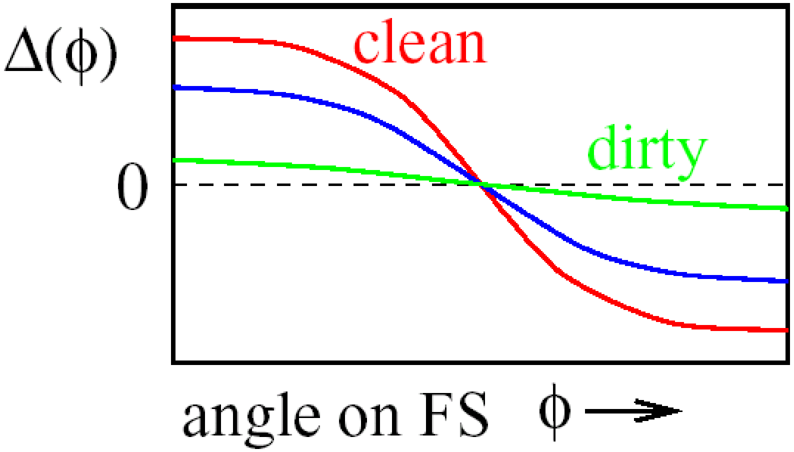
Scattering mixes gaps at different points at the FS



anisotropic s-wave

d-wave



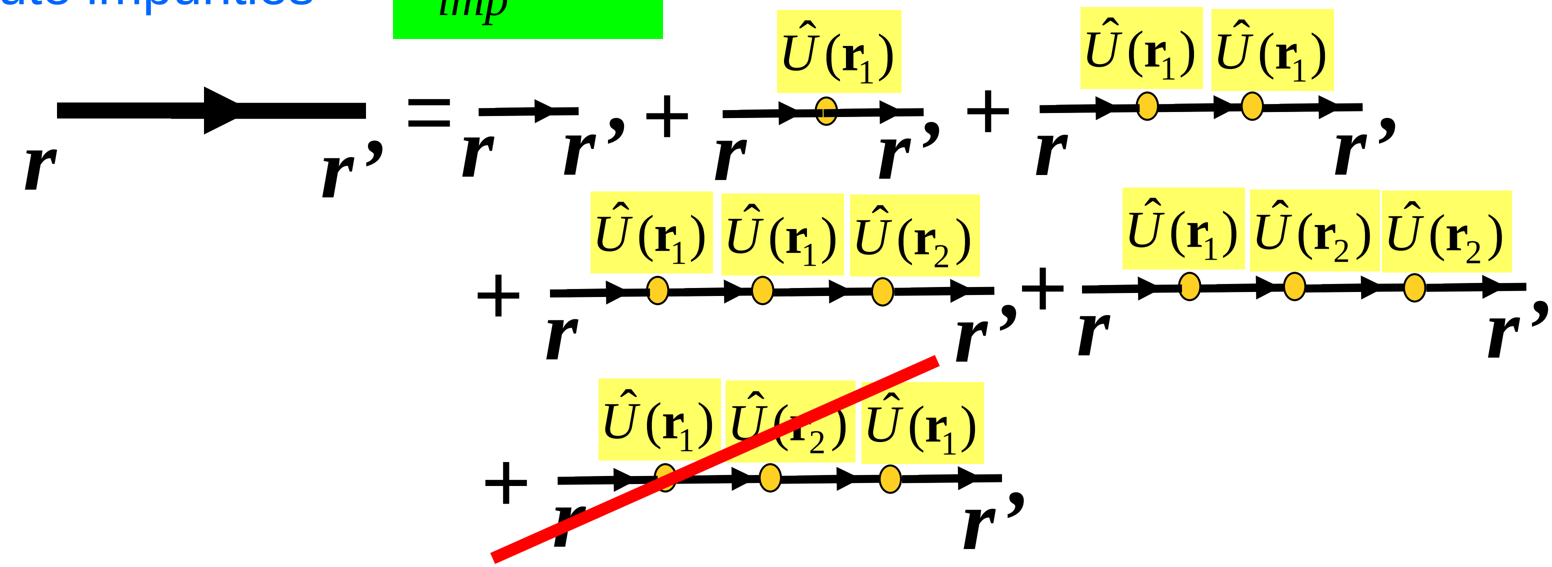


Anisotropy smeared out,
T_c slightly suppressed

Gap and T_c suppressed

Self-consistent approximation

dilute impurities $n_{imp} << 1$

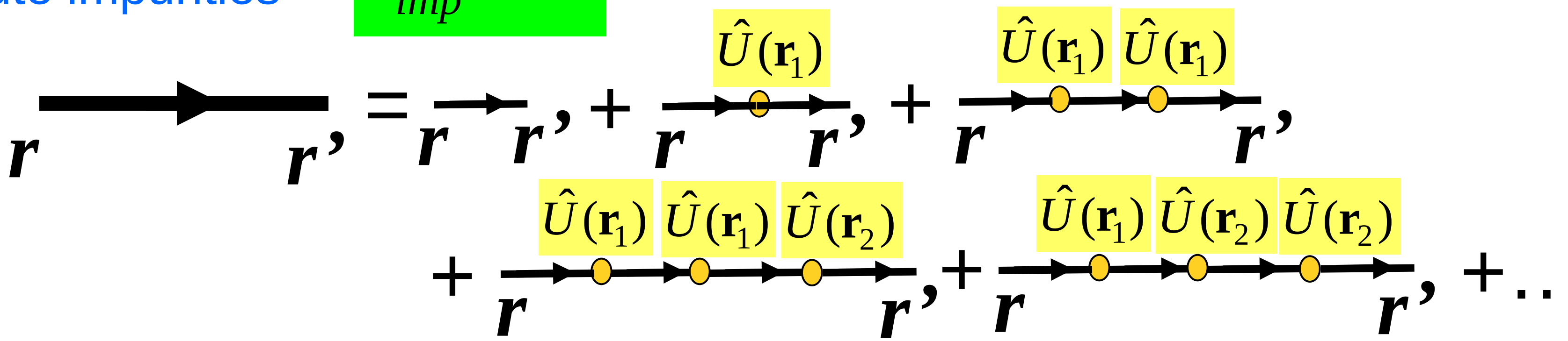


improbable: ignore

Then average over random positions of all impurities

Self-consistent approximation

dilute impurities $n_{imp} << 1$



Self-consistent T-matrix

$$\hat G^{-1}(\mathbf{k},\omega) = i\omega_n - \xi(\mathbf{k})\tau_3 - \Delta_0\sigma_2\tau_2 - \hat\Sigma \equiv i\tilde\omega - \tilde\varepsilon(\mathbf{k})\tau_3 - \tilde\Delta\sigma_2\tau_2$$

$$\hat\Sigma(\mathbf{p},\omega) = n_{\mathrm{imp}}\hat T_{\mathbf{p},\mathbf{p}}$$

$$\hat T_{\mathbf{p},\mathbf{p}'} = \hat U_{\mathbf{p},\mathbf{p}'} + \int d\mathbf{p}_1 \hat U_{\mathbf{p},\mathbf{p}_1}\hat G(\mathbf{p}_1,\omega)\hat T_{\mathbf{p}_1,\mathbf{p}'}$$

Gap and order parameter are not the same.

Full Green's function with scattering on *all other impurities*: need self-consistency

P. Hirschfeld et al., S. Schmitt-Rink et al. 1986

Abrikosov-Gorkov theory

Isotropic s-wave. Weak scatterers: Born approximation (2nd order)

Potential scattering does not affect T_c or gap: Anderson's theorem

Abrikosov-Gorkov theory

Isotropic s-wave. Weak scatterers: Born approximation (2[nd] order)

Potential scattering does not affect T_c or gap: Anderson's theorem

(Weak) magnetic scattering destroys superconductivity and the gap

$$\alpha_s = n_{imp} N_0 J^2 S(S+1)$$

FM coupling or small effective AFM coupling

Normal state scattering rate

single parameter: impurity concentration and strength appear together. Only in Born

Abrikosov, Gorkov, 1960

$$\ln \frac{T_c}{T_{c0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\alpha_s}{2\pi T_{c0}}\right)$$

General equation: needs correct definition of α

Abrikosov-Gorkov theory

Isotropic s-wave. Weak scatterers: Born approximation (2nd order)

Potential scattering does not affect T_c or gap: Anderson's theorem

(Weak) magnetic scattering destroys superconductivity and the gap

$$\alpha_s = n_{imp} N_0 J^2 S(S+1)$$

FM coupling or small effective AFM coupling

Normal state scattering rate

Gap for excitations (pole of Green's function) vanishes at

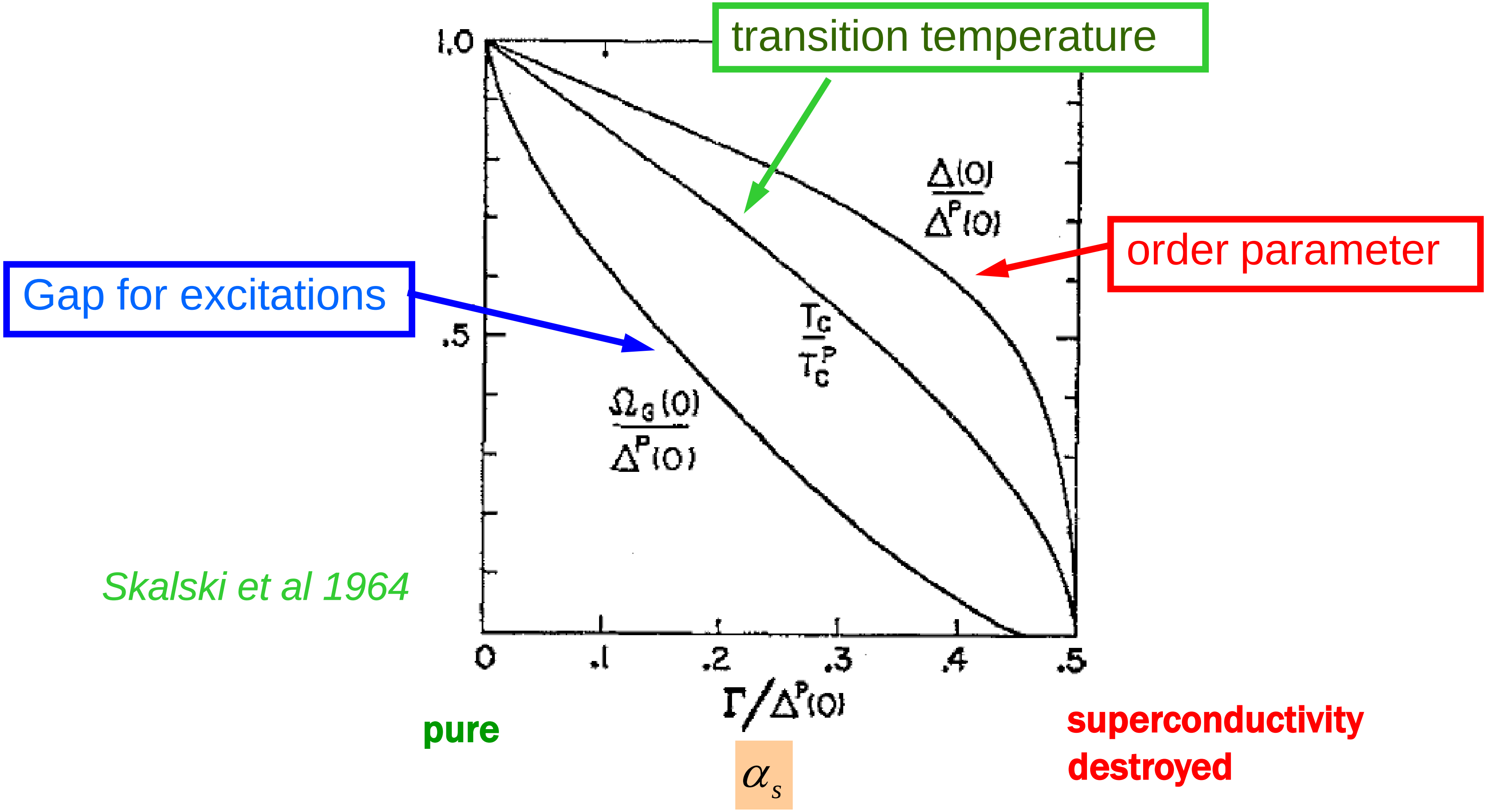
Order parameter (solution of self-consistency equation) exists up to

$$\alpha_s = \alpha_g = \Delta_0 \exp(-\pi/4) \quad < \quad \alpha_s = \alpha_c = \Delta_0/2 \approx 1.1\alpha_g$$

There exists a regime of gapless superconductivity!

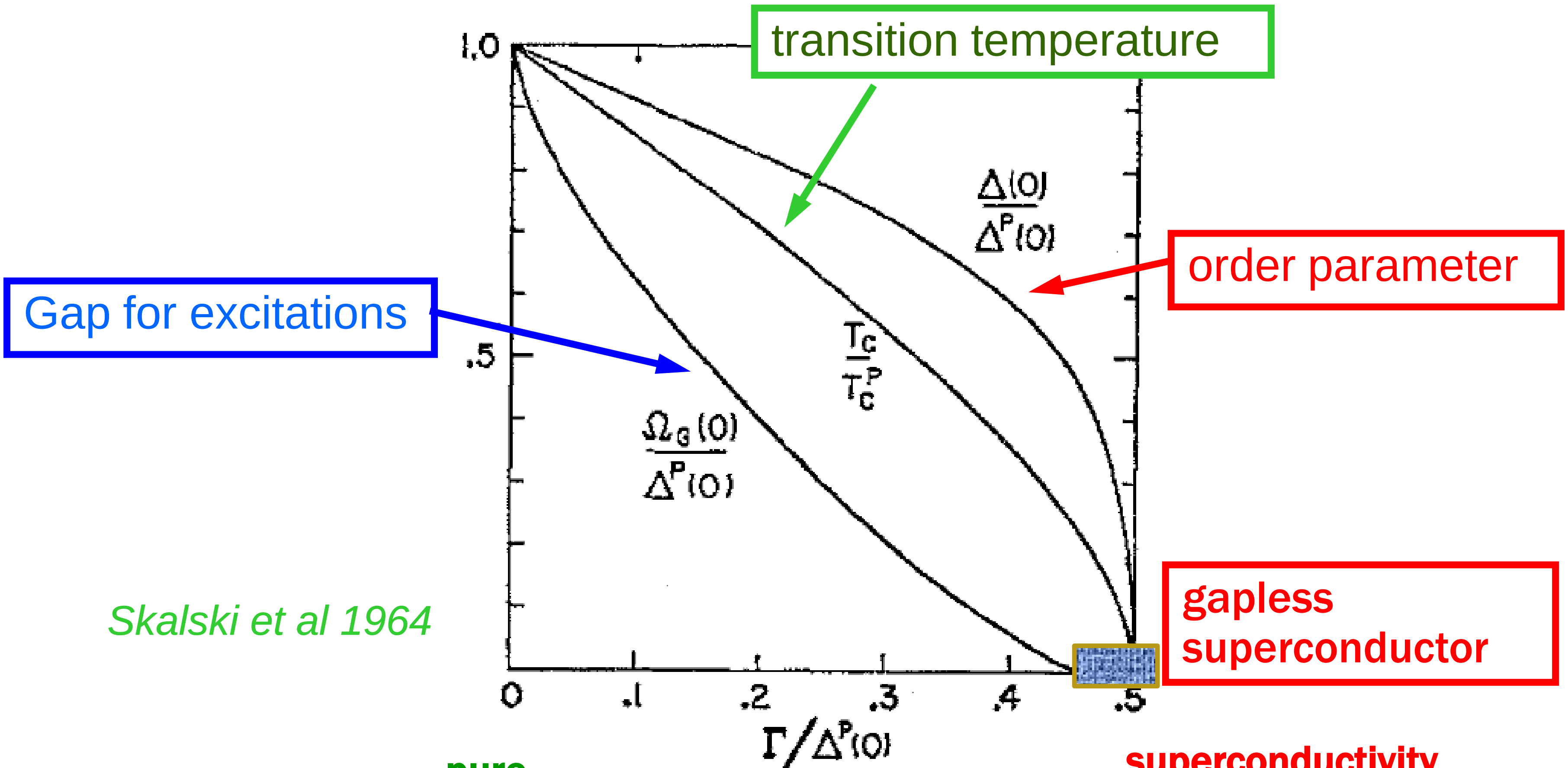
Abrikosov, Gorkov, 1960

s-wave, weak magnetic scattering



Skalski et al 1964

s-wave, weak magnetic scattering



Skalski et al 1964

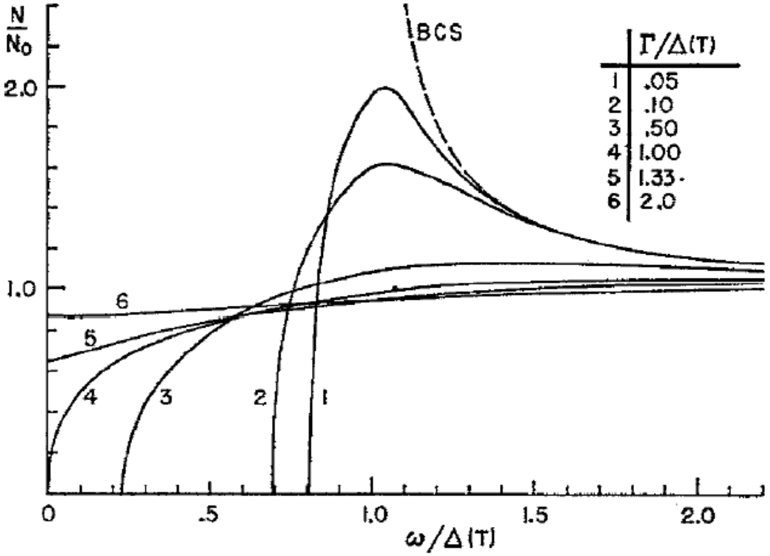
pure

α_s

superconductivity destroyed

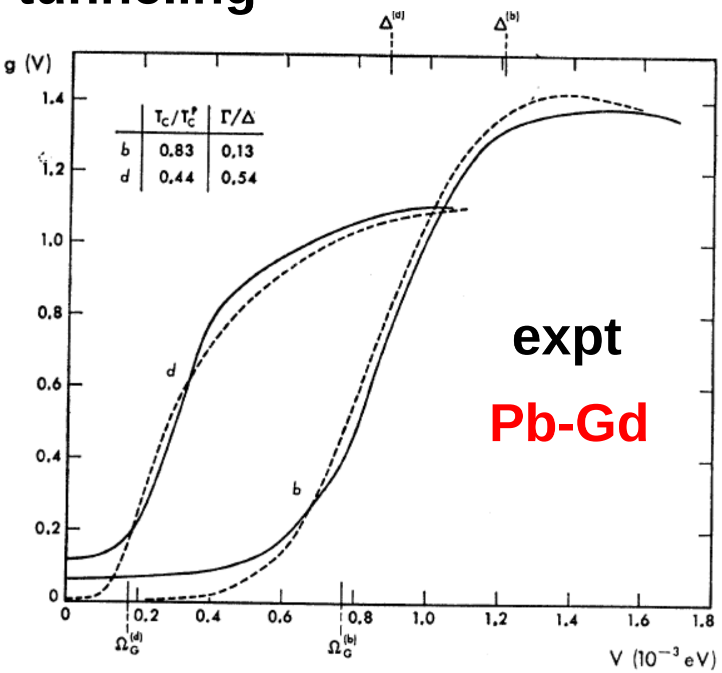
Comparison with experiment

theory



Skalski et al 1964

tunneling



expt

Pb-Gd

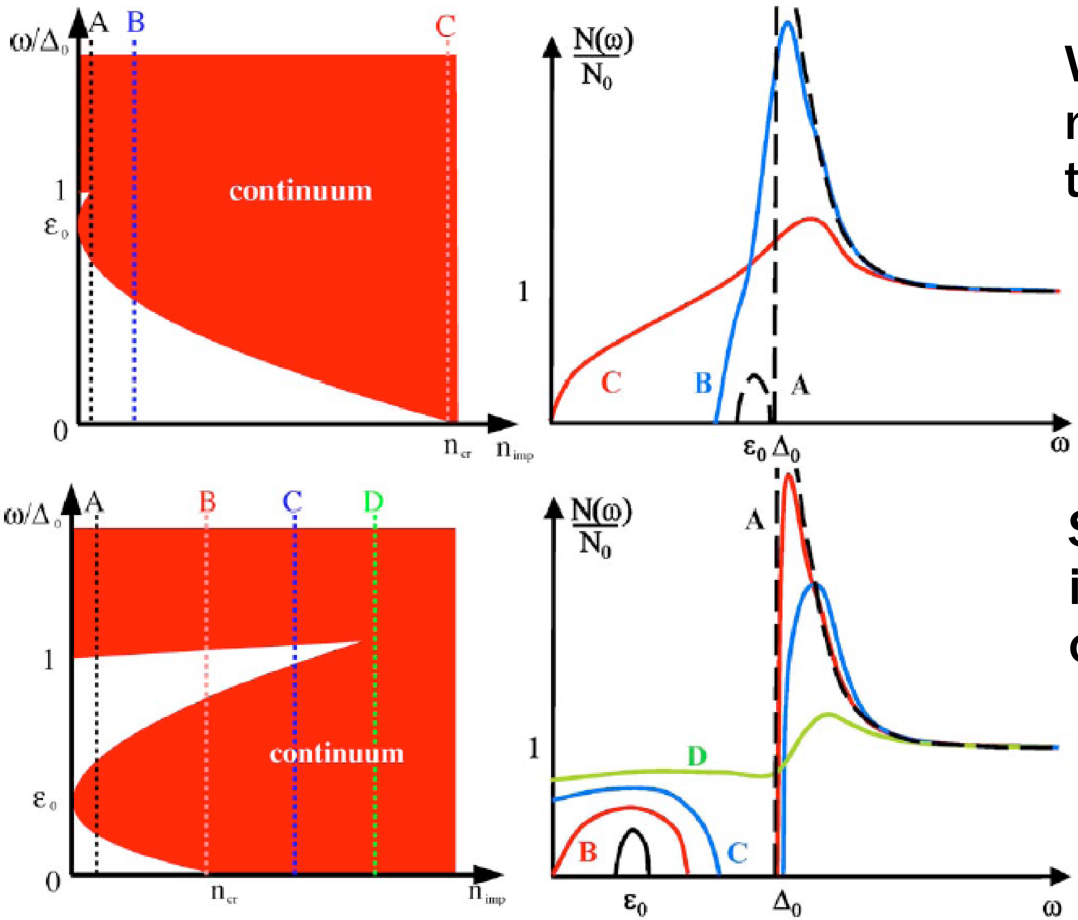
M. Woolf and F. Reif, 1965

Shiba bands

Abrikosov-Gorkov: smearing out of the gap edge

$$\alpha_s = n_{imp} N_0 J^2 S(S+1)$$

Growth of impurity band from the position of the bound state: hopping

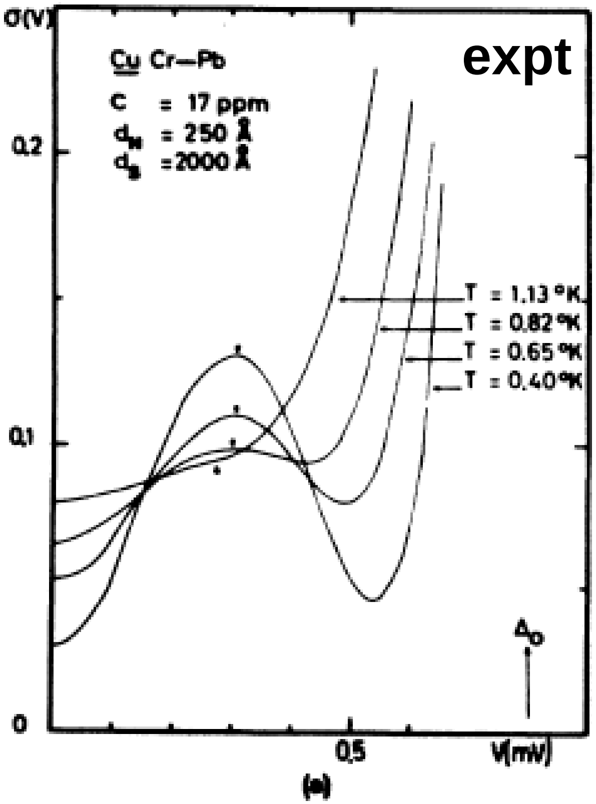


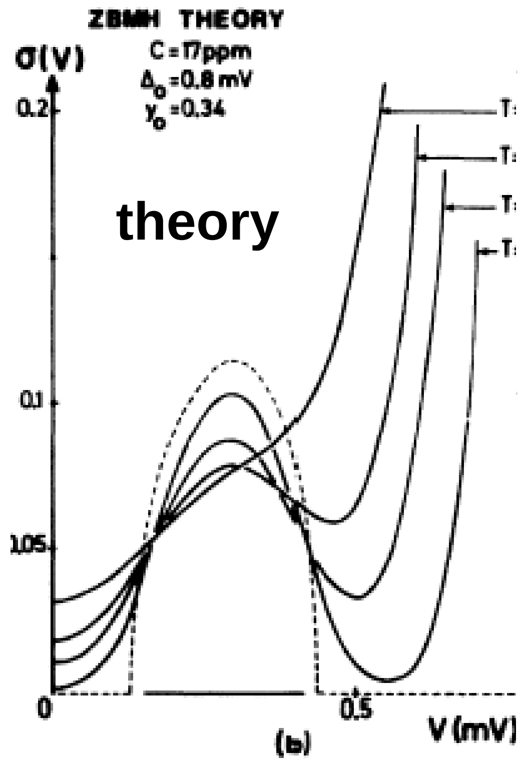
Weak scattering: bound state near the gap edge, smearing of the gap

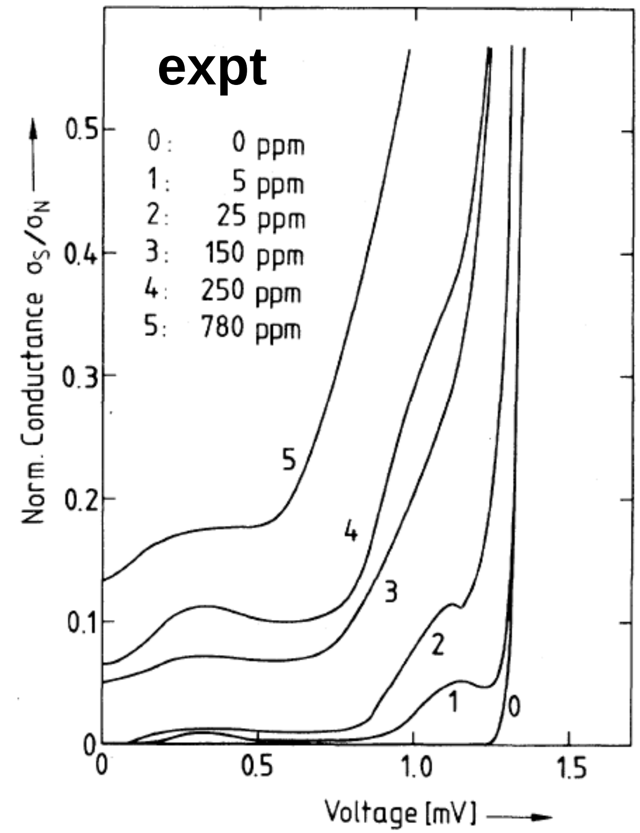
Strong scattering: growth of impurity band from the position of the bound state in the gap

A. Balatsky, IV, J-X. Zhu 2006

Experiment







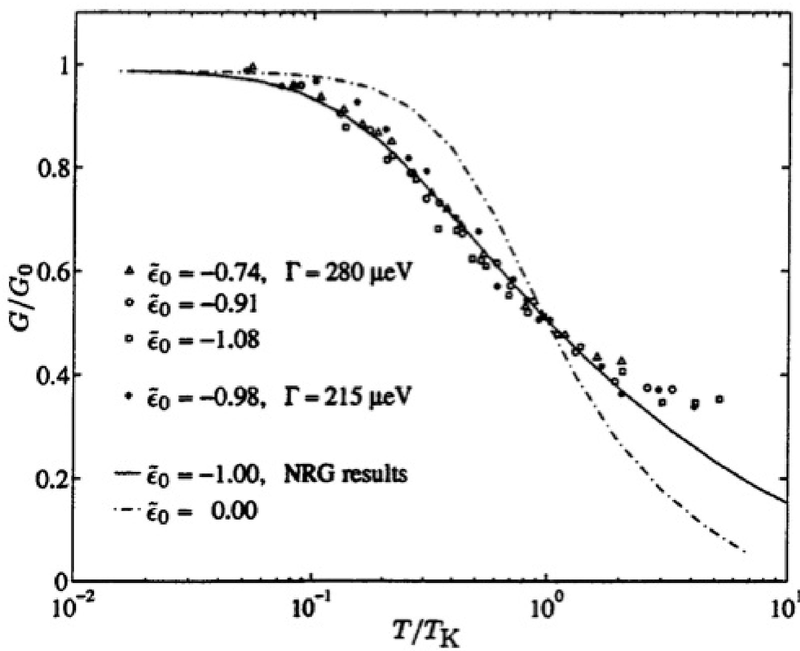
L. Dumoulin et al., 1977

W. Bauriedl et al., 1981

Many impurities: Kondo

Reminder: quantum effects mean that scattering depends on energy/temperature, is strongest for $T \sim T_K$

Conduction electron moment at the imp site



D. Goldhaber-Gordon et al 1998

Many impurities: Kondo

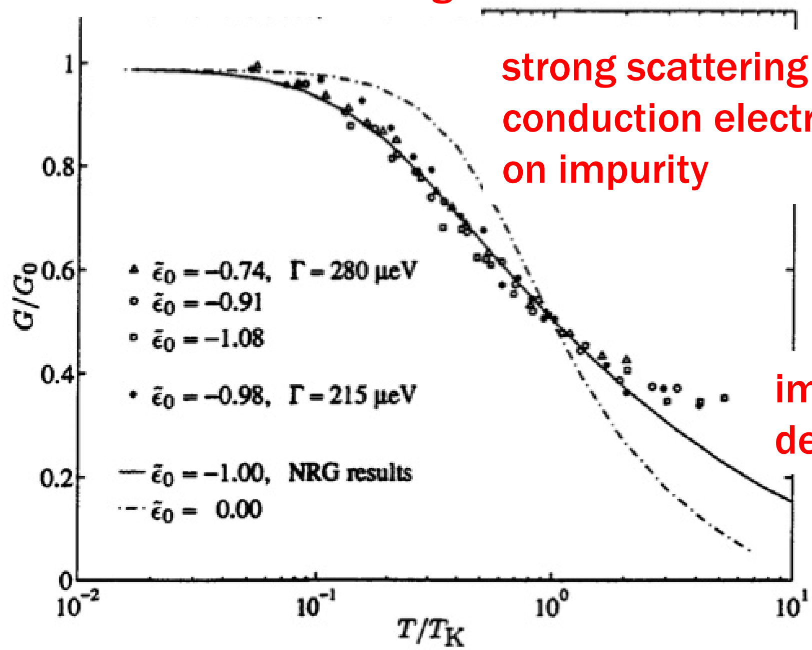
Reminder: quantum effects mean that scattering depends on energy/temperature, is strongest for $T \sim T_K$

impurity spin nearly screened: weak scattering

strong scattering of conduction electrons on impurity

impurity spin nearly decoupled: weak scattering

Conduction electron moment at the imp site



D. Goldhaber-Gordon et al 1998

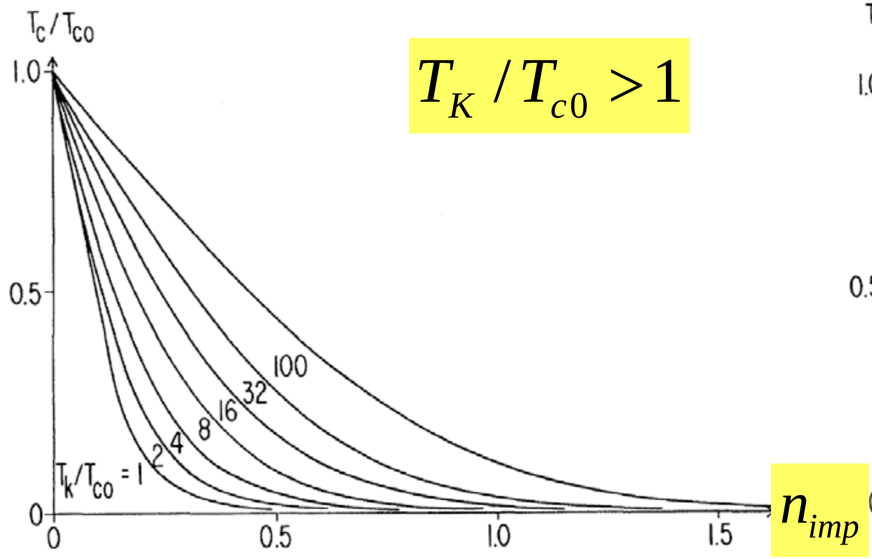
Scattering rate depends on the ratio of T_c/T_K

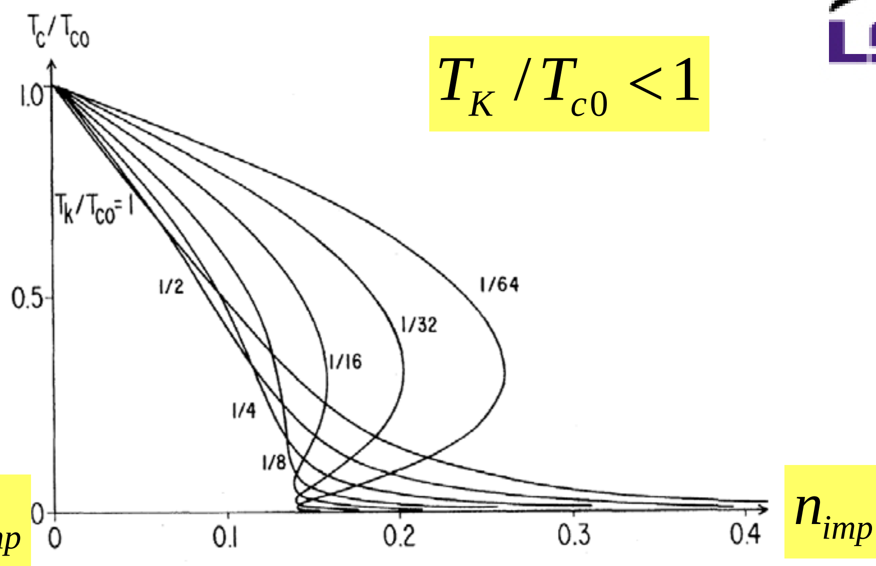
$$\alpha \approx n_{imp} \frac{\pi^2 S(S+1)}{\ln^2 T_c/T_K + \pi^2 S(S+1)}$$

J. Zittartz and E. Müller-Hartmann 1971

Determine T_c self-consistently

Many impurities: Kondo II





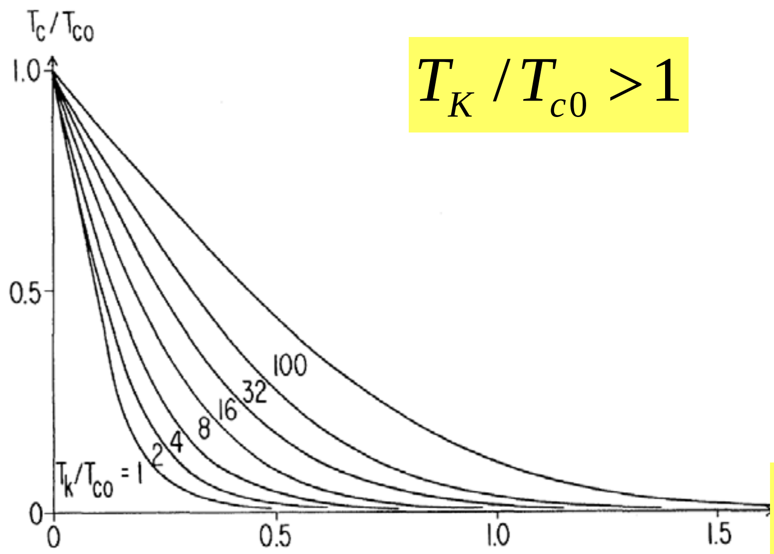
At $T=0$ fully screened impurity:
no pairbreaking

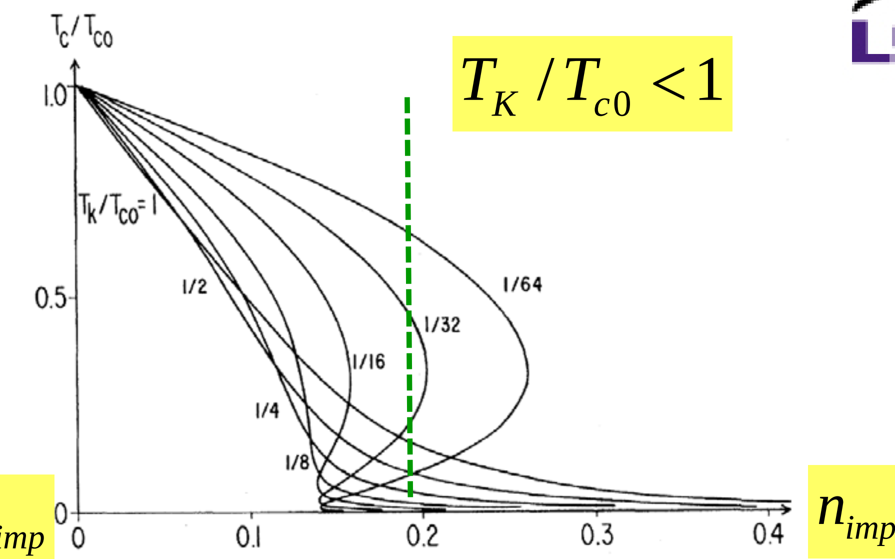
Lower T_{c0} : less efficient scattering

Approximation questionable

J. Zittartz and E. Müller-Hartmann 1971

Many impurities: Kondo II





At $T=0$ fully screened impurity:
no pairbreaking

Lower T_{c0} : less efficient scattering

Approximation questionable

J. Zittartz and E. Müller-Hartmann 1971

Reentrance:

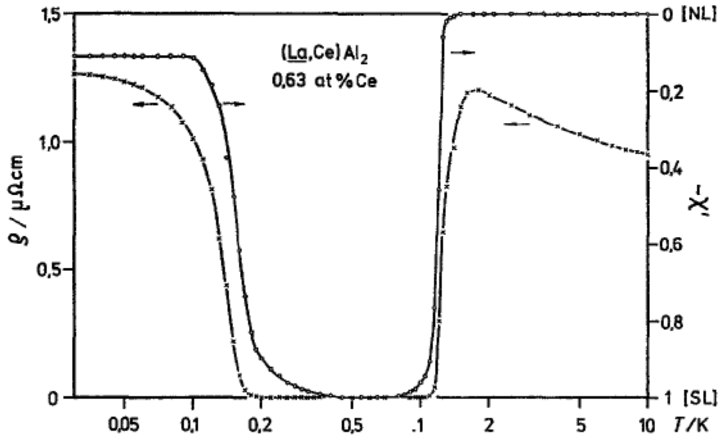
a) Superconducting at $T_c > T_K$

b) Approach T_K : scattering increases, back to normal

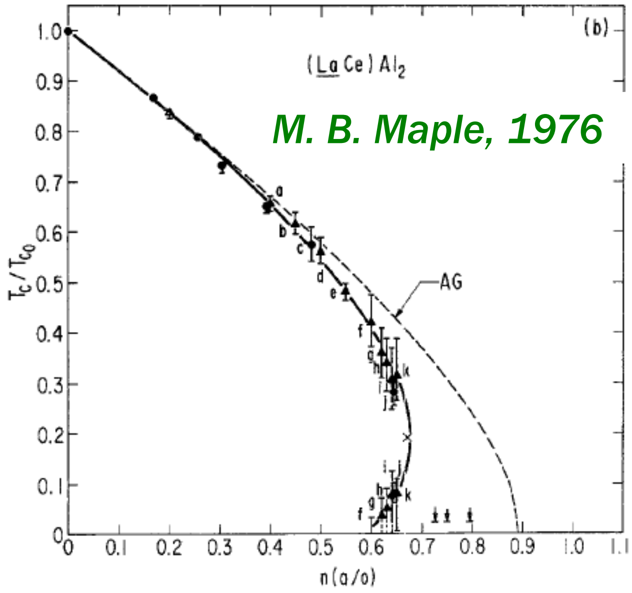
c) At $T < T_K$ screening, scattering decreases:,back to superconductor

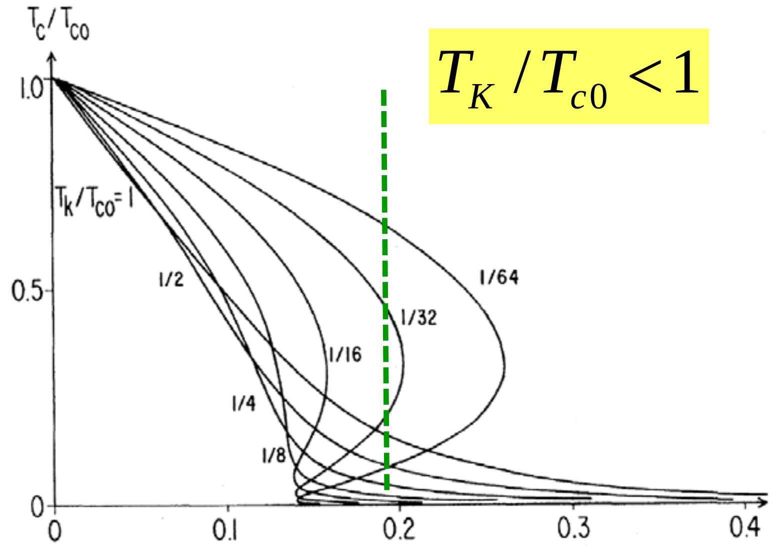
Many impurities: Kondo II

K. Winzer, 1973



M. B. Maple, 1976





$T_K / T_{c0} < 1$

n_{imp}

Reentrance:

a) Superconducting at $T_c > T_K$

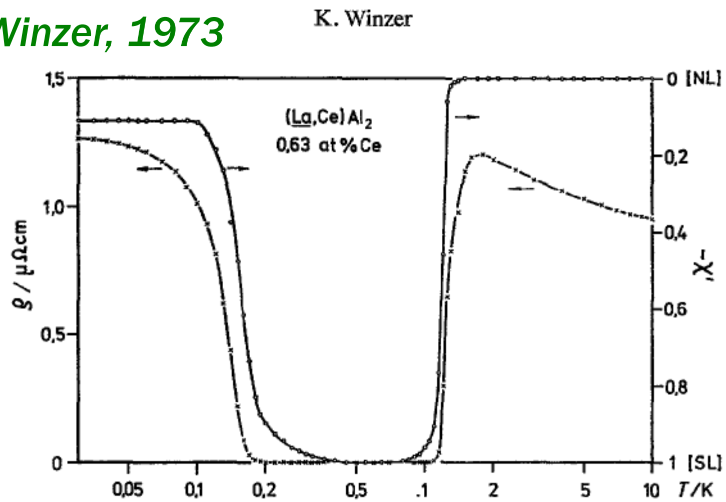
b) Approach T_K : scattering increases, back to normal

c) At $T < T_K$ screening, scattering decreases:,back to superconductor

J. Zittartz and E. Müller-Hartmann 1971

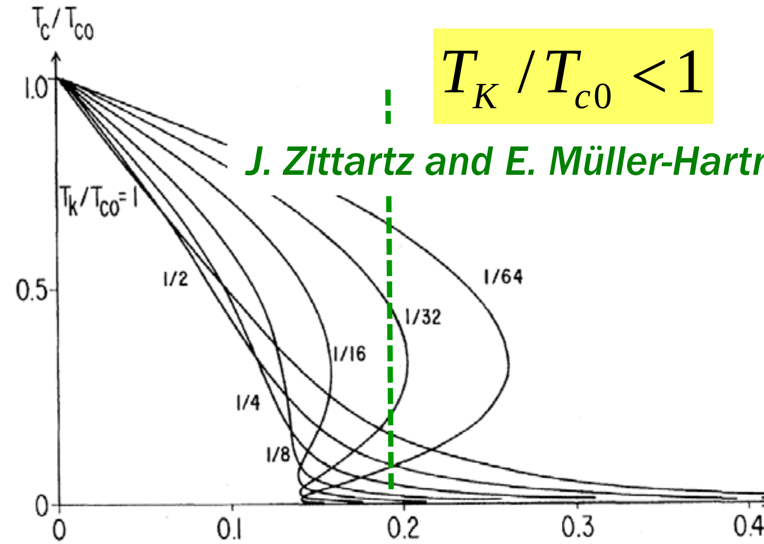
Many impurities: Kondo II

K. Winzer, 1973



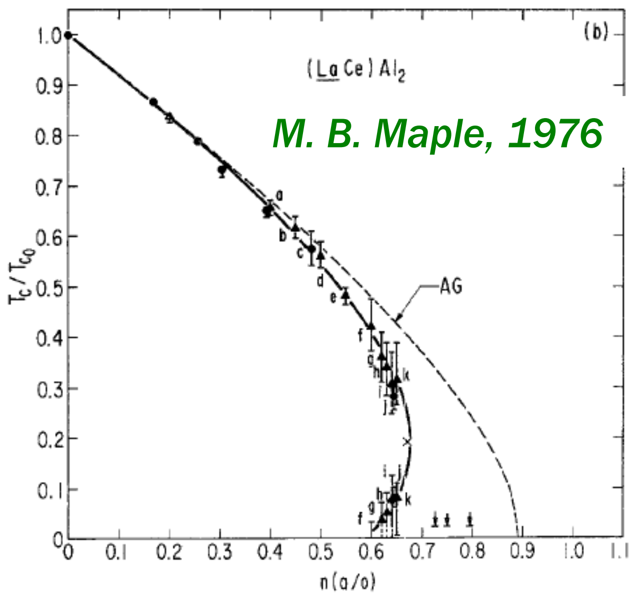
$T_K / T_{c0} < 1$

J. Zittartz and E. Müller-Hartmann 1971



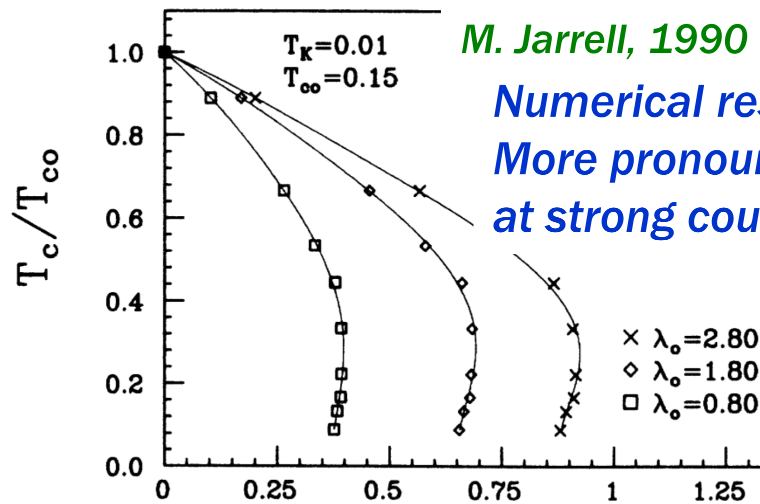
n_{imp}

M. B. Maple, 1976



M. Jarrell, 1990

Numerical results: More pronounced at strong coupling



n_{imp}

Dirty superconductors with nodes

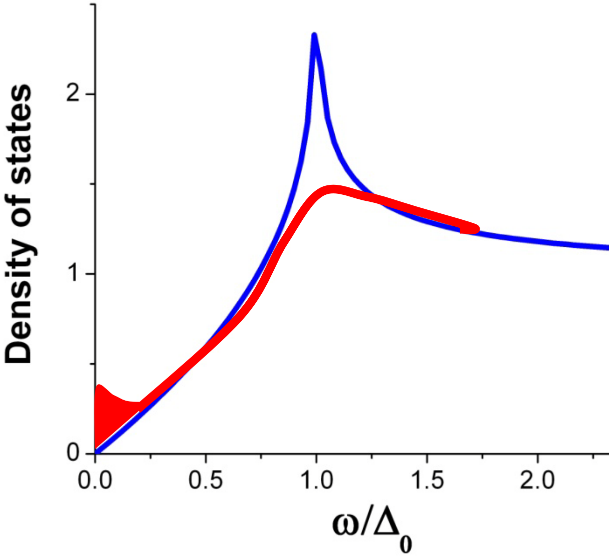
All impurities are pairbreaking Γ **scattering rate in the normal state**

Always gapless (in self-consistent T-matrix)

Born limit (weak scattering)

$$\frac{N_{sc}(0)}{N_0} \approx \frac{\Delta_0}{\Gamma} \exp\left[-\frac{\Delta_0}{\Gamma}\right]$$

L. Gorkov and P. Kalugin, 1985,
T. Rice and K. Ueda, 1985



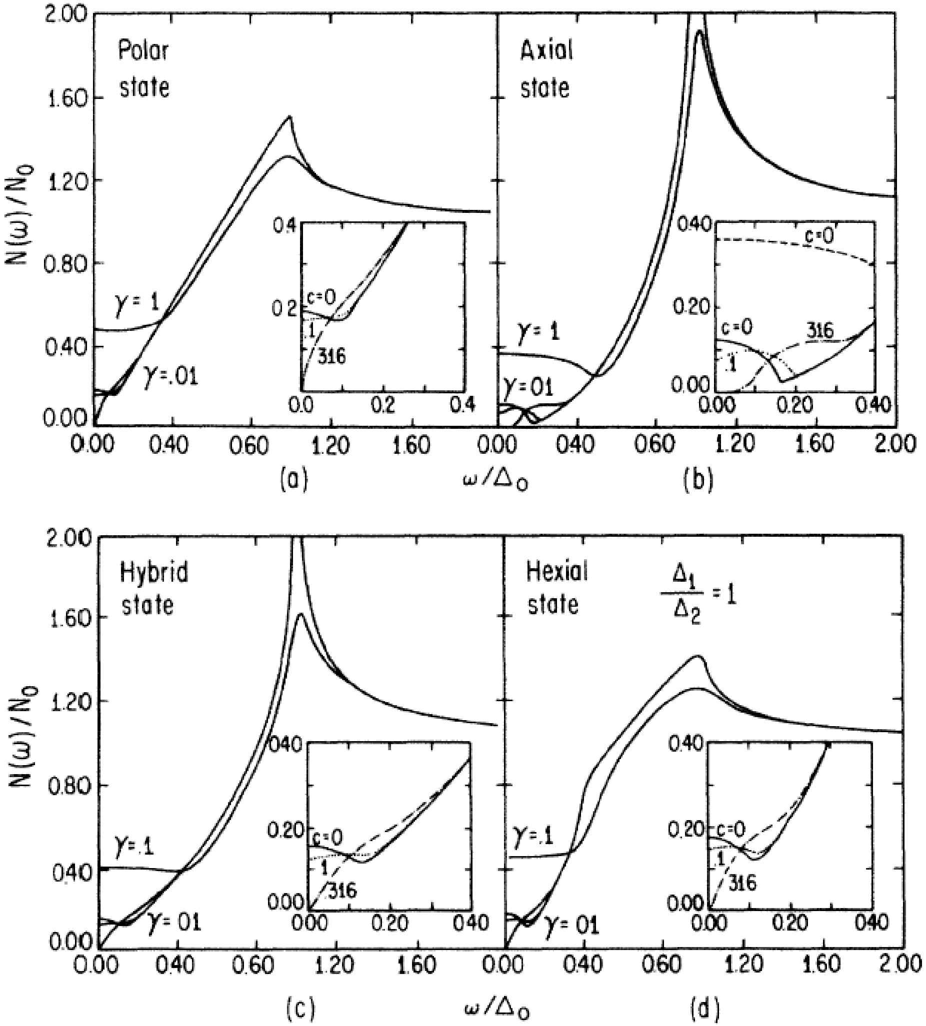
Unitarity limit (strong scattering)

$$\frac{N_{sc}(0)}{N_0} \approx \sqrt{\frac{\Gamma}{\Delta_0}}$$

P. Hirschfeld et al., 1986,
S. Schmitt-Rink et al, 1986

Gapless behavior in nodal SC

P. Hirschfeld et al., 1989,



Finite DOS at ***ω=0*** (gapless)

Impurity bandwidth

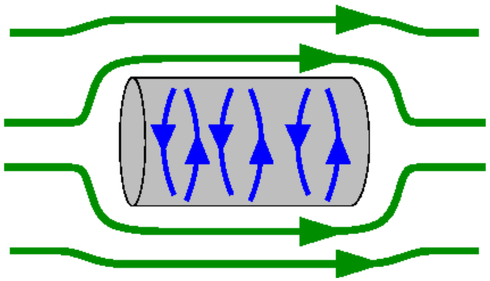
$$\frac{\gamma}{\Delta_0} \approx \frac{N_{sc}(0)}{N_0}$$

Born: small

Unitarity: large

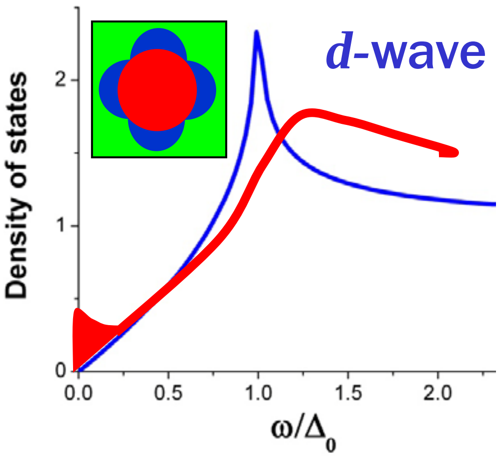
Can be experimentally measured

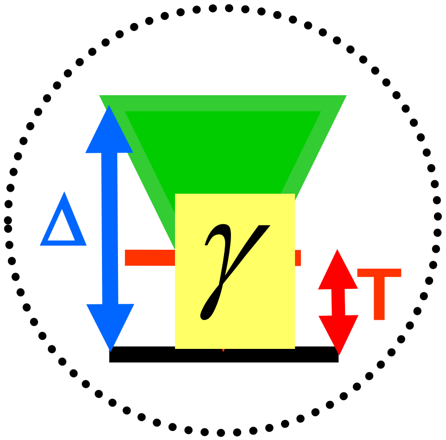
Penetration depth in superconductors



Magnetic field is screened at the length

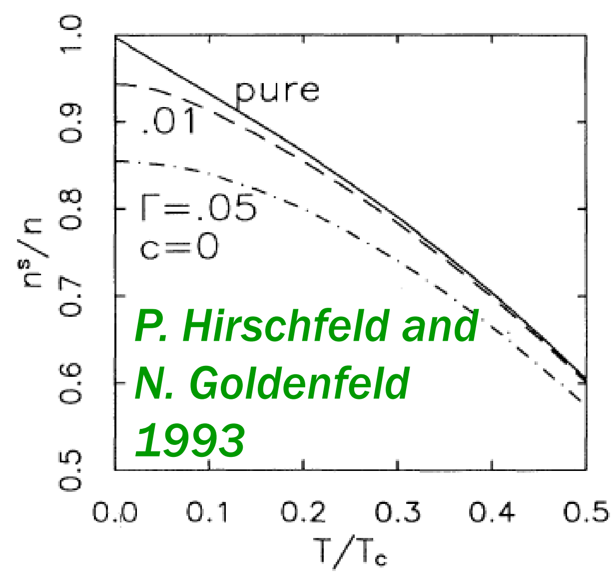
$\lambda_L^{-2} \propto n_s$ density of superconducting electrons



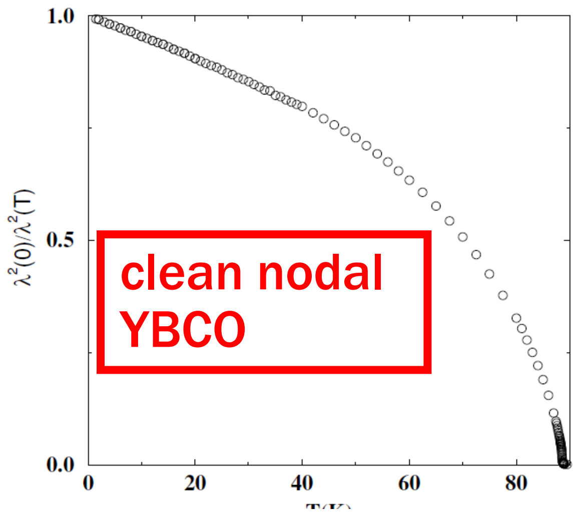


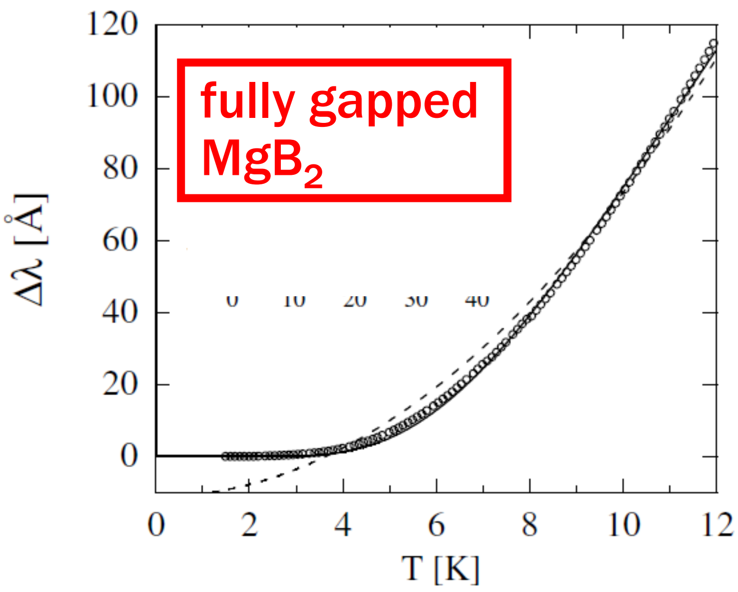
$\Delta\lambda_L^{-2} \propto T$ at low T

$\Delta\lambda_L^{-2} \propto T^2/\gamma$ at low T

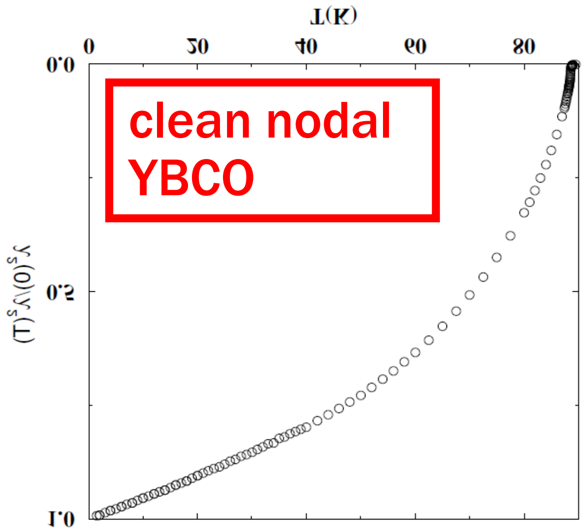


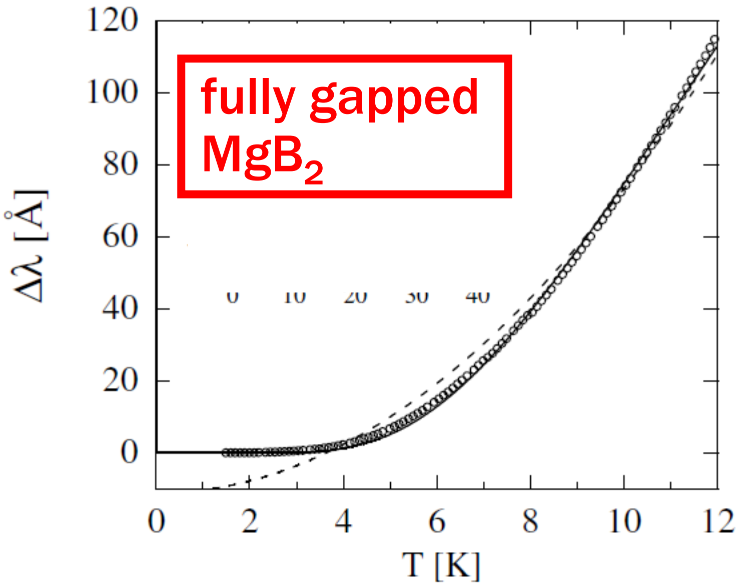
Penetration depth in superconductors



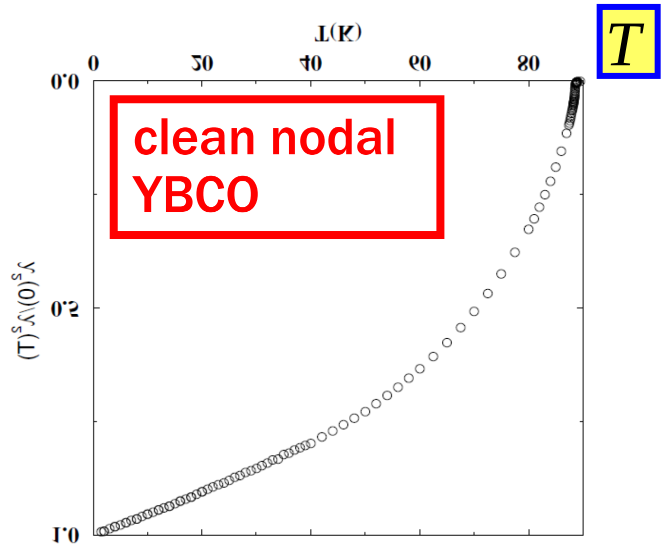


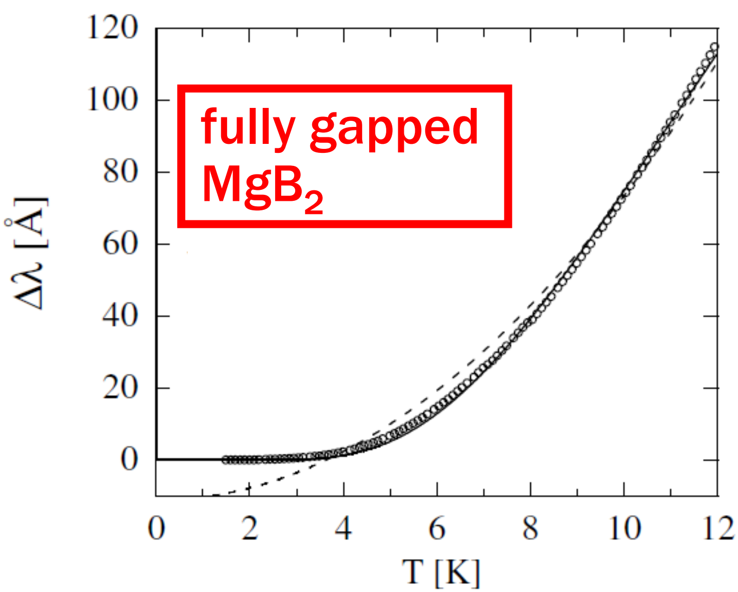
Penetration depth in superconductors



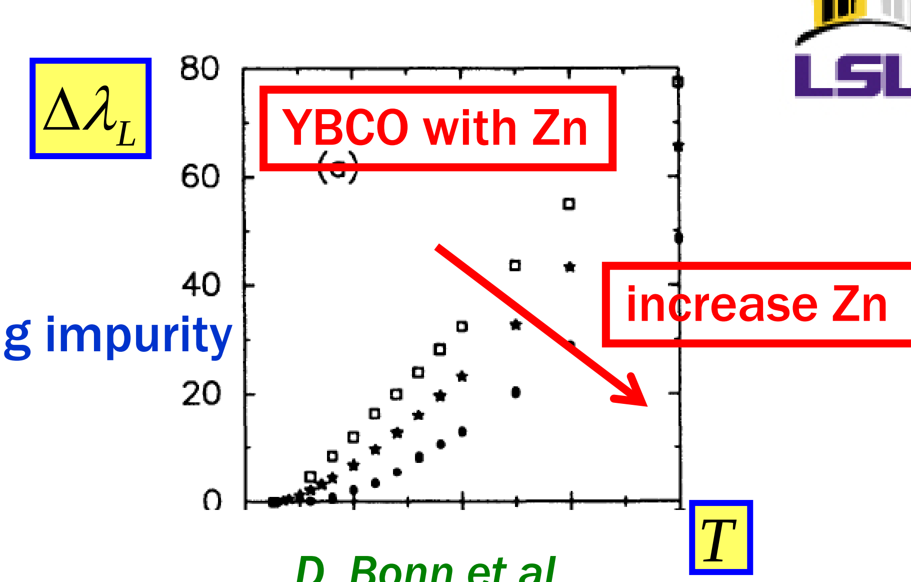


Penetration depth in superconductors



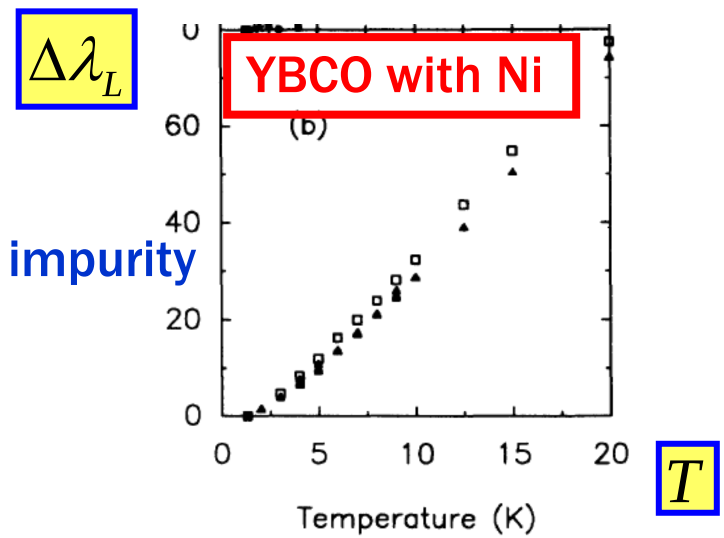


strong impurity



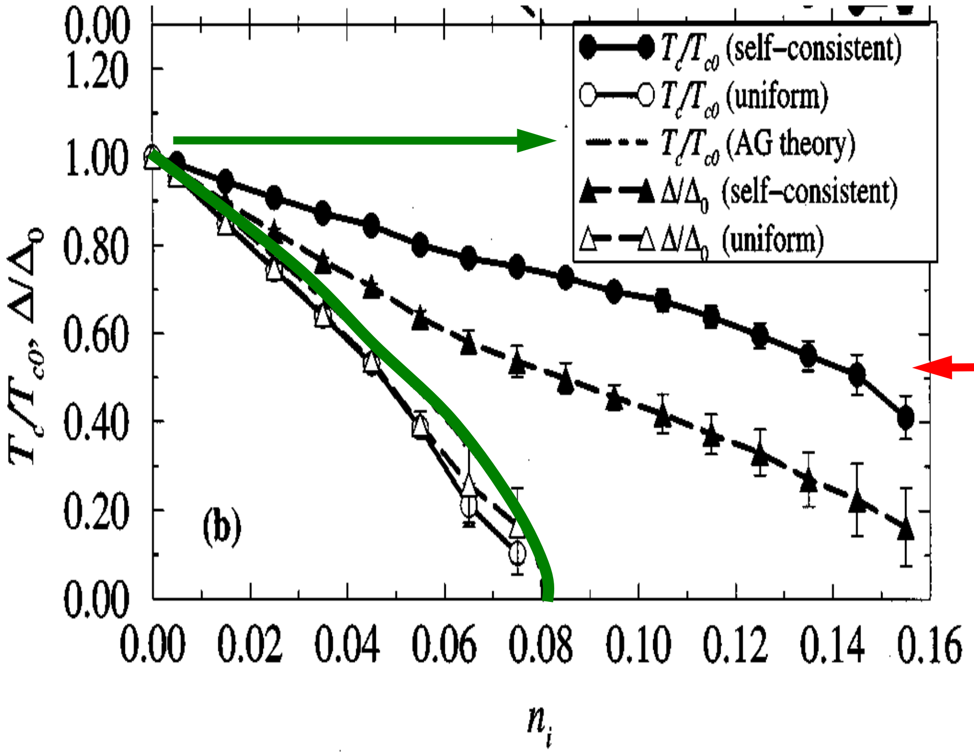
D. Bonn et al,

weak impurity



Transition temperature suppression

Non-magnetic impurities suppress unconventional superconductivity just as magnetic impurities suppress isotropic pairing



AG theory assumes uniform suppression of the gap in the bulk

For short coherence length, a local suppression ("swiss cheese" superconductor) may be better

M. Franz et al. 1997

Message: part IV

Impurity bands in superconductors:

- s-wave: due to magnetic impurities
- d-wave: due to any impurities

Transition temperature suppressed by these impurities in a similar fashion for both cases.

Gapless superconductors:

- s-wave: above critical concentration
- d-wave: at any concentration

not counting tails
small for Born

Final Summary

Impurity bound/resonant states grow into impurity bands

- s-wave: due to magnetic impurities
- d-wave: due to any impurities

Screening of the local moment competes with pairing: from local moment + pairs to local singlet + unpaired electron

Understanding of single impurity Kondo in s-wave systems, open questions (pseudogap Kondo, quantum criticality) in d-wave.

Re-entrant superconductivity in Kondo s-wave superconductors

Impurity-controlled physics at low T in nodal systems

And now what happens if we have Kondo ion on each site?