QIMP11, May 29th – June 10th, 2011, Dresden

# Far from equilibrium and time-dependent phenomena

## for electron transport in quantum dots

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# Outline

Introduction

Part I: Single electron transport in quantum dots

Electron and spin in quantum dots

Time-resolved single electron detection

Single electron manipulation

Interaction with photons and phonons

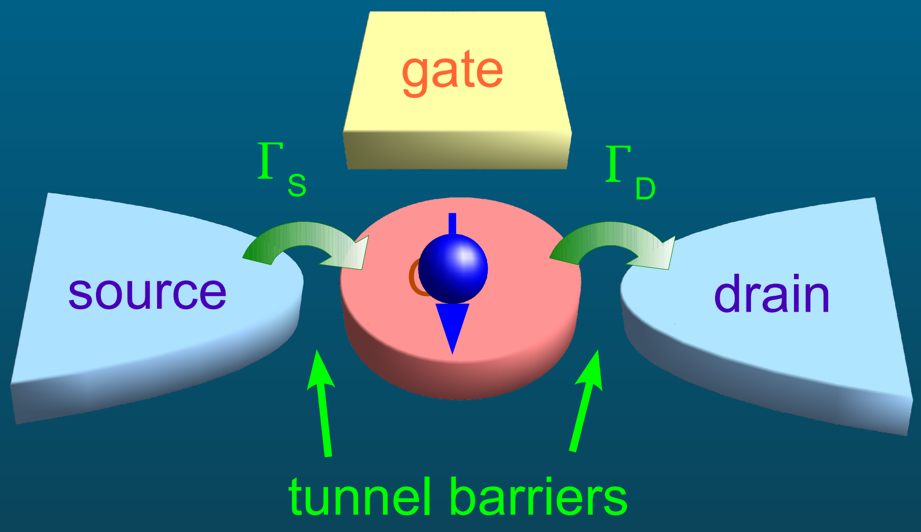
Part II: Kondo effect in quantum dots

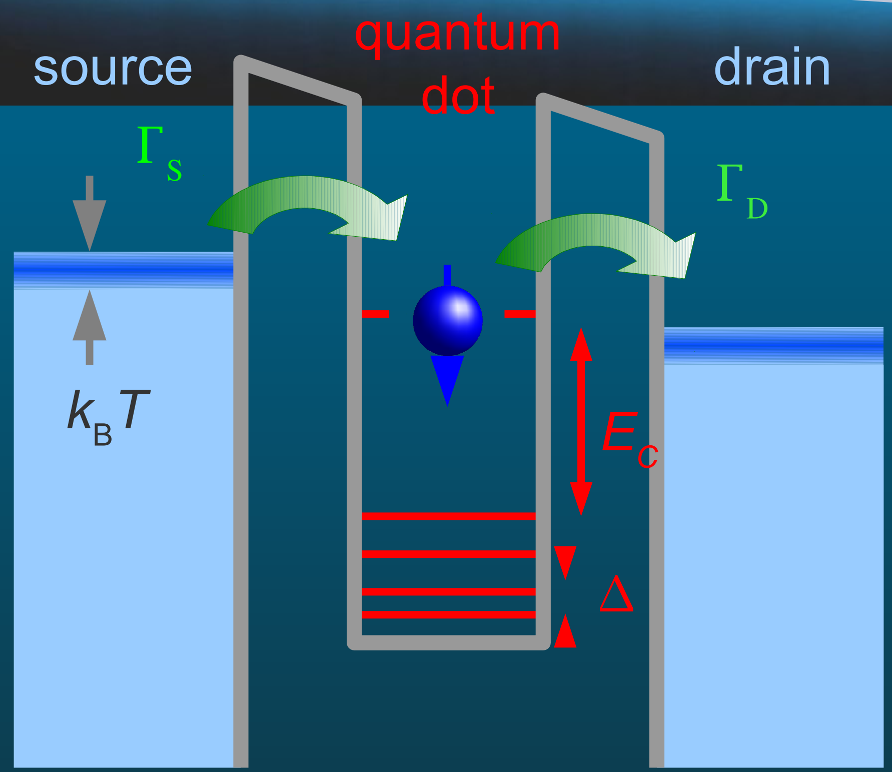
# Introduction

# Challenges for quantum electronic transport

- Low-frequency linear transport in non interacting systems is well understood
  - Landauer-Büttiker theory $$G = \frac{e^2}{h} \sum_n T_n$$
- Understanding the experiments requires to go beyond!
  - non-equilibrium effects (large bias voltage, current noise)
  - role of electron-electron interactions
  - interaction with the environment ⇒ finite coherence time
  - high-frequency response (adiabatic or non-adiabatic regime)
  - role of the electron spin
- Quantum dots as an “ideal” playground to provide answers

# Transport in quantum dots




- Small island
  - large capacitance C
    charging energy $E_C = e^2/C$
  - quantum confinement
    level spacing $\Delta \sim \hbar^2/(m^* r^2)$

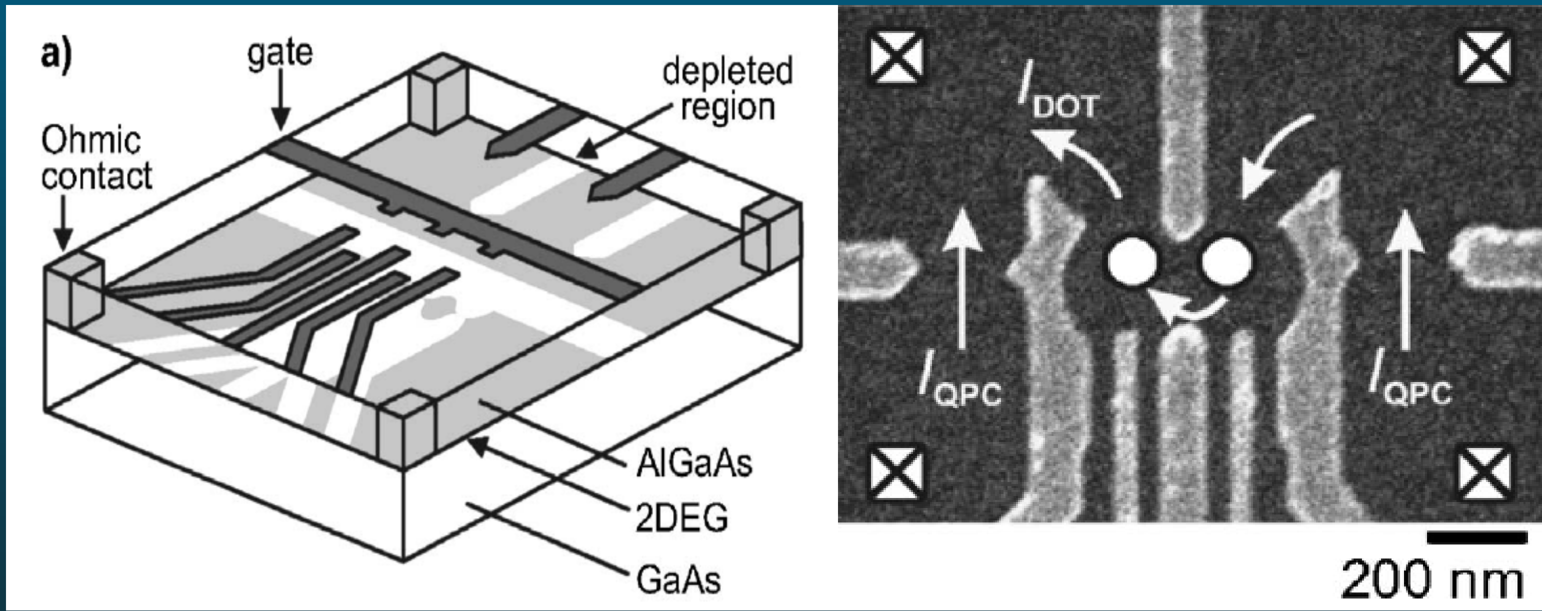
$$k_B T \ll \Delta < E_C$$

transport through a single atomic level

trapped electron = quantum impurity connected to Fermi leads

# Fabrication of semiconductor quantum dots

- Most successful up to now: GaAs heterostructures

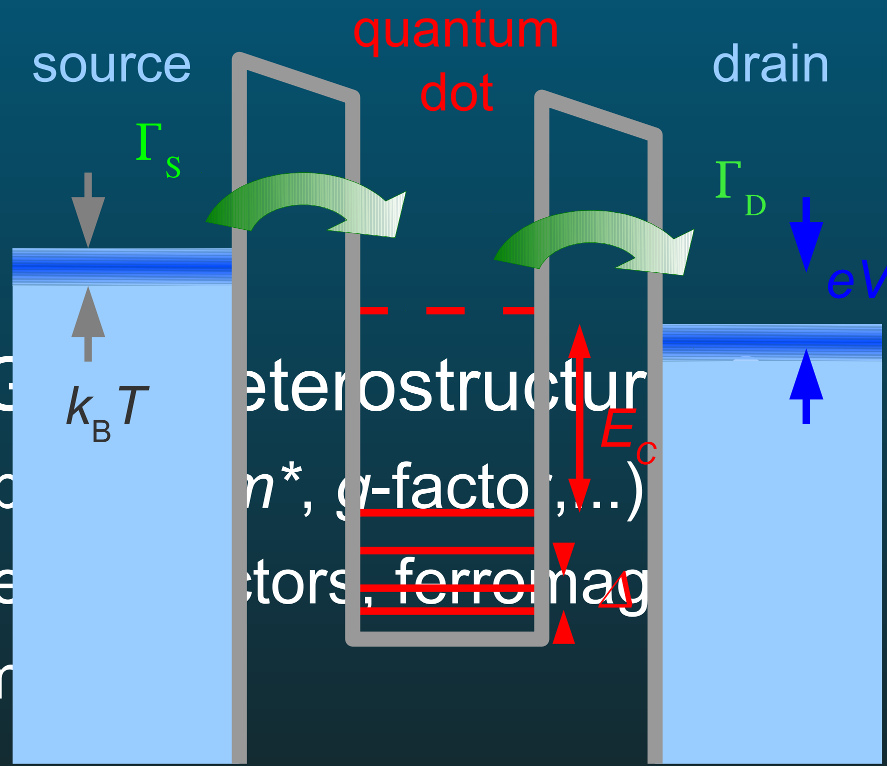



R. Hanson *et al.*, Rev. Mod. Phys. **79**, 1217 (2007)

Most of the demonstrative experiments on quantum dots were performed on this system.

# Fabrication of semiconductor quantum dots

- Most successful up to now: GaAs heterostructures
  - extensive tuning of parameters
    - number of electrons *N*
    - confinement potential *Δ*
    - coupling to the leads $\Gamma_S$, $\Gamma_D$
    - bias voltage $V_{SD}$
- What is difficult to achieve with G… eterostructur…
  - change the intrinsic electronic prop… m*, g-factor,...)
  - coupling with other materials (supe… tors, ferromag…
  - new geometries (interaction with m…
  - optically active quantum dots



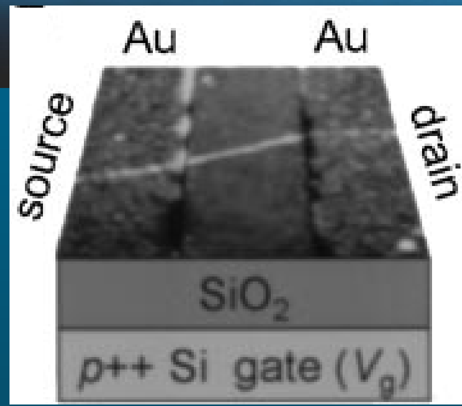
# Fabrication of semiconductor quantum dots

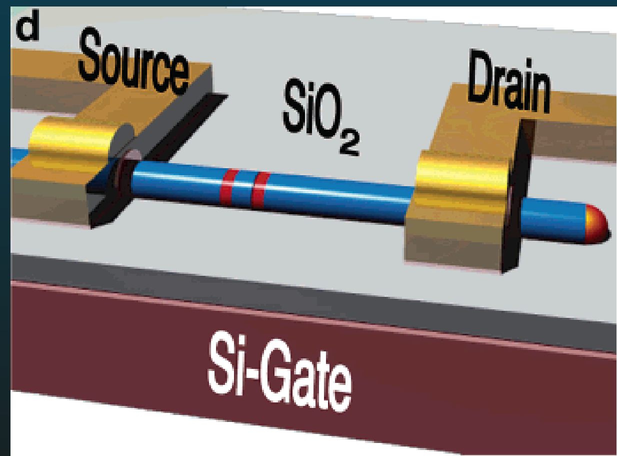
- Necessity of tuning the material properties
  - change the intrinsic electronic properties
    - effective mass ⇒ broader range of level spacing
    - spin-orbit interaction (InAs, InSb: strong SOI)
    - zero nuclear spin (Si, C) ⇒ long spin coherence time
  - coupling with other materials
    - superconductors
    - ferromagnetic materials
  - new geometries
    - suspended nanostructures (nanowire, nanotubes)
    - heterogeneous integration
  - optically active quantum dots (see lecture A. Imamoglu)

$$\Delta \approx \frac{\hbar^2}{m^* r^2}$$

# Fabrication of semiconductor quantum dots

- Carbon-based nanostructures
  - carbon nanotubes, fullerene
    M. Bockrath *et al.*, Science **275**, 1922 (1997)
    H. Park *et al.*, Nature **407**, 57 (2000)
  - graphene
    L. A. Ponomarenko *et al.*, Science **320**, 356 (2008)
- Semiconductor nanowire
  - InP, Si, InAs, Ge, InSb
    S. De Franceschi *et al.*, Appl. Phys Lett. **83**, 244 (2003)
    Z. Zhong *et al.*, Nano Lett. **5**, 1143 (2003)
    M. T. Björk *et al.*, Nano Lett. **4**, 1621 (2004)
    Y. Hu *et al.*, Nature Nanotechnol. **2**, 622 (2007)
    H. A. Nilsson *et al.*, Nano Lett. **9**, 3151 (2009)



# Take-away message (1)

High tunability of semiconductor quantum dots for transport through quantum impurities

electronic properties

material properties

interaction with the environment

next: what can we probe in transport experiments?

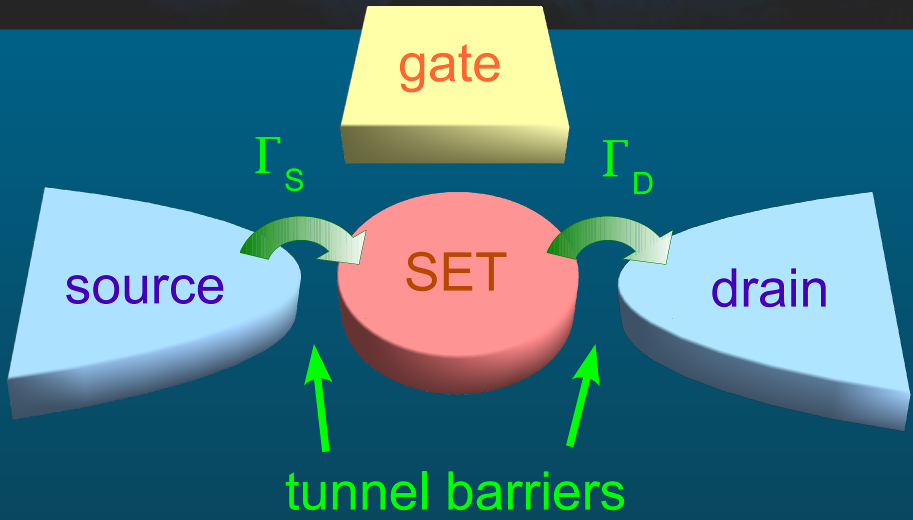
# Part I

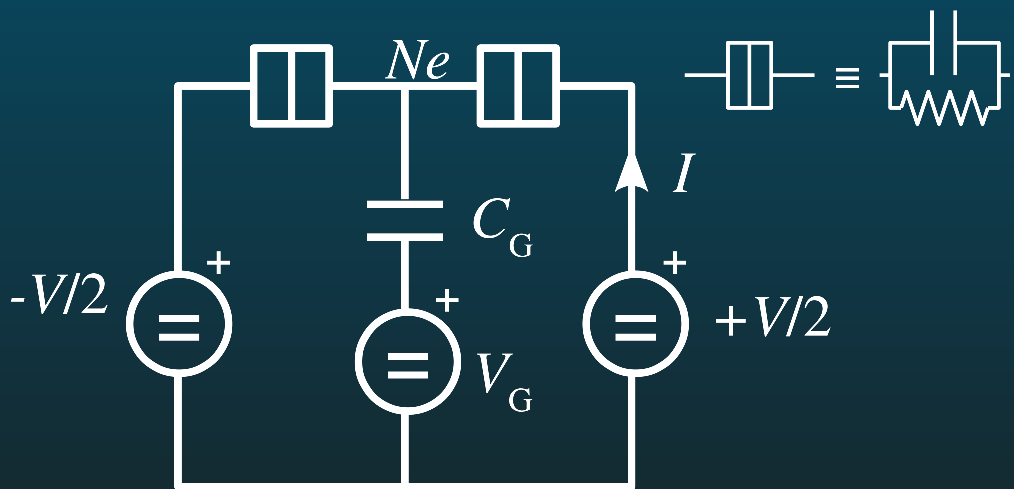
# Single electron transport in quantum dots

1. Transport mechanisms in quantum dots
2. Time-resolved single electron detection
3. Single electron manipulation
4. Interaction with photons
5. Interaction with phonons

# Single electron transistor (SET)

review: *Single Charge Tunneling*, ed. Graber & Devoret, Plenum Press (1992)




$$-Ne = Q_g + Q_L + Q_R$$

$$C = C_L + C_R + C_g$$

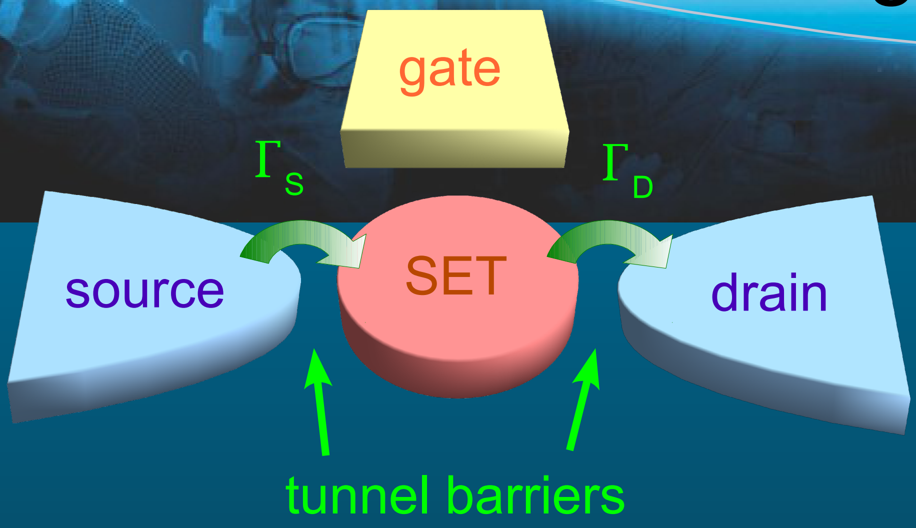
$$V_g = \frac{Q_L}{C_L} - \frac{Q_g}{C_g} = \frac{Q_R}{C_R} - \frac{Q_g}{C_g}$$

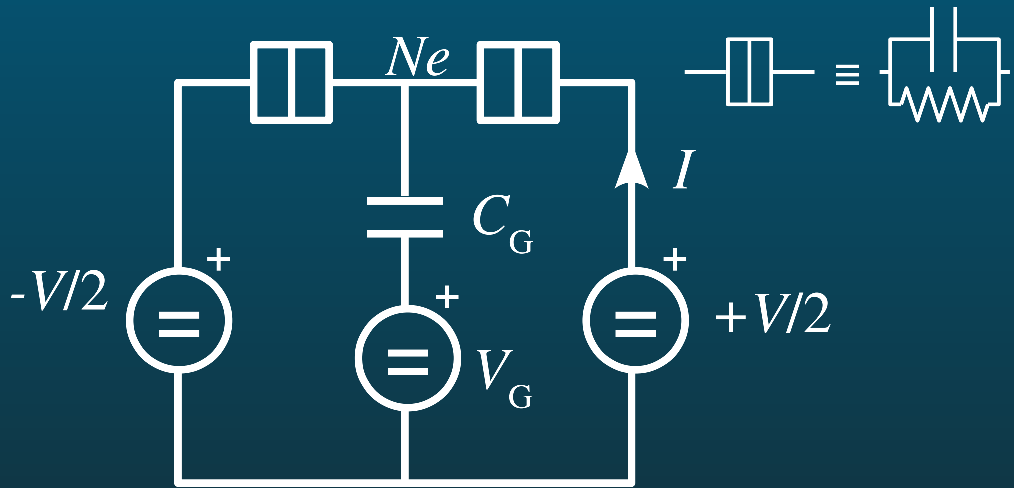
$$E_{cl}(N, V_g) = \frac{Q_L^2}{2C_L} + \frac{Q_R^2}{2C_R} + \frac{Q_g^2}{2C_g}$$

$$\approx \frac{(Ne - C_g V_g)^2}{2C}$$

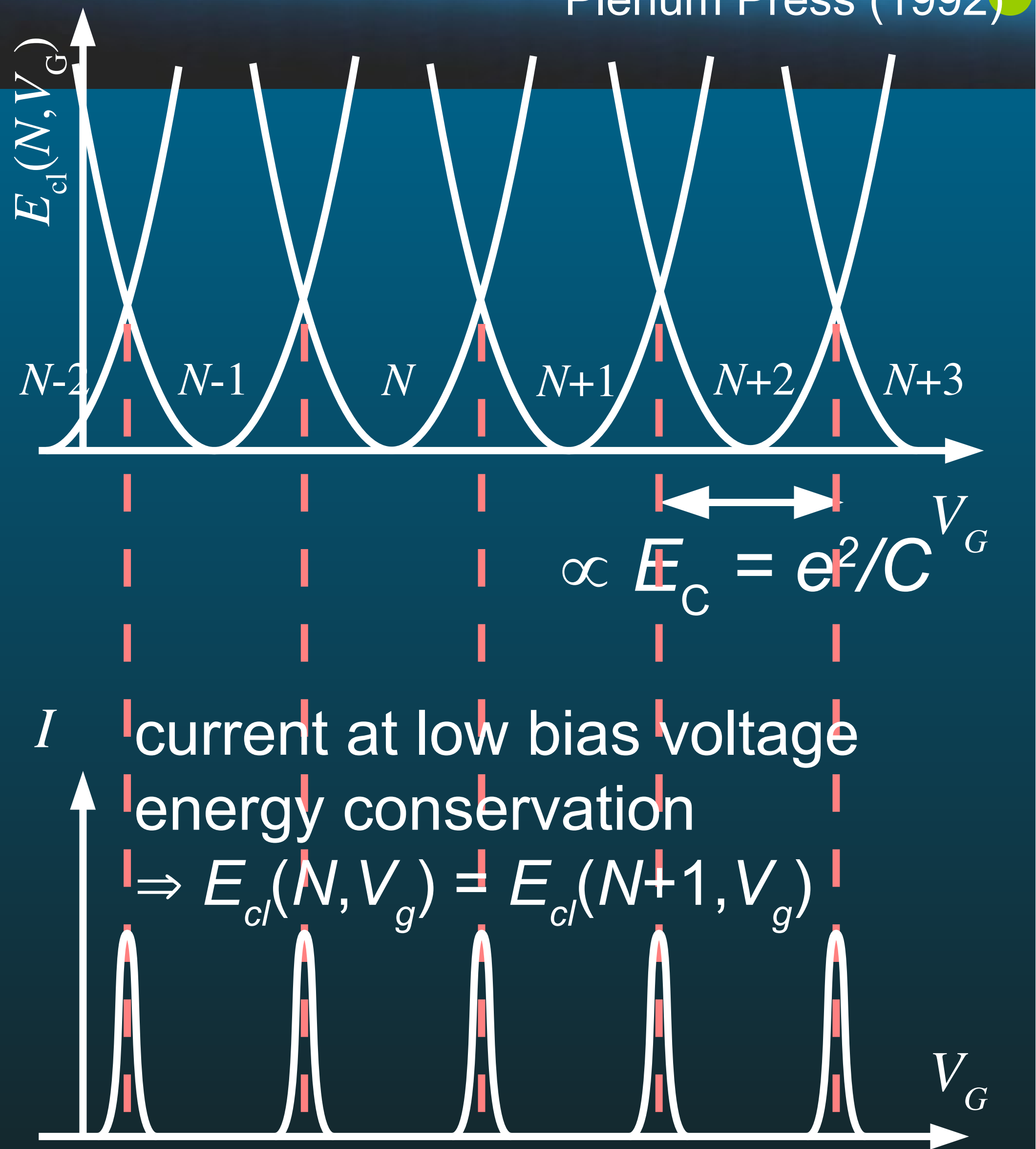
# Single electron transistor (SET)

review: *Single Charge Tunneling*, ed. Graber & Devoret, Plenum Press (1992)



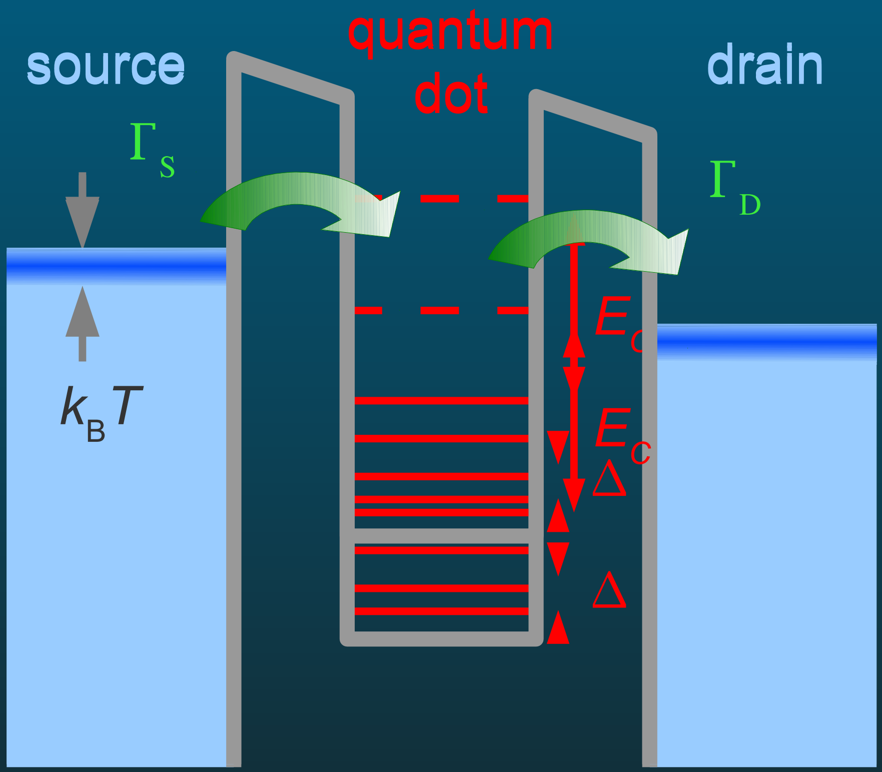



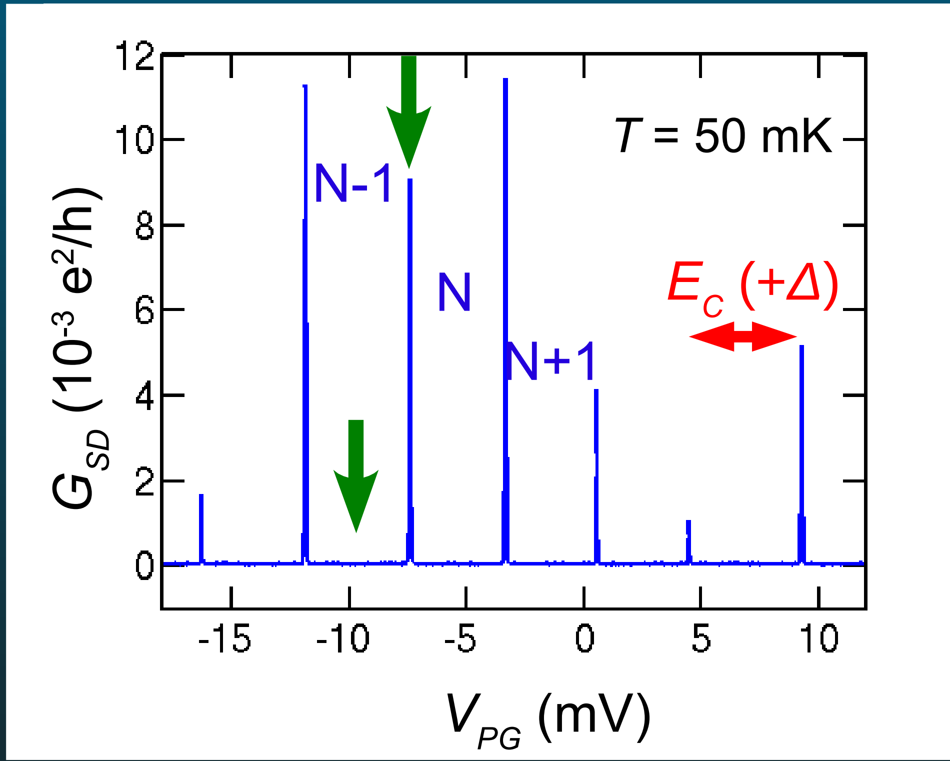

$$E_{cl}(N, V_g) \approx \frac{(Ne - C_g V_g)^2}{2C}$$



# Transport in quantum dots

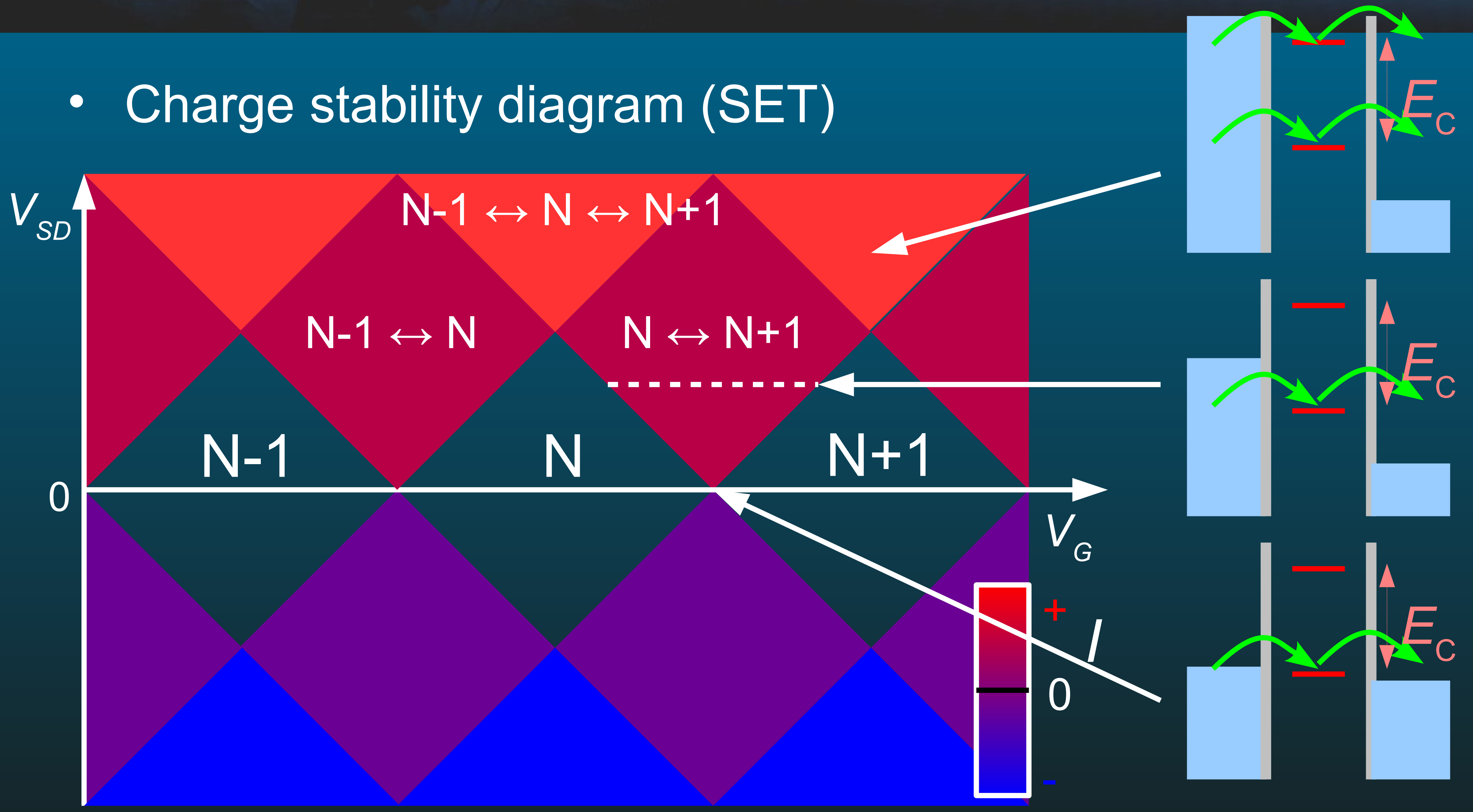
$$k_B T \ll \Delta < E_C$$



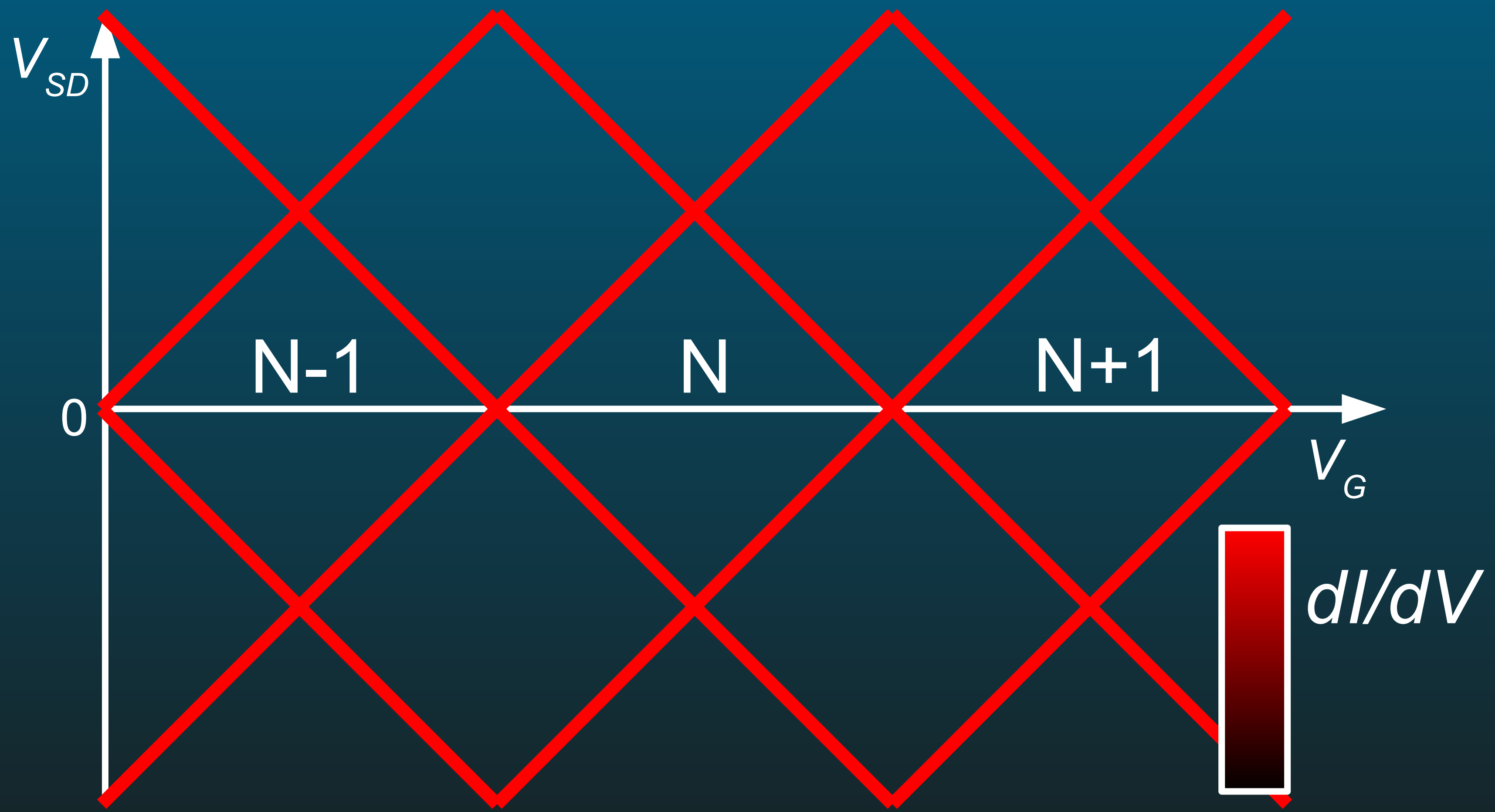
# High bias spectroscopy

- Charge stability diagram (SET)



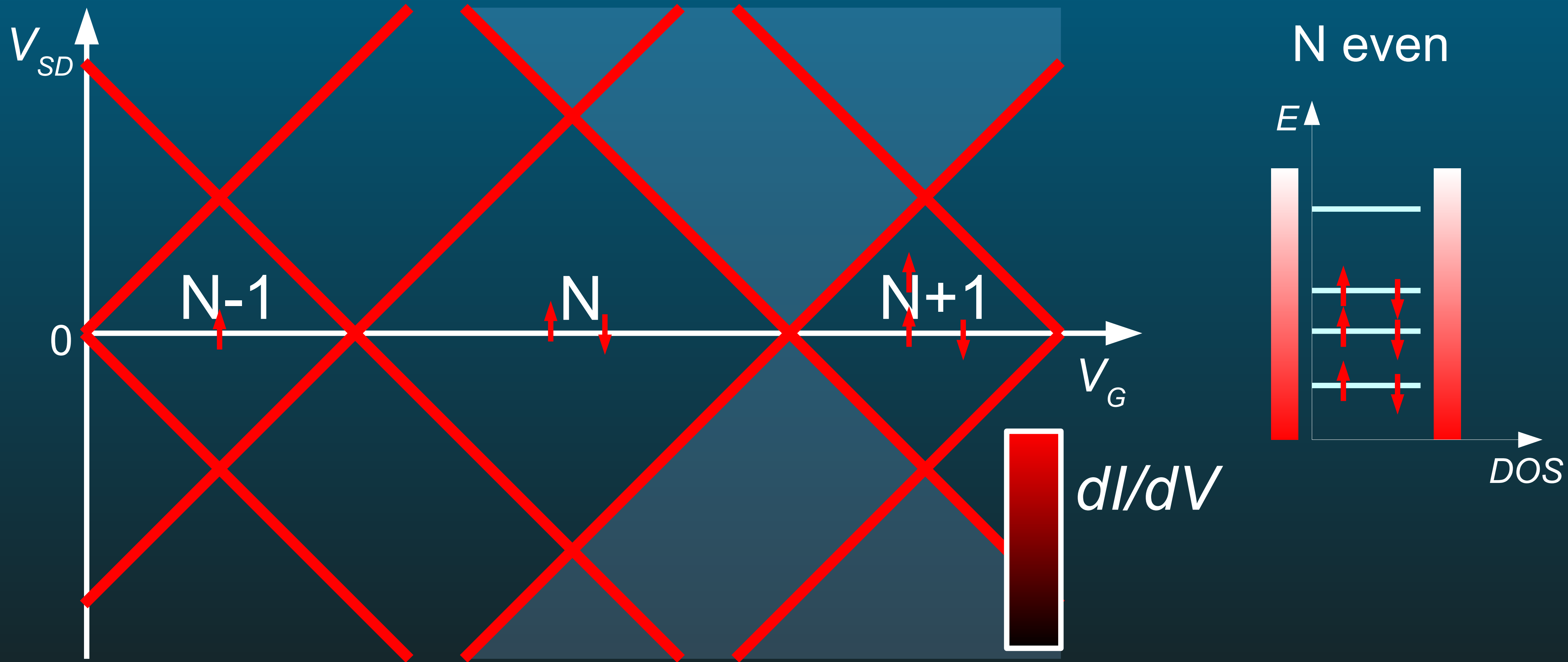
# High bias spectroscopy

- Charge stability diagram (SET)



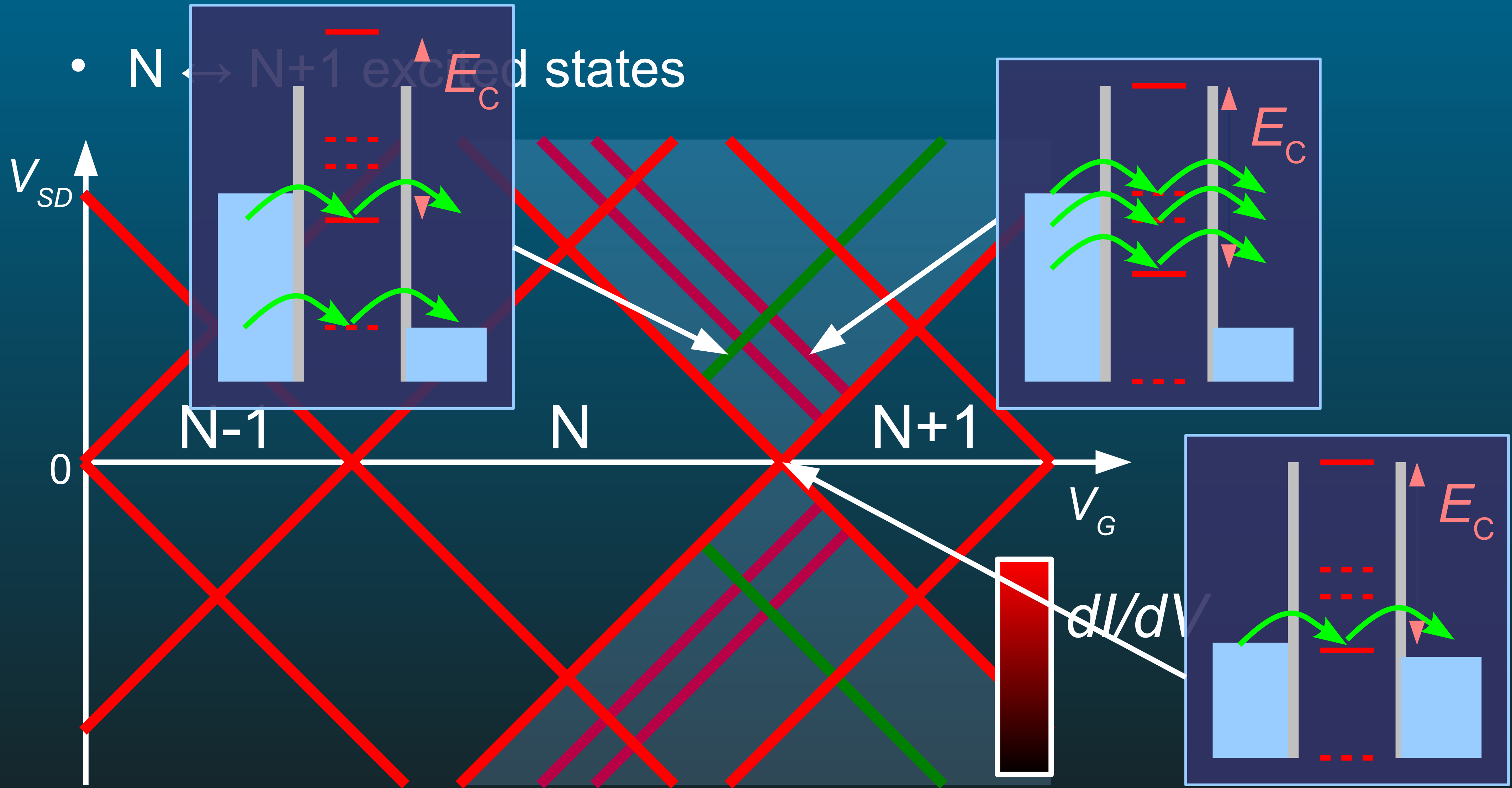
# High bias spectroscopy

- Charge stability diagram (QD): spin filling



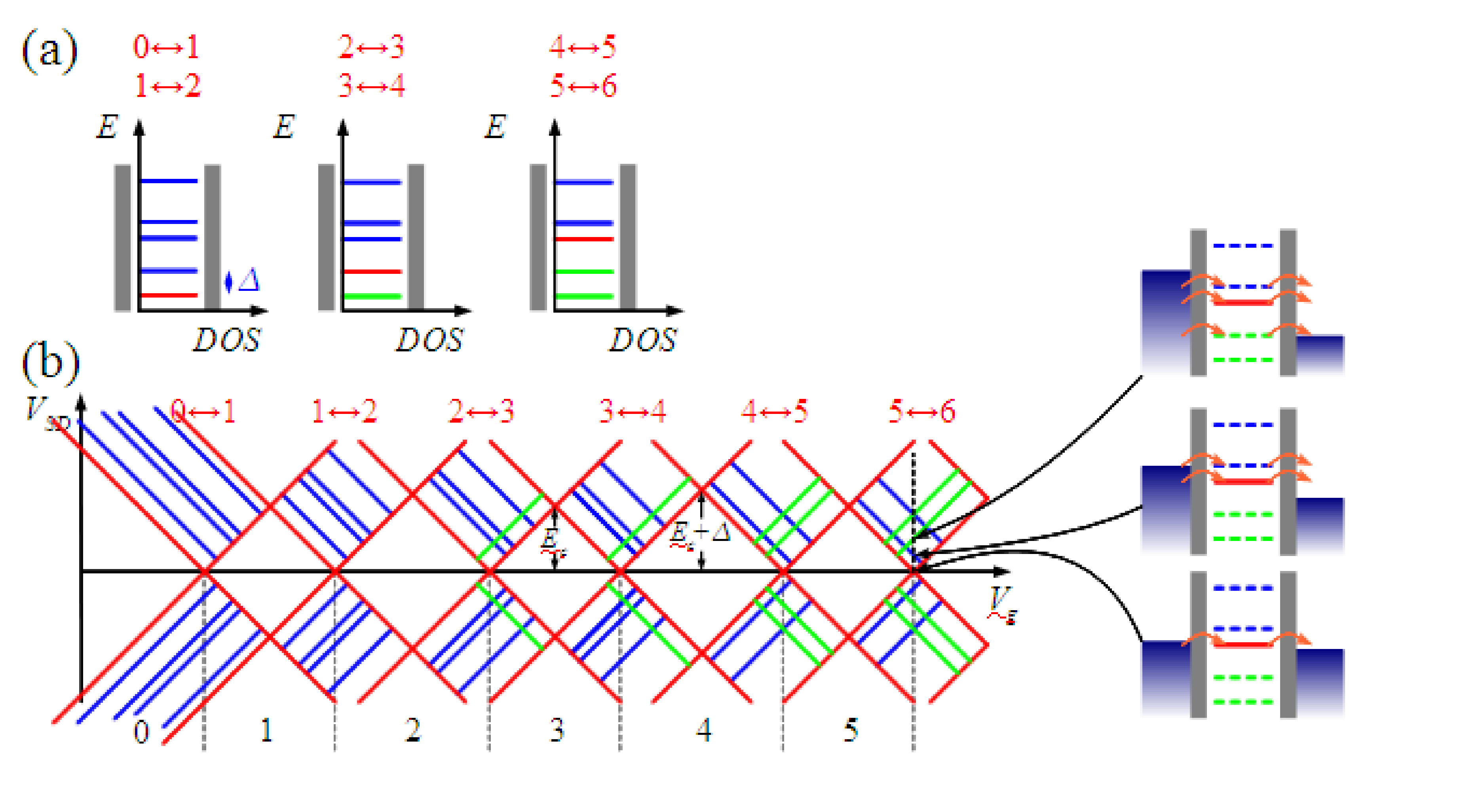
# High bias spectroscopy

- N ↔ N+1 excited states



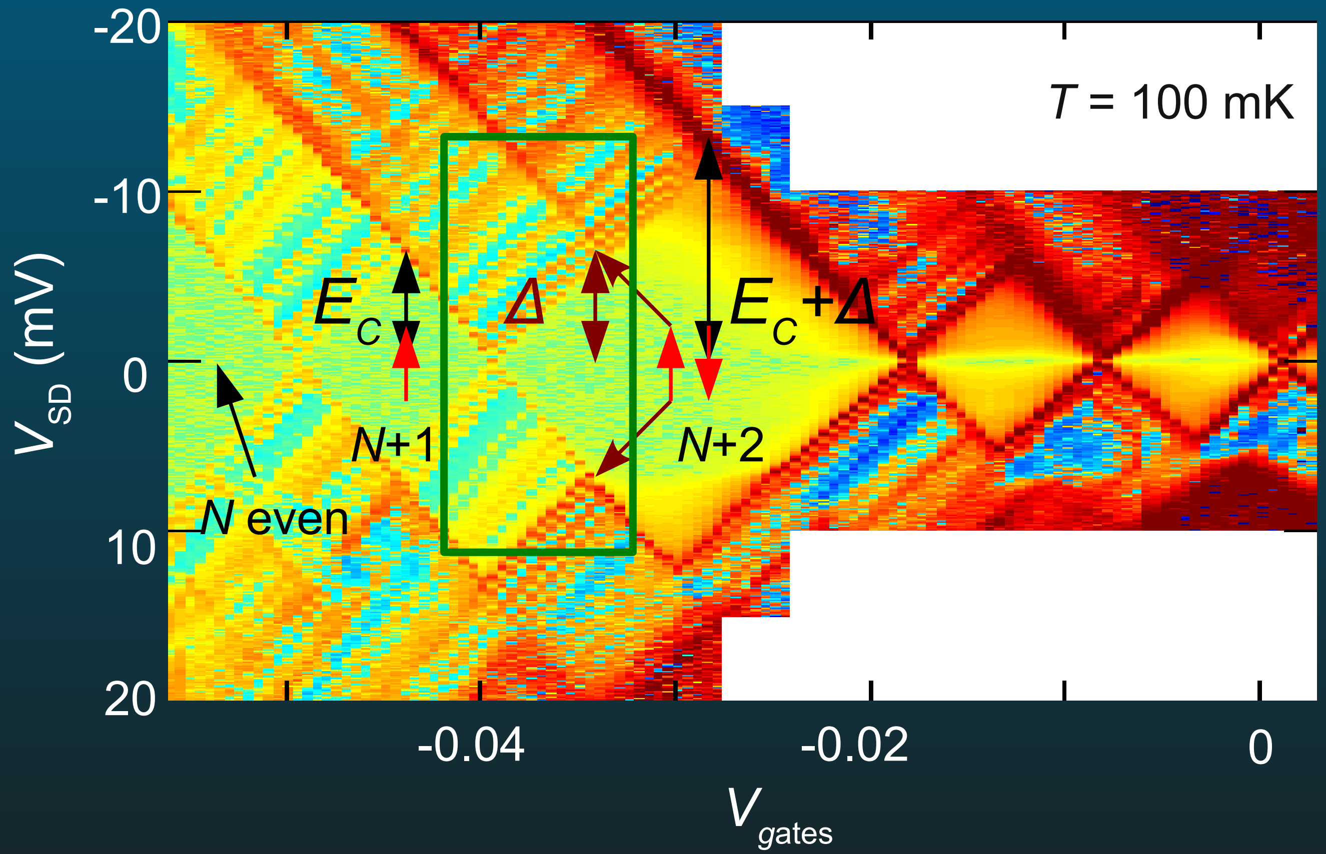
# High bias spectroscopy

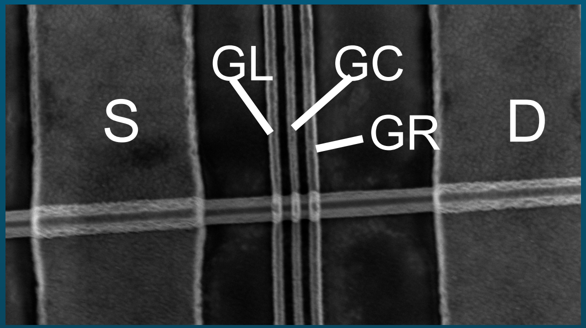
- Constant interaction model: $E_c$ independent of $N$

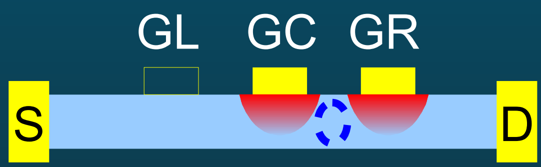


# High bias spectroscopy

- Spectroscopy of an InAs nanowire QD



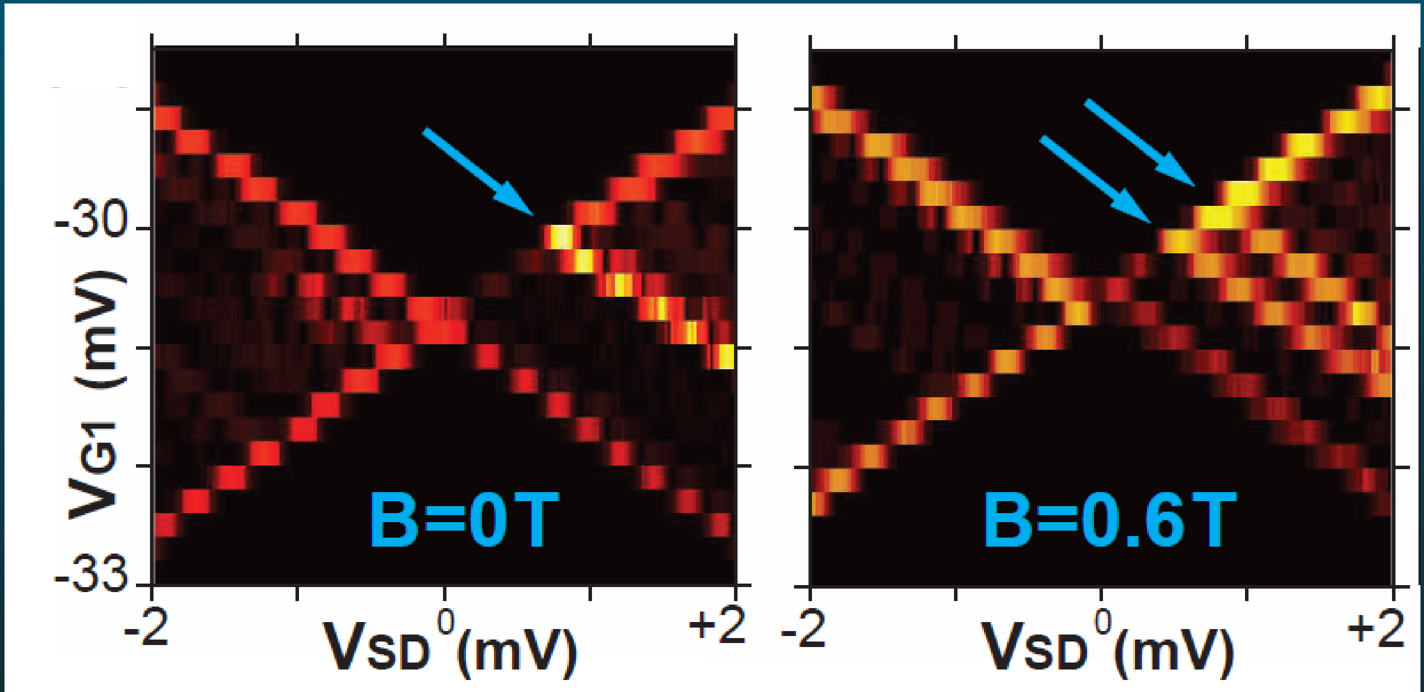




$E_C \approx \Delta \approx 6$ meV gives a QD radius of 20 nm

# Spin spectroscopy

- At high magnetic field: splitting of the degenerate spin states → can be used as a spin filter




$$\Delta E_Z = g^* \mu_B B$$

$|g^*| = 5.5$, due to quantum confinement (bulk InAs, $|g^*| = 15$)

see also: R. Hanson *et al.*, Phys. Rev. Lett. **91**, 196802 (2003)

# Signature of spin-orbit interaction

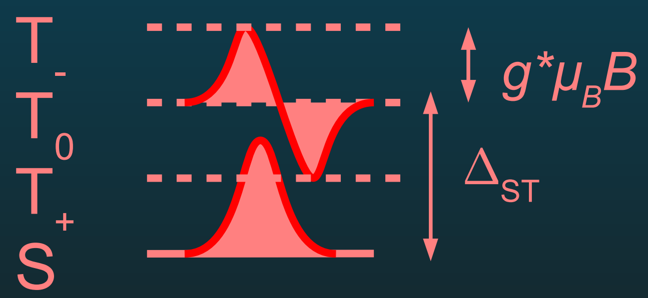
- Spin-orbit Hamiltonien: coupling of the spin and orbital degrees of freedom

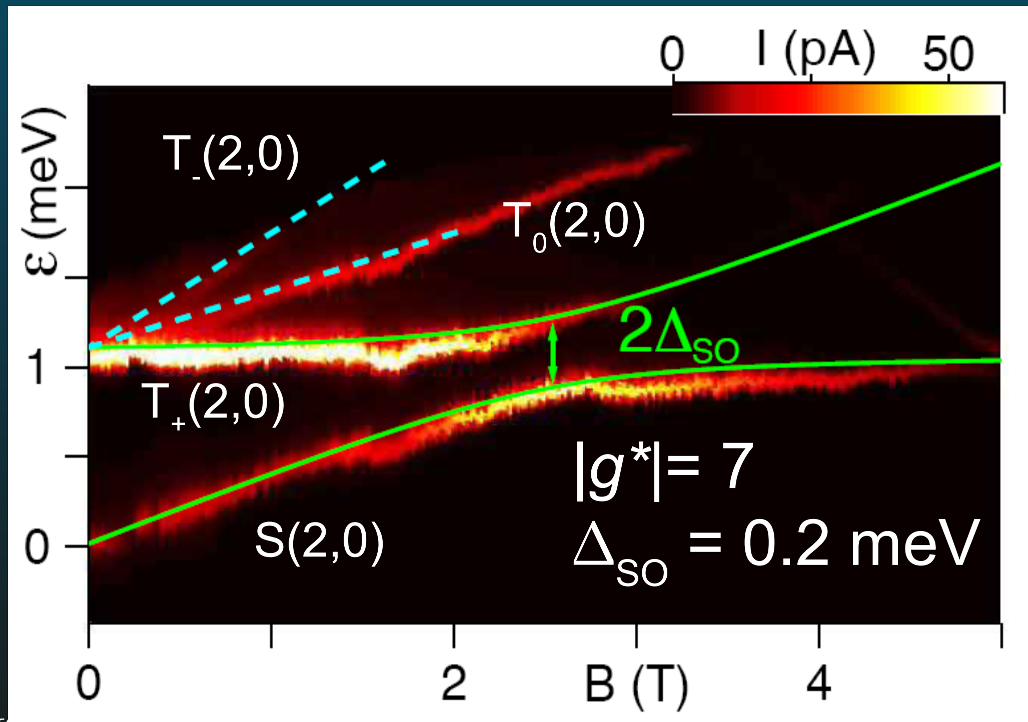
$$H_{SO} = -\mu_B \boldsymbol{\sigma} \cdot \left( \frac{\boldsymbol{p} \times \boldsymbol{E}}{2mc^2} \right) = -\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}_{eff}$$

- Mixing of spin states in InAs quantum dots

C. Fasth *et al.*, PRL **98**, 266801 (2007)
A. Pfund *et al.*,PRB **76**, 161308(R) (2007)

2 electrons states



# Single electron transport mechanisms

- Sequential tunneling model
  - master equation approach
    Beenakker, Phys. Rev. B **44**, 1646 (1991)

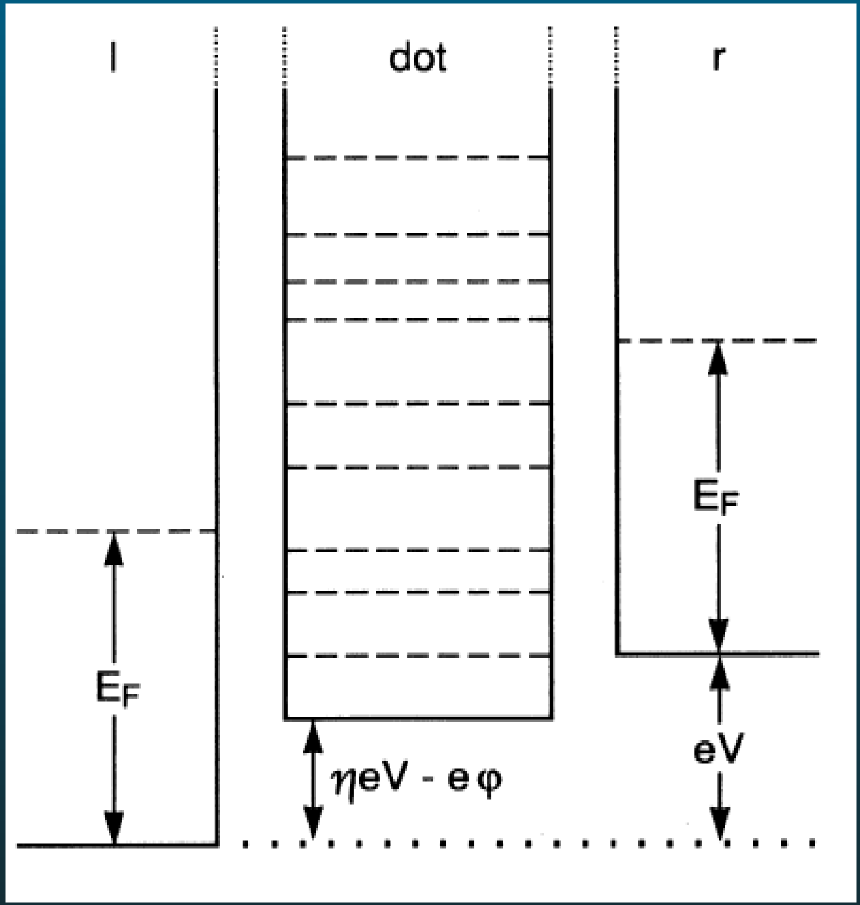
$$\frac{d}{dt}\left| p,t \right\rangle = -\hat{L}\left| p(t) \right\rangle$$

$p_n$ = probability to find the system in a state n

$$L_{mn} = \delta_{n,m}\gamma_n - \Gamma_{m\leftarrow n}$$

$$\gamma_n = \sum_{m\neq n} \Gamma_{m\leftarrow n}$$

$\Gamma_n$ = transition rate from state n to state m

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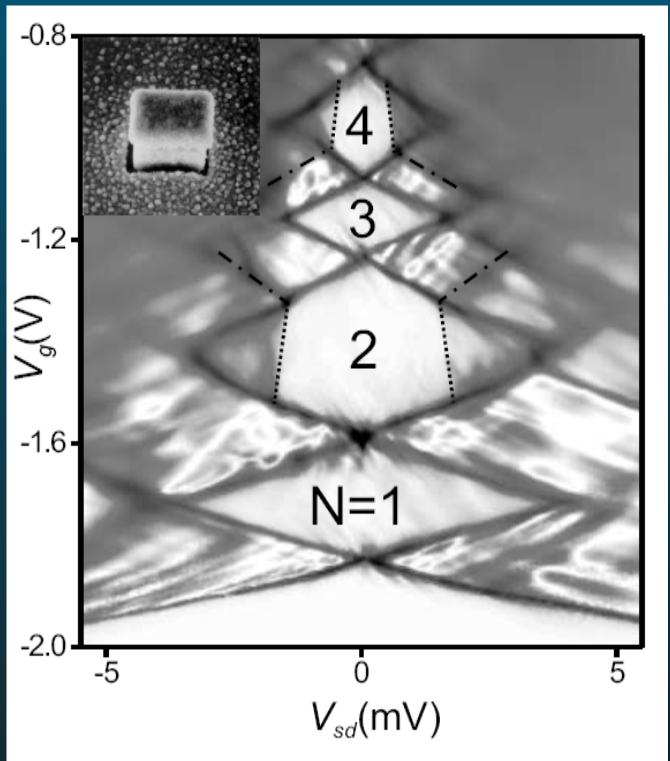
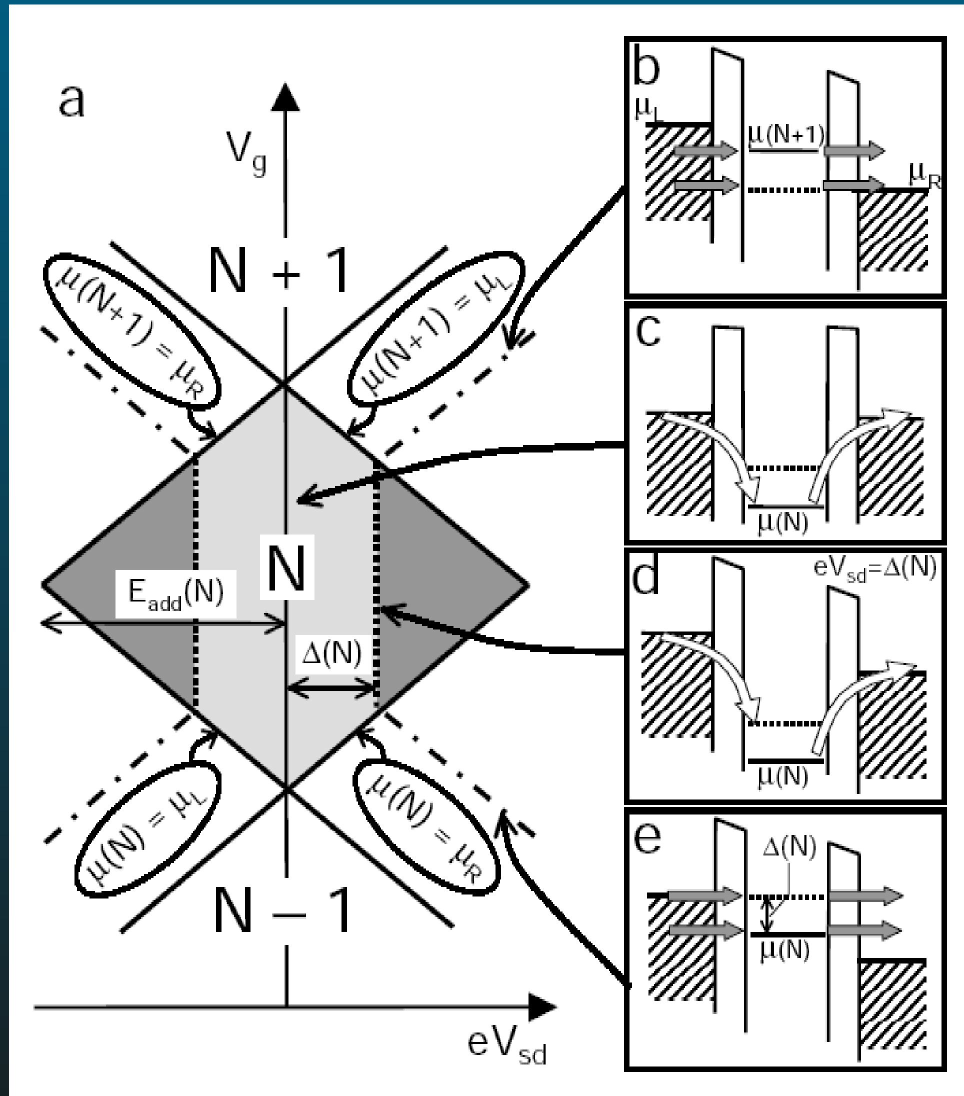
# Single electron transport mechanisms

- Higher order processes:
  - elastic and inelastic cotunneling




S. de Franceschi *et al.*, PRL 86, 878 (2001)



# Take-away message (2)

Transport experiment can probe the quantum structure of the quantum dot

electron and spin states...

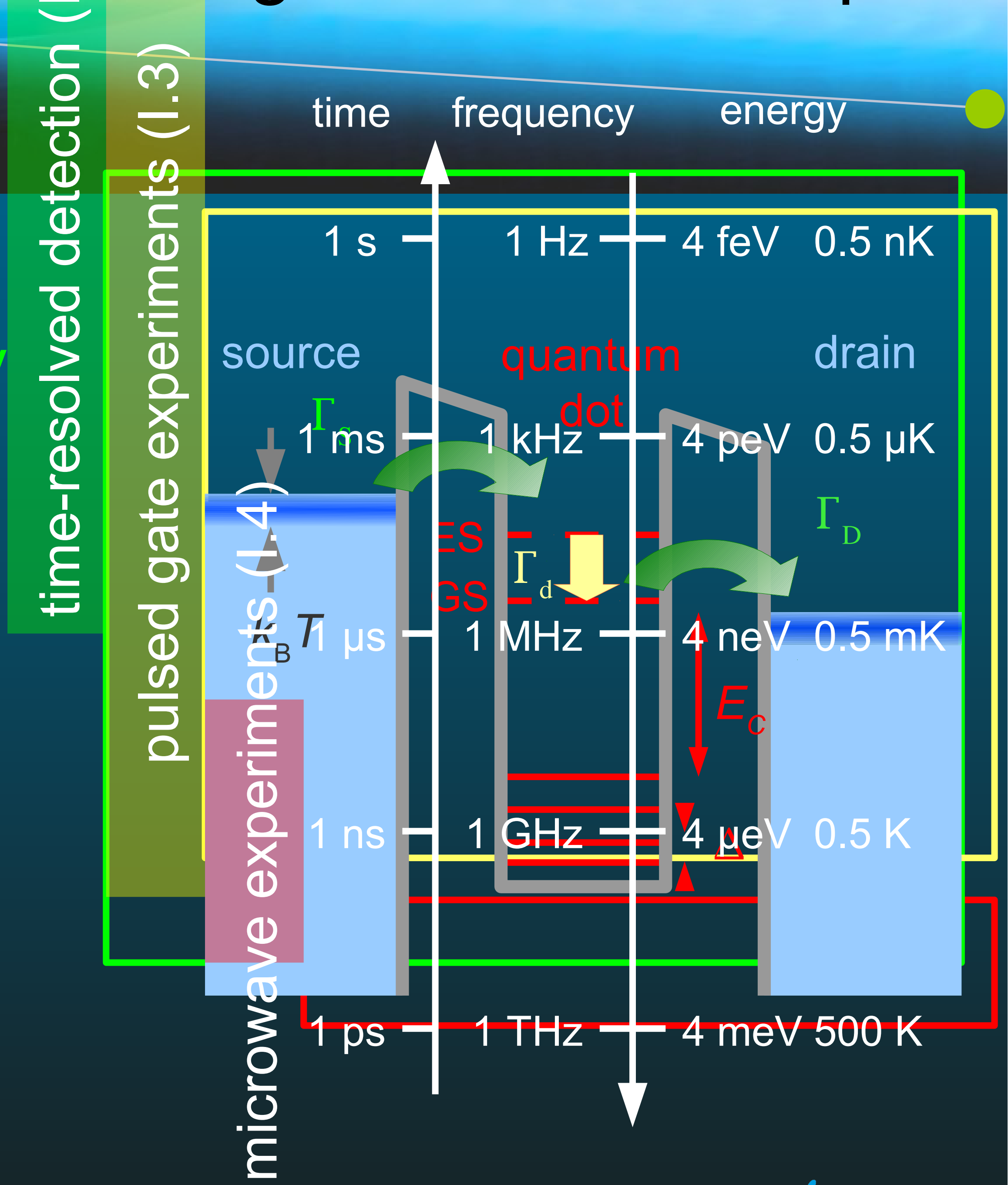
… assuming the constant interaction model !

transport mechanism via sequential  co-tunneling

next: can we access the transport time-scales?

# Time scales for single electron transport

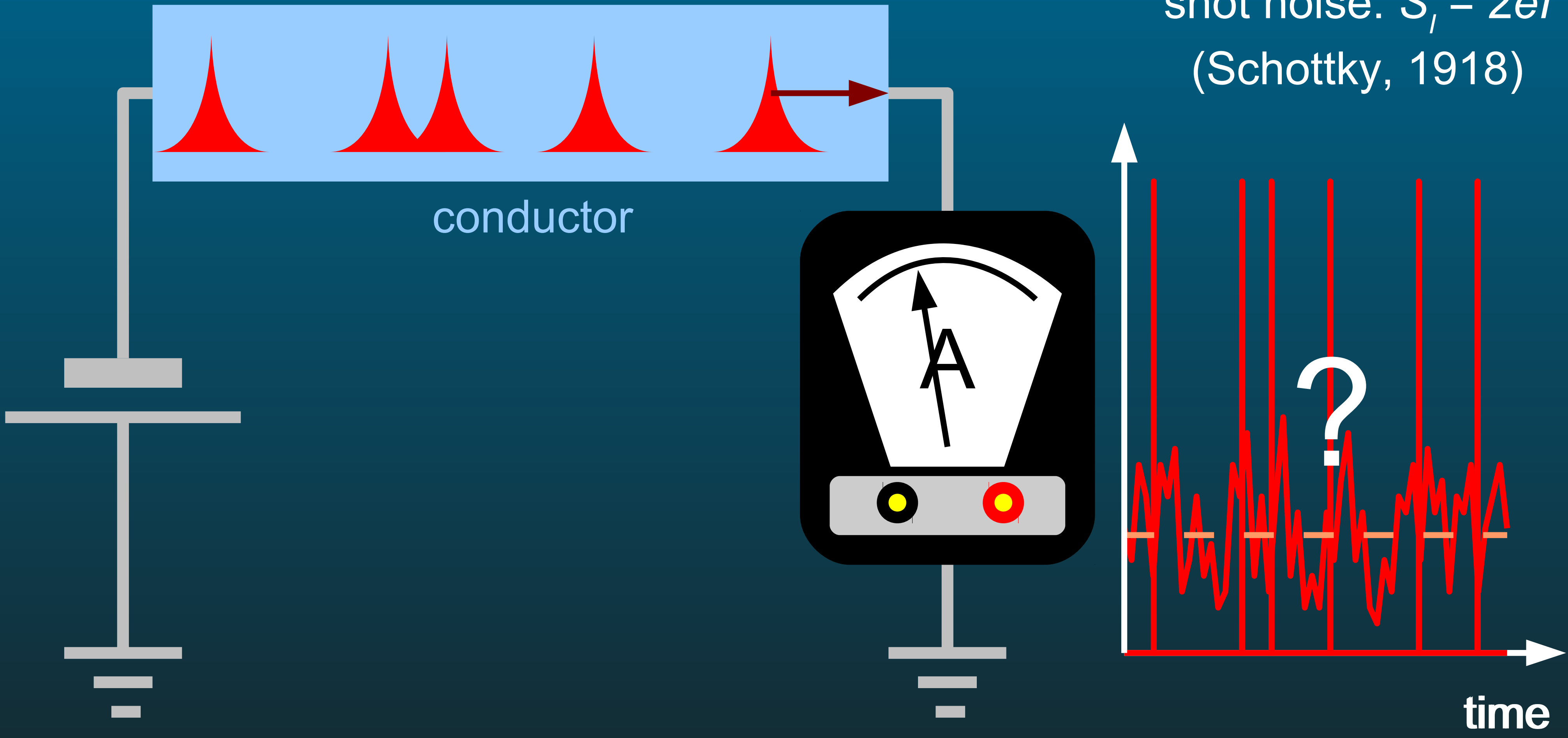
- Inverse tunneling rates
  $1/\Gamma_S$, $1/\Gamma_D$ = 10 ps – infinity
  - time scale for a trapped electron to escape
- Charge or spin decay time
  $1/\Gamma_d$ = few ns – 1 second
  - coherent manipulation
- $h/E_C$, $h/\Delta$ = 1 – 100 ps
  - non-adiabatic transistion



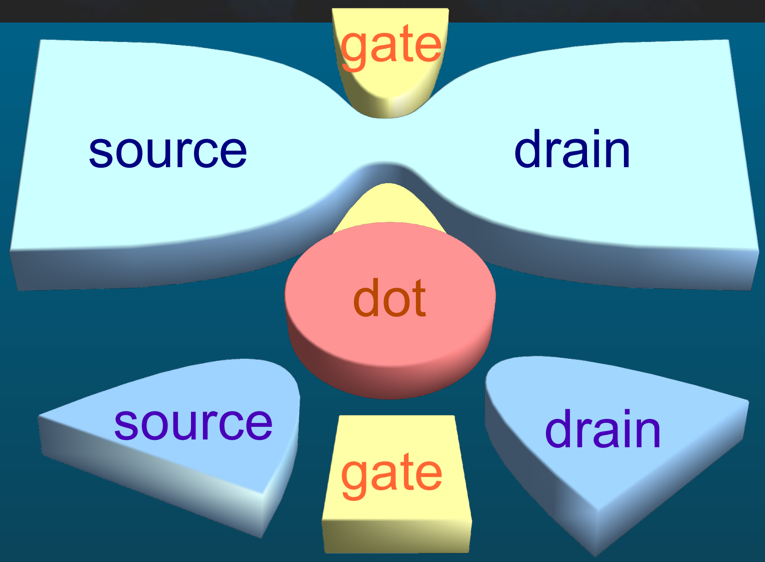
# 2. Time-resolved single electron detection

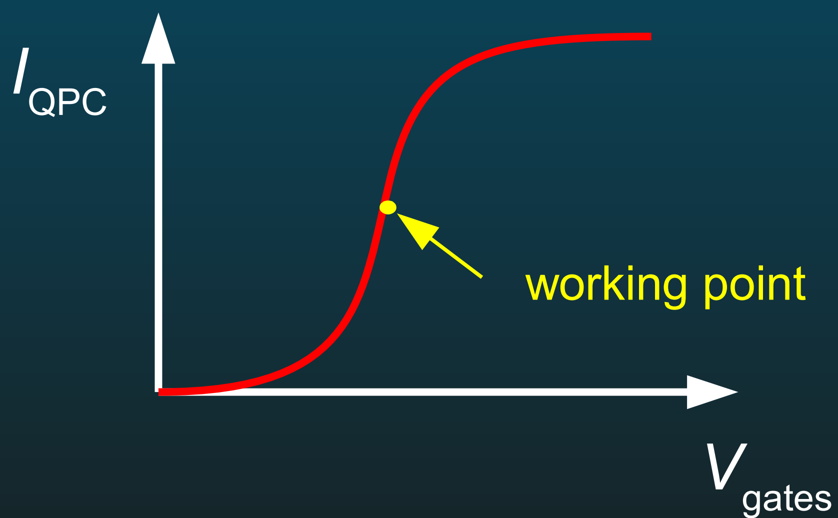
shot noise: $S_I = 2eI$

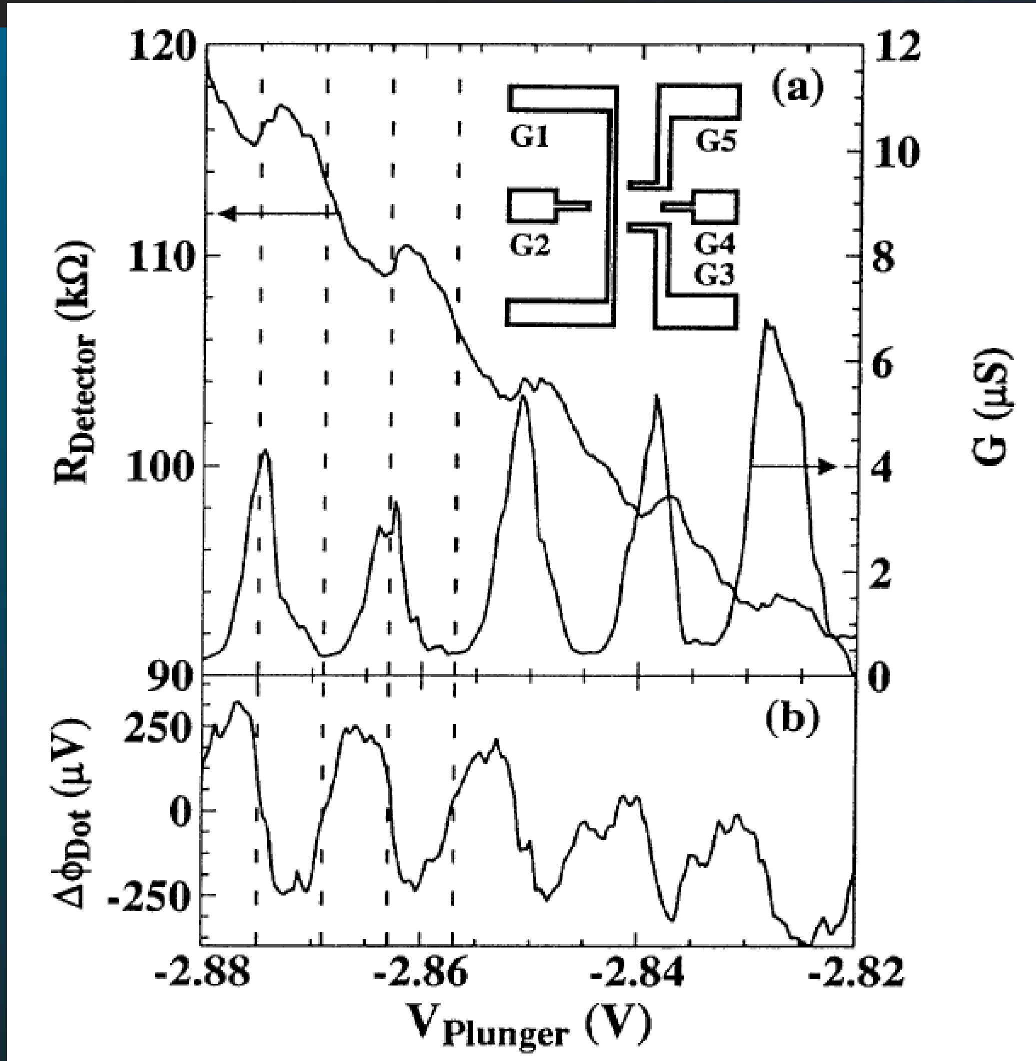
(Schottky, 1918)



# Single charge detection with a quantum point contact




M. Field *et al.*, PRL **70,** 1311 (1993)

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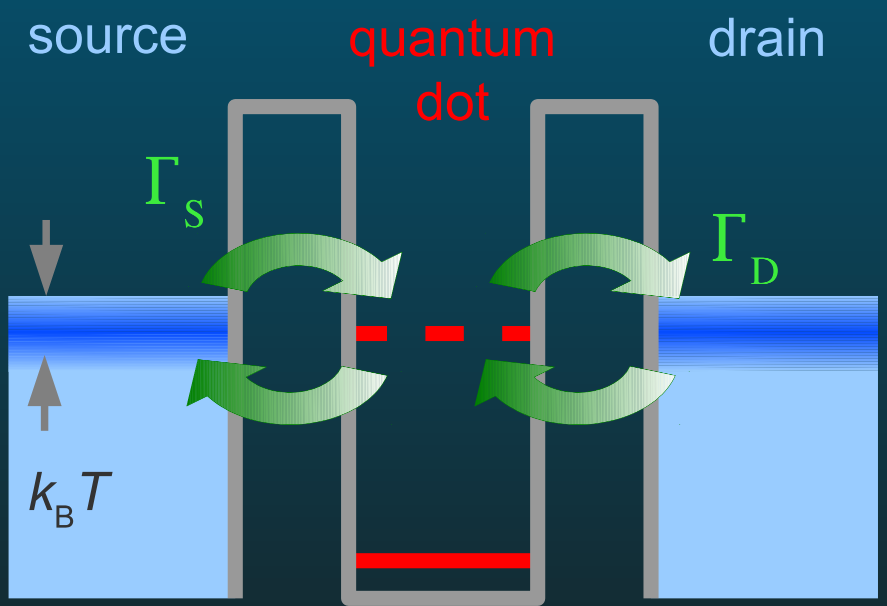
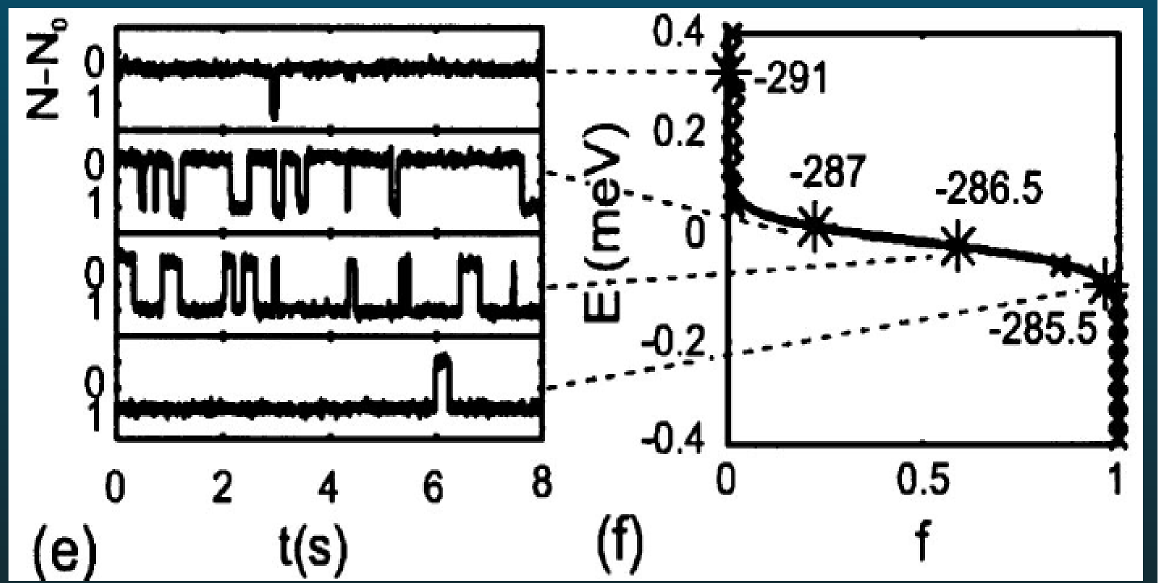
# Time-resolved single electron detection

- Thermal fluctuations between leads and dot

  W. Lu *et al.*, Nature **423**, 422 (2003)
  R. Schleser *et al.*, APL **85,** 2005 (2004)
  L. Vandersypen *et al.*, APL **85**, 4394 (2004)

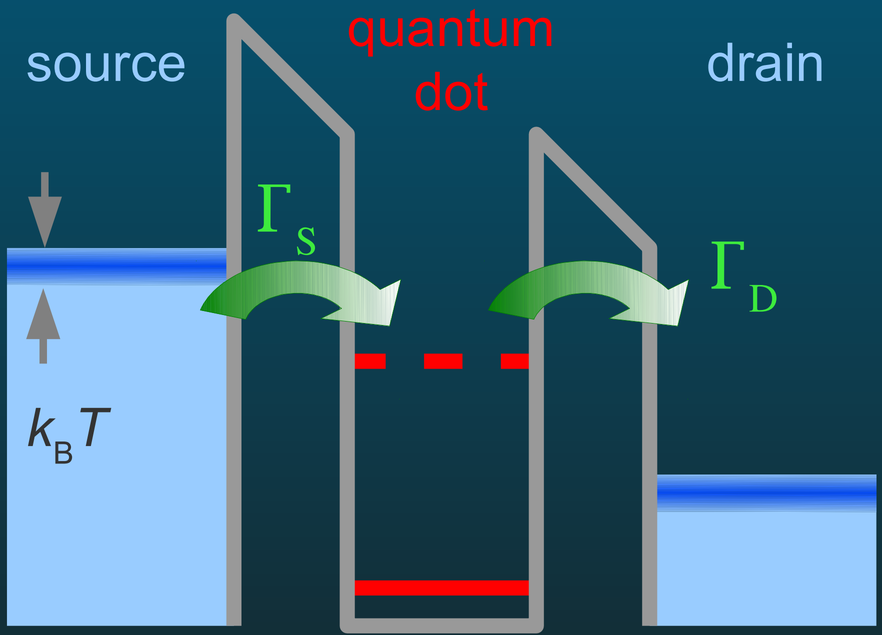


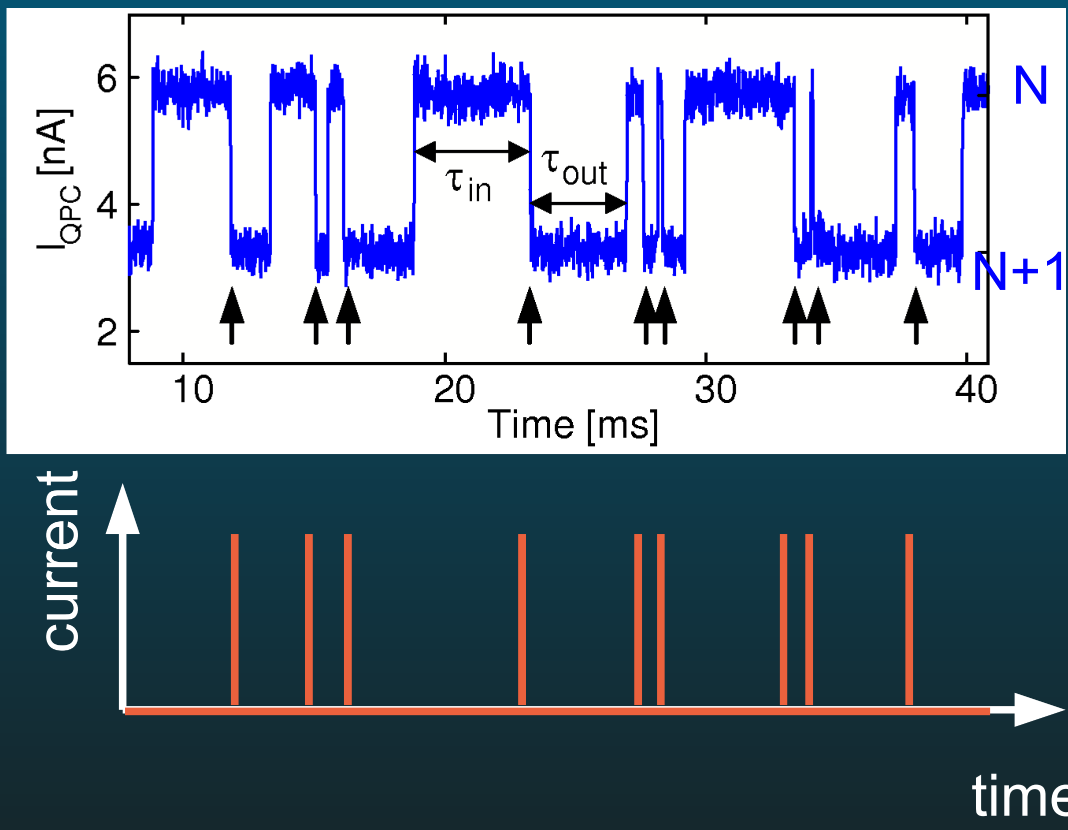




R. Scheser *et al.*, APL **85**, 2005 (2004)

# Time-resolved detection of single electron transport

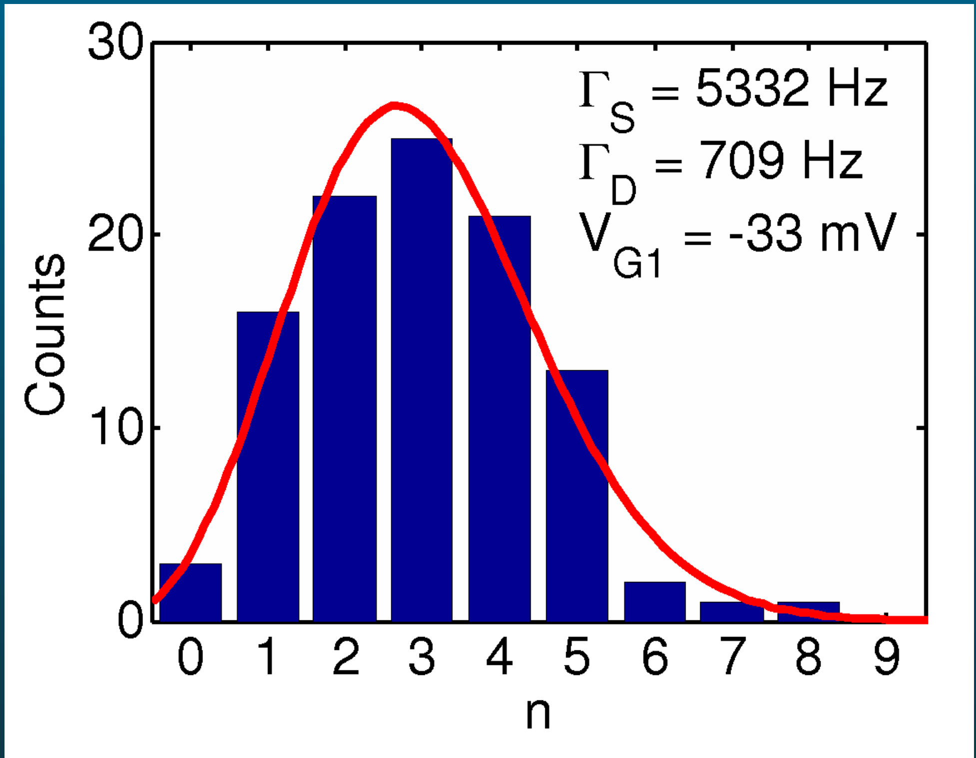
- Large bias voltage ⇒ directional flow
  S. Gustavsson, RL *et al.*, PRL **96,** 076605 (2006)

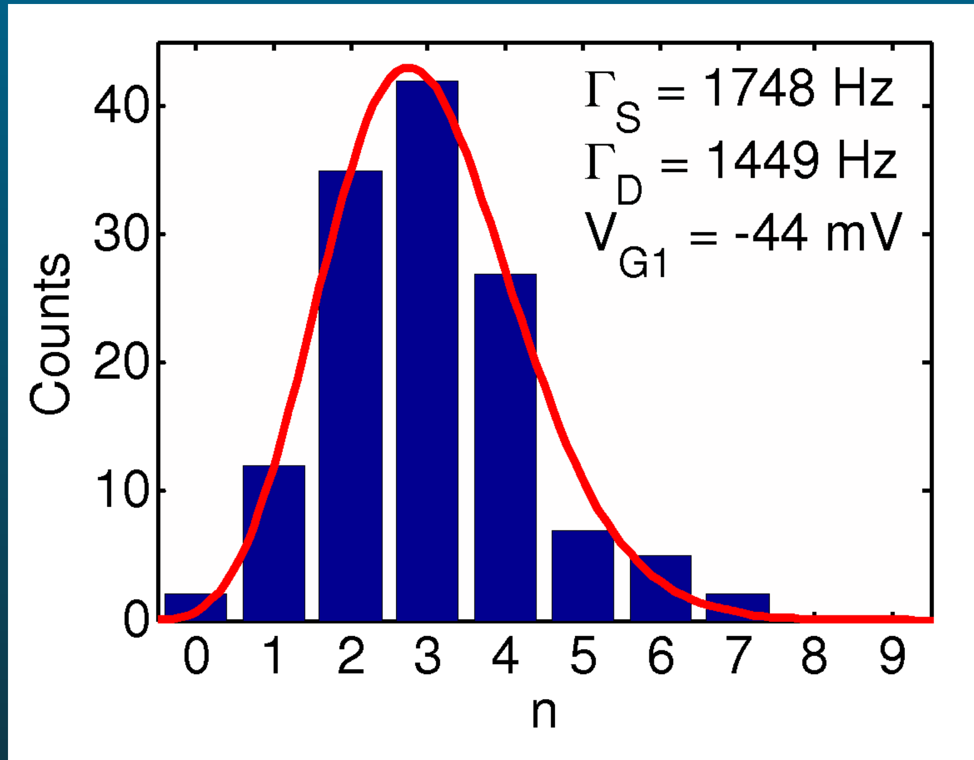


# Histograms of current fluctuations

S. Gustavsson, RL *et al.*, PRL **96,** 076605 (2006)




- Poisson distribution for asymmetric coupling
- Sub-Poisson distribution for symmetric coupling

Theory: Hershfield *et al.*, PRB **47,** 1967 (1993)
Bagrets & Nazarov, PRB **67,** 085316 (2003)

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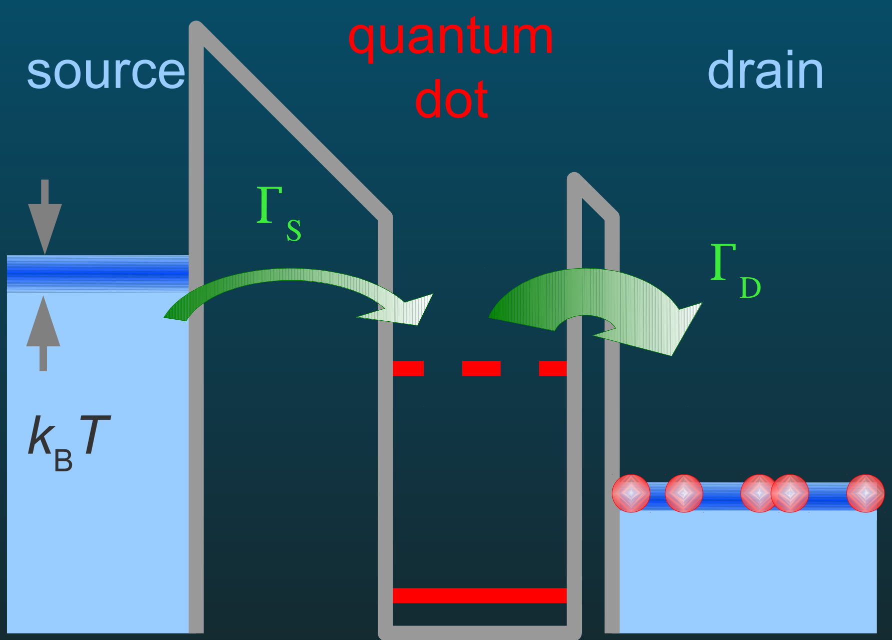
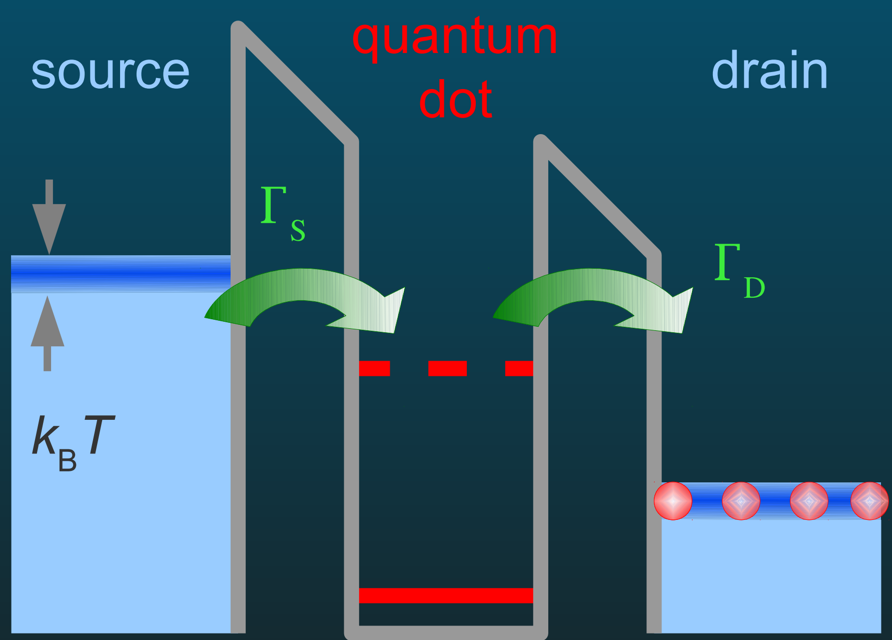
# Histograms of current fluctuations

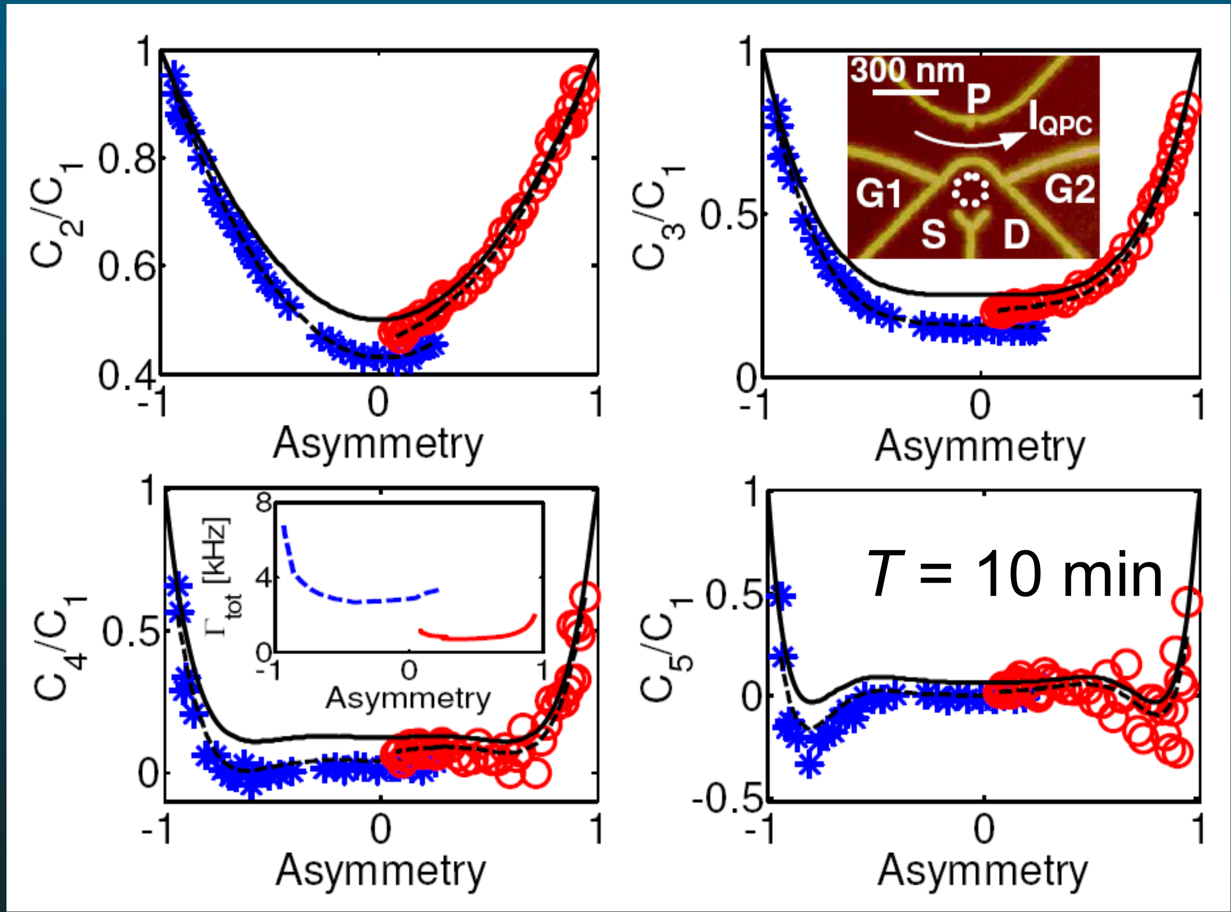
- Asymmetric coupling
  - statistics dominated by the thicker barrier
- Symmetric coupling
  - Coulomb blockade "orders" the electrons



# Experimental measurement of the full counting statistics

- More than noise: access to the full counting statistics (distribution function)
  - $I = e\mu/t_0$,
    $\mu = <n>$
  - $S_I = 2e^2\mu_2/t_0$,
    $\mu_2 = <(n-<n>)^2>$
  - $S_I^3 = e^3\mu_3/t_0$,
    $\mu_3 = <(n-<n>)^3>$
  - and many more...



# take-away message (3)

Real-time measurement of single electron transport in quantum dots

determination of the full counting statistics (current noise)

but still limited to the sequential tunneling regime

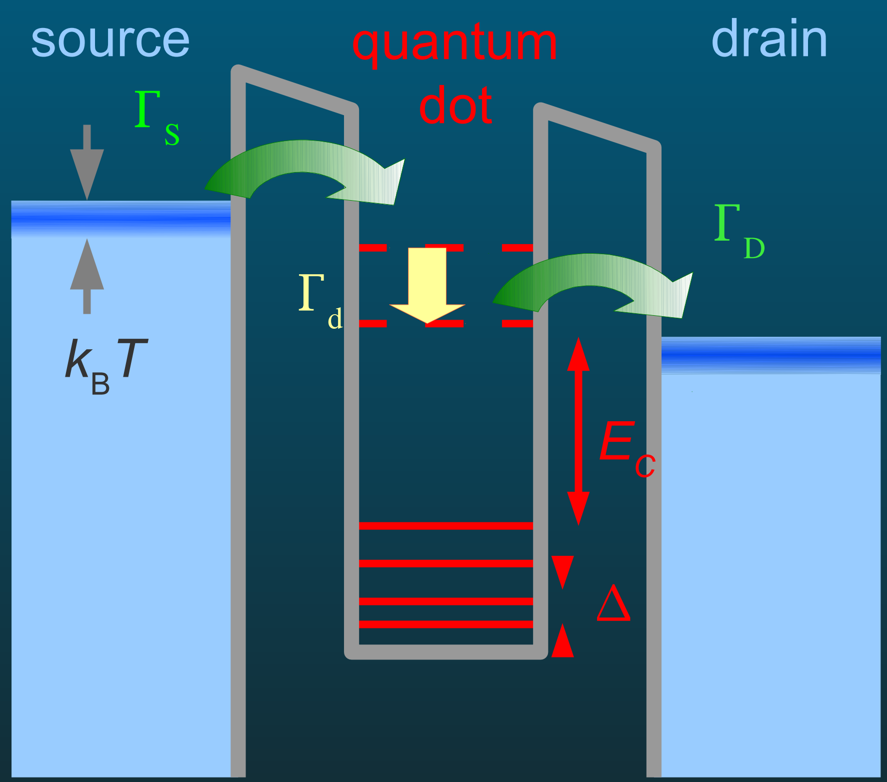
next: manipulating electron states in real time

# 3. Single electron manipulation

- Fast gate sweep
  - rise time: $\tau \sim$ 100 ps – 10 ns
- Adiabatic regime

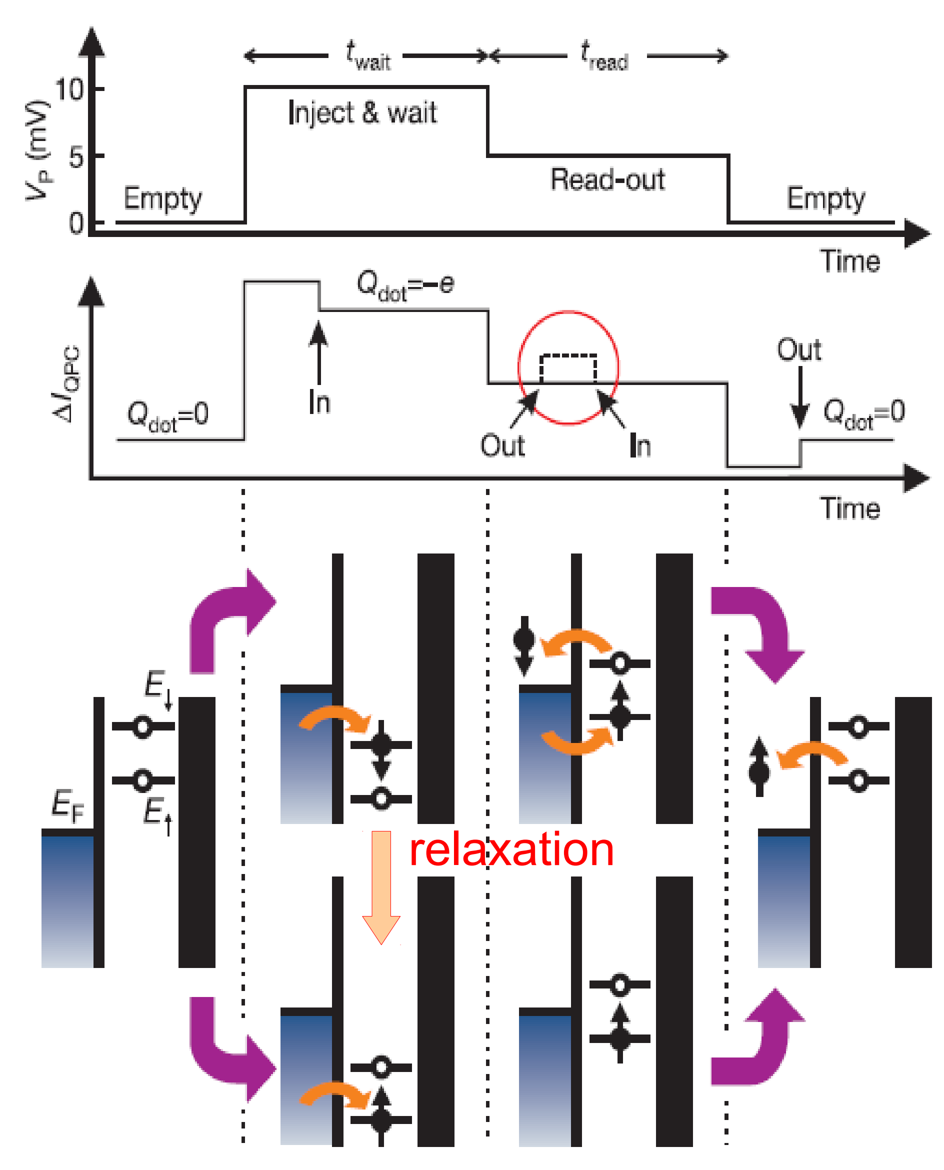
$$\tau \gg h/\Delta$$

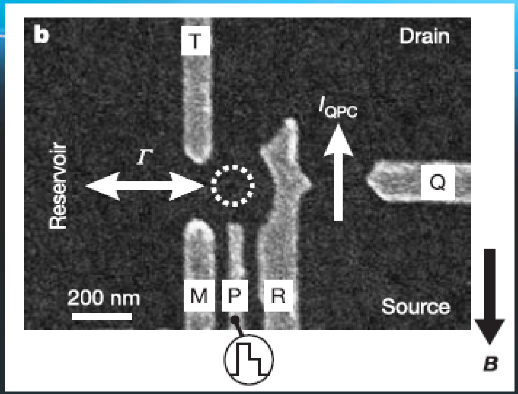
$$\tau < 1/\Gamma_d, 1/\Gamma_S, 1/\Gamma_D$$

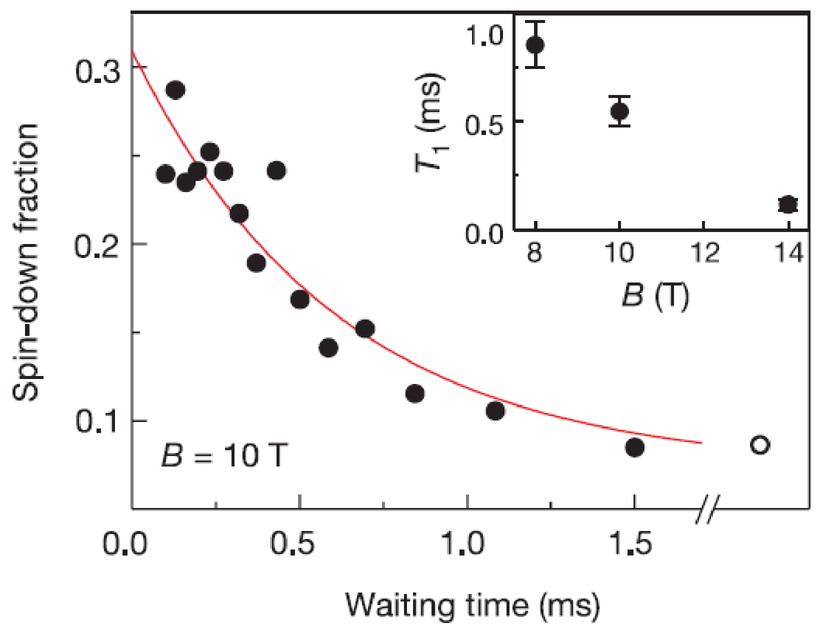


# Measurement of the spin relaxation time

J. Elzerman *et al.*, Nature **430**, 431 (2004)




limited by the spin-orbit interaction

more recently: $T_1$ up to 1 second at 1 Tesla

S. Amasha *et al.*, Phys. Rev. Lett. **100**, 046803 (2008)

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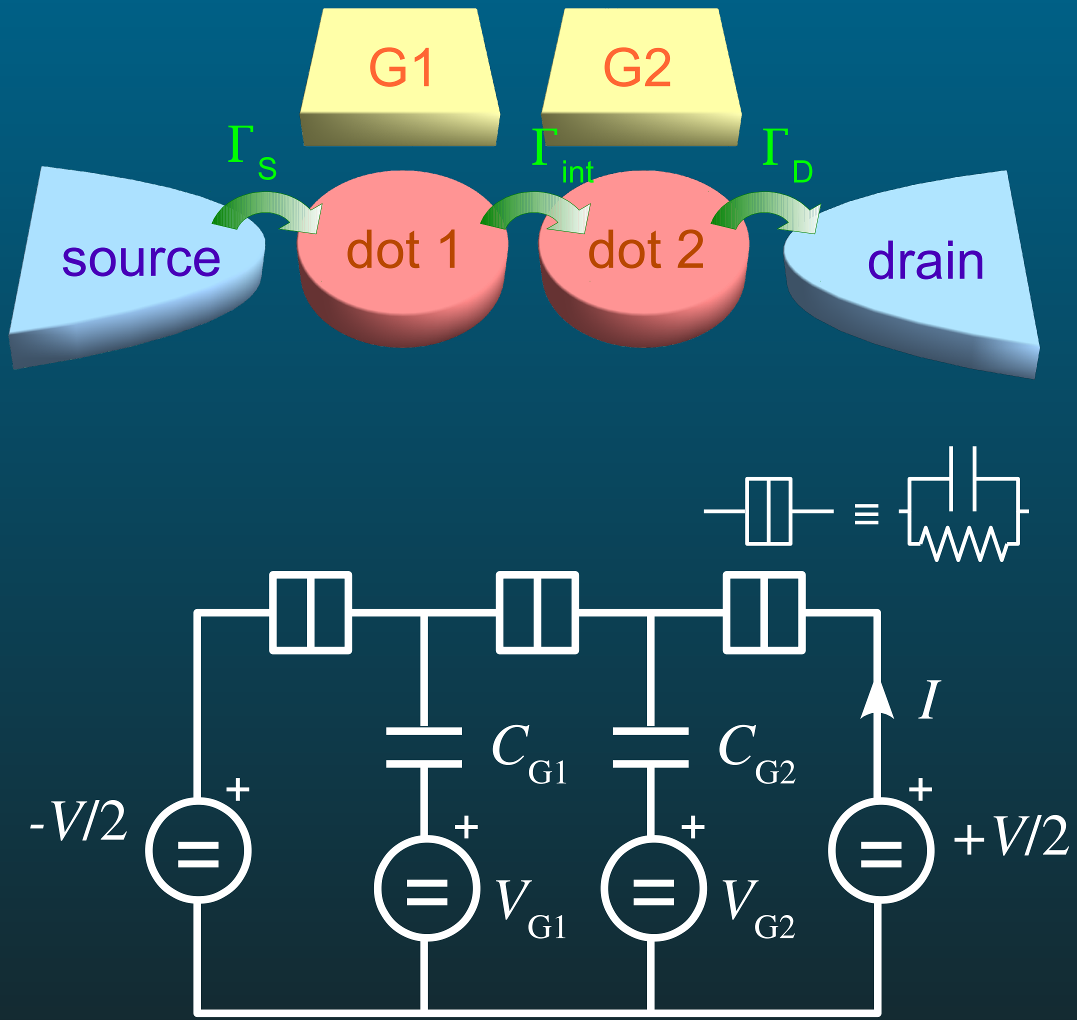
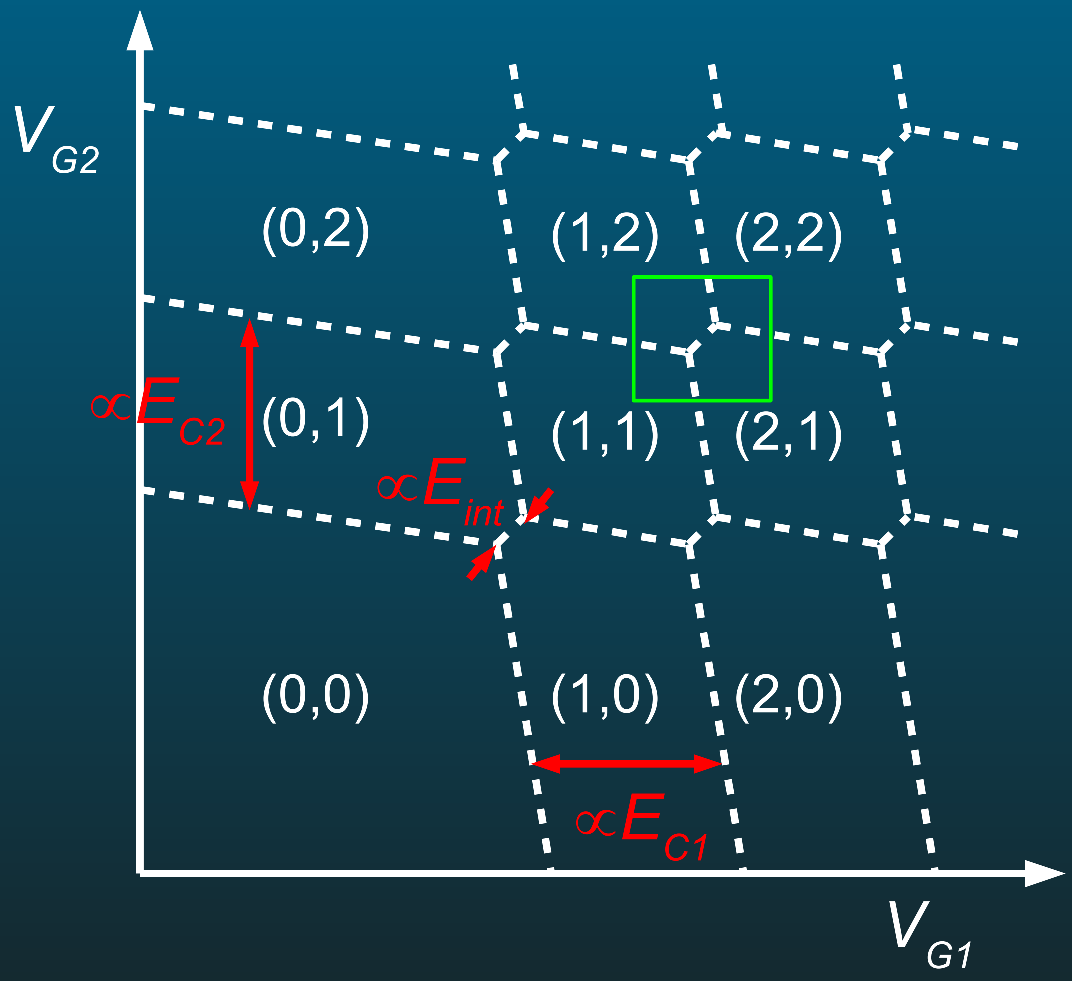
# Double quantum dot

review: W.G. van der Wiel *et al.*, Rev. Mod. Phys. **75**, 1 (2003)

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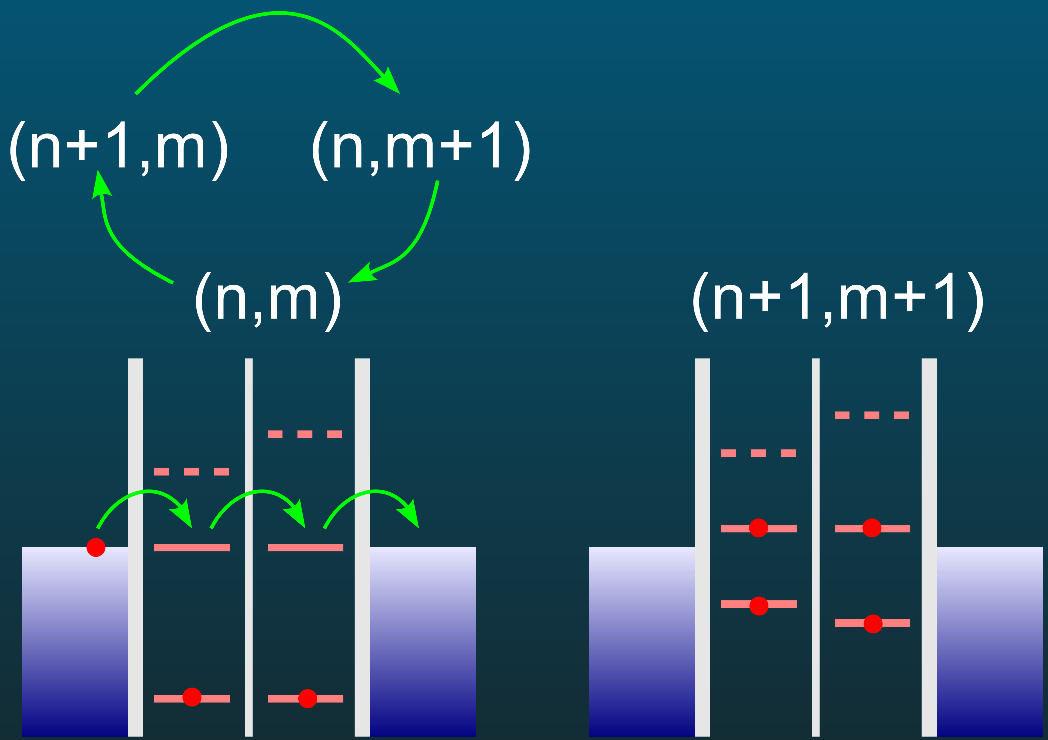
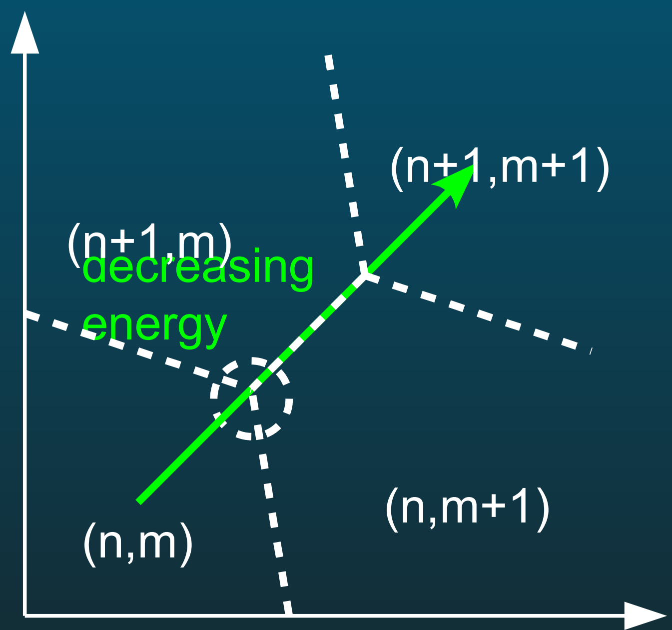
# Electron transport through a double quantum dot

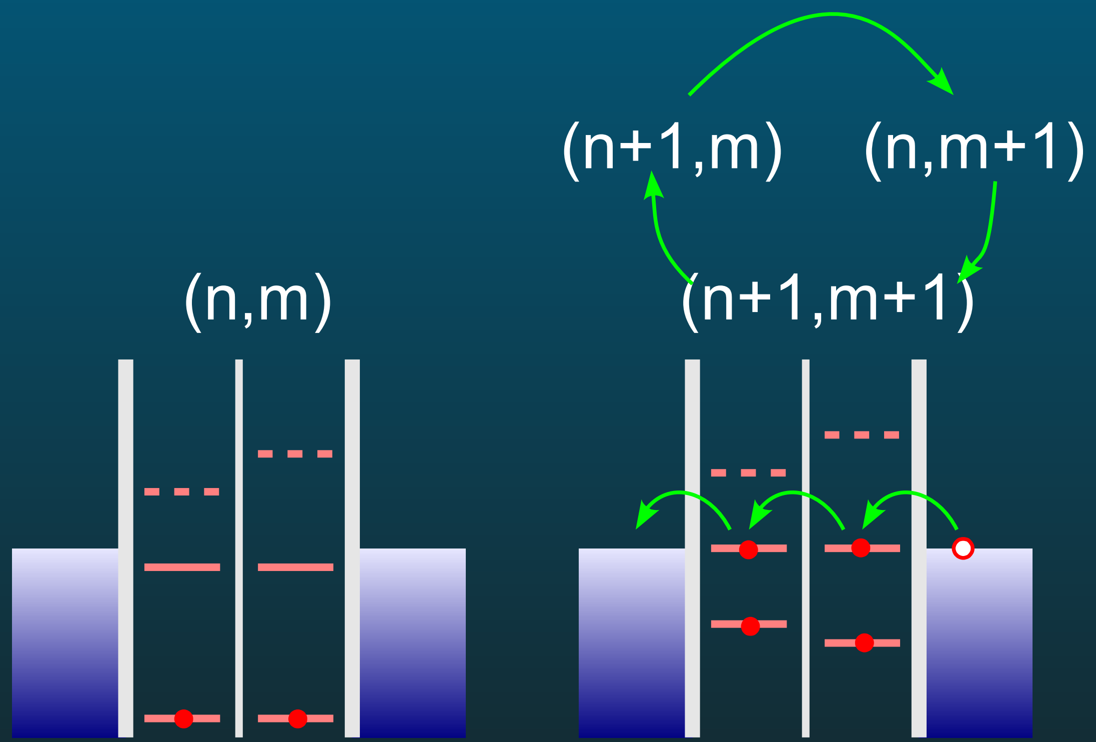
- Small bias voltage: current at degeneracy points
  - electron cycle

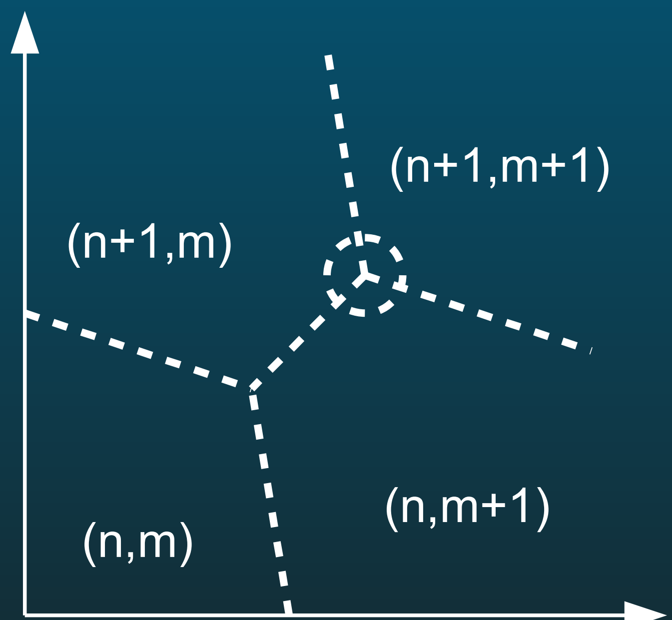


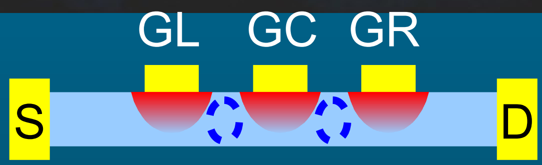
# Electron transport through a double quantum dot

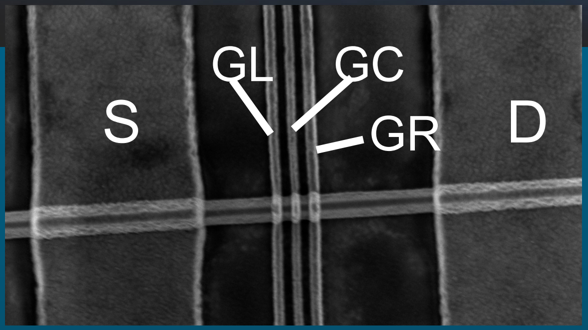
- Small bias voltage: current at degeneracy points
  - hole cycle

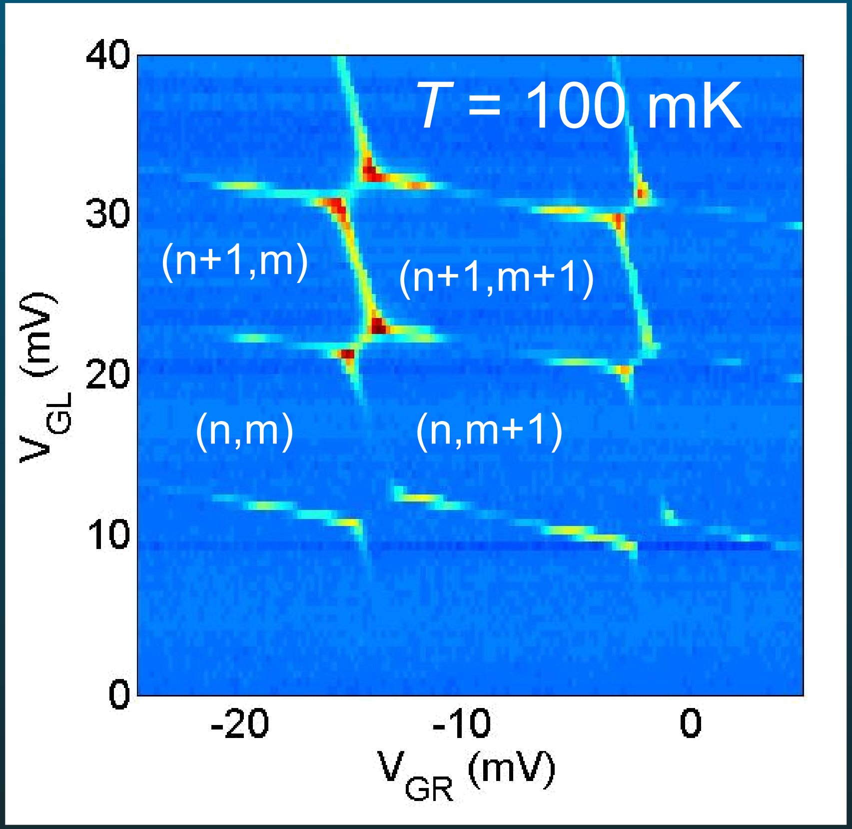


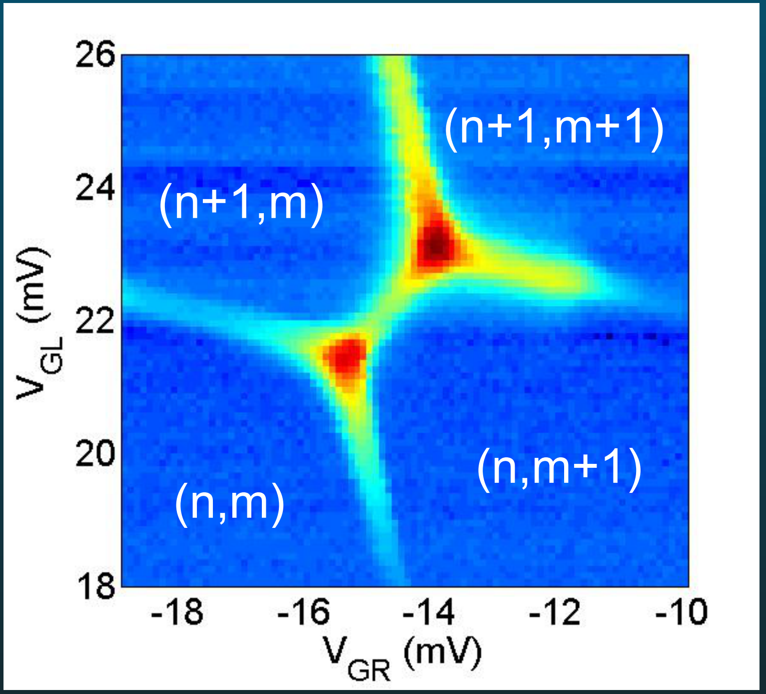



# Electron transport through a double quantum dot



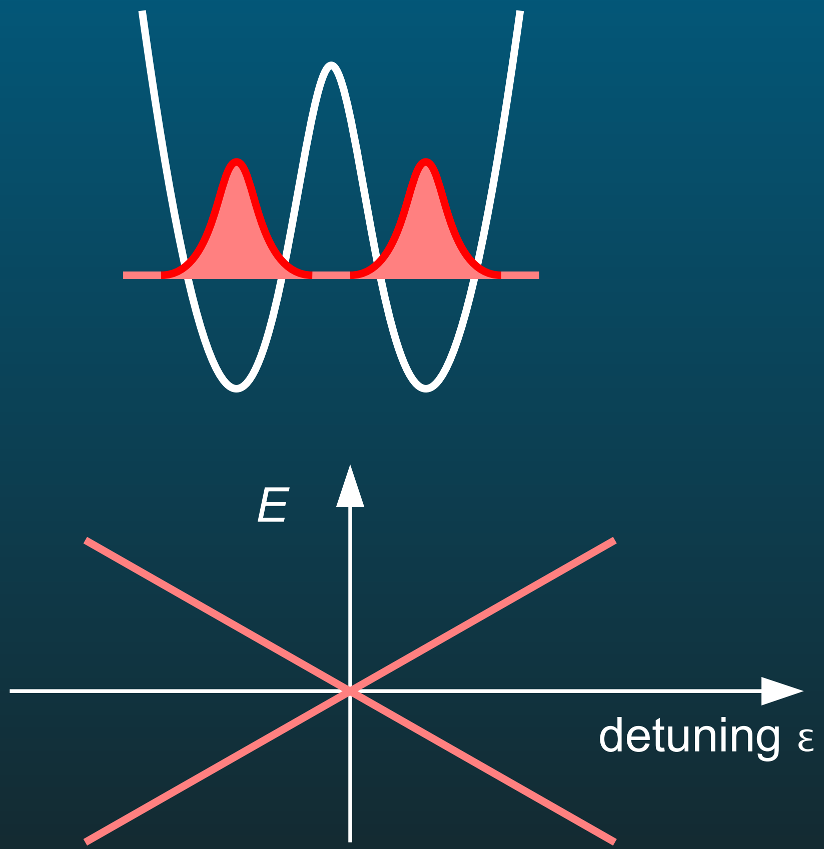



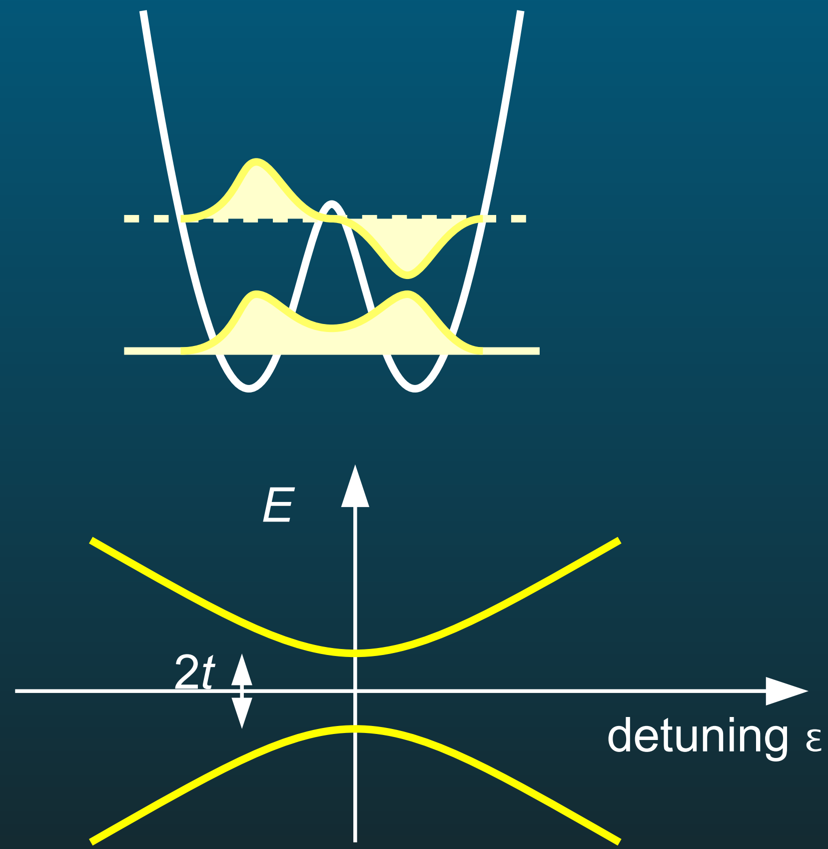



# Inter-dot tunnel coupling

- weak coupling

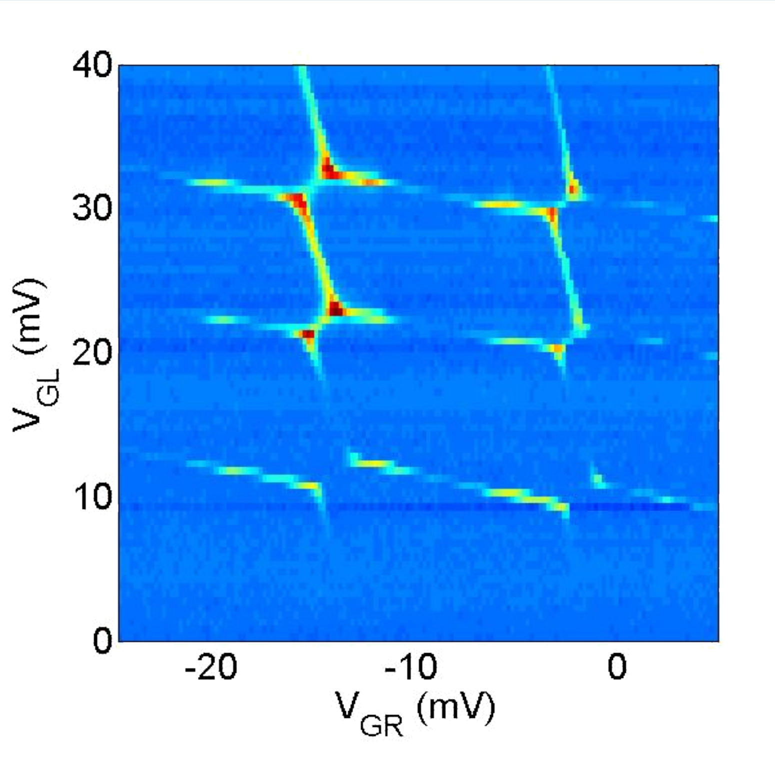


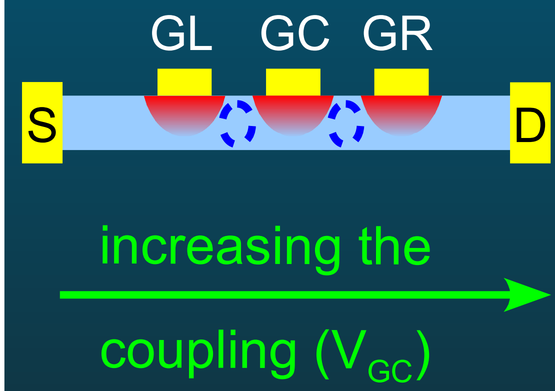

- strong coupling

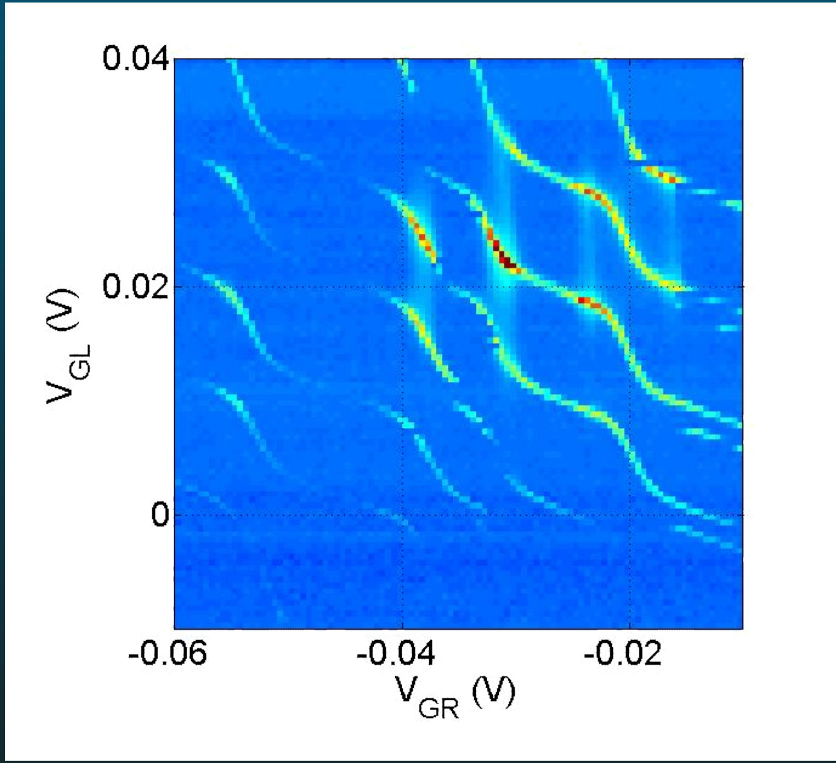


# Inter-dot tunnel coupling

- weak coupling
- strong coupling

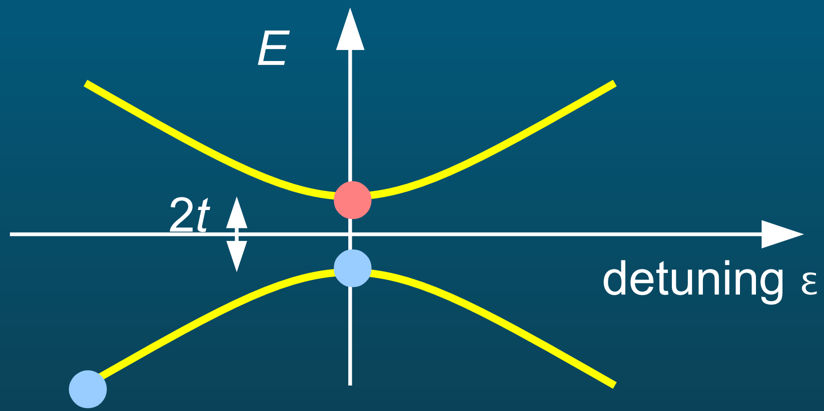


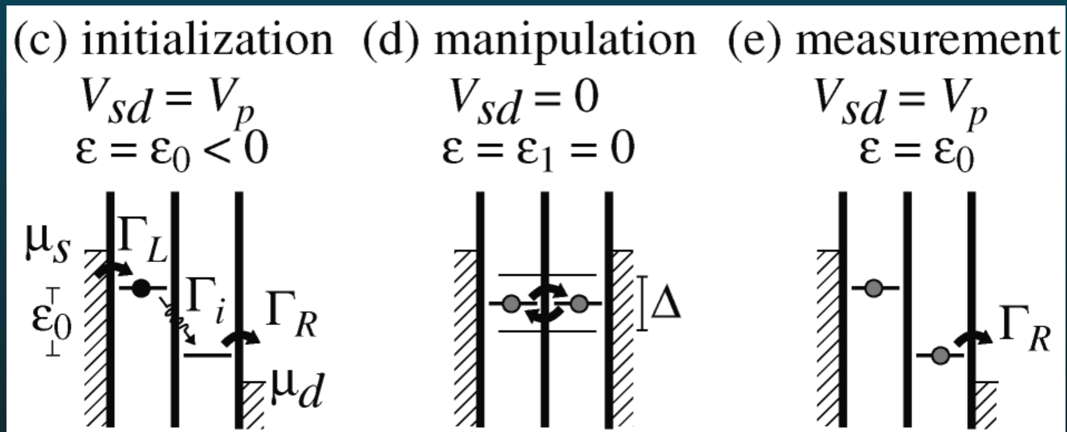



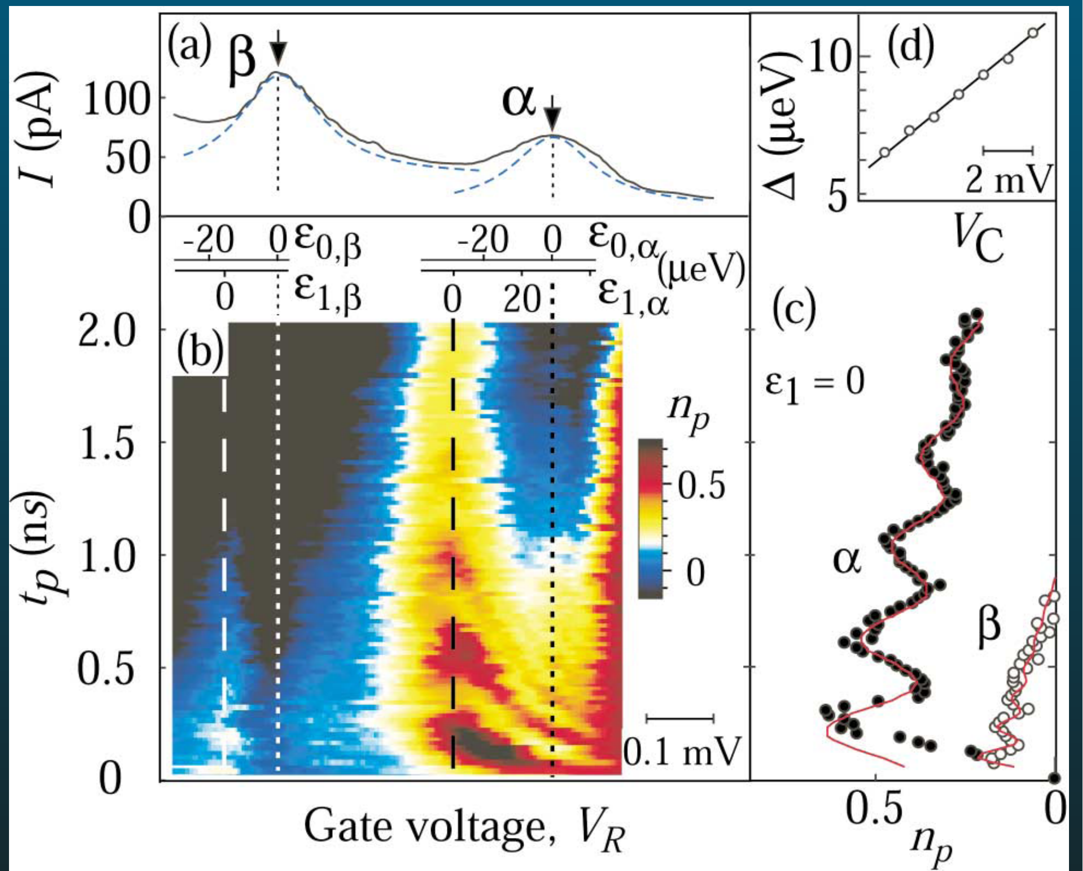



# Coherent manipulation of charges

- Coherent evolution in a double quantum dot




T. Hayashi *et al.*, Phys. Rev. Lett. **91**, 226804 (2003)

# Take-away message (4)

Manipulation of quantum states on time-scales smaller than the relaxation and coherence times

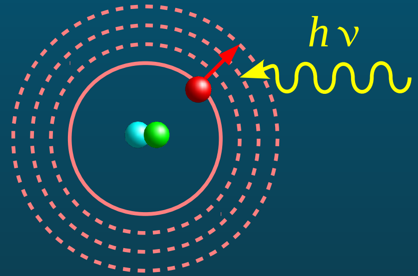
quantitative investigation of relaxation and decoherence

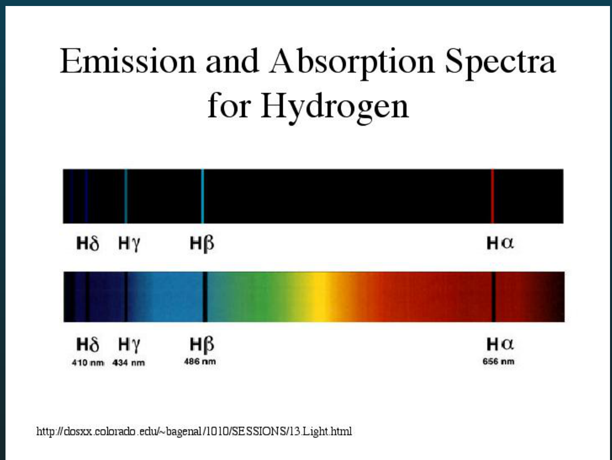
coherent manipulation of quantum states

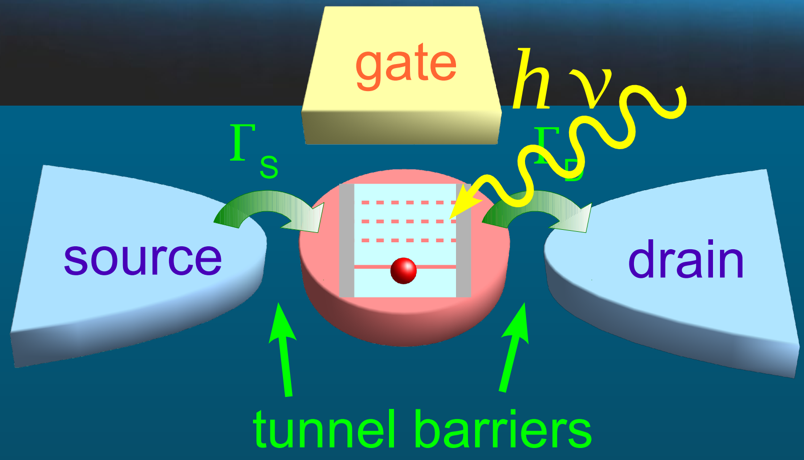
next: non-adiabatic manipulation at high frequency

# 4. Interaction with photons

- Absorption of light by an (artificial) atom through electronic transition




- Quantum dots in GaAs
  - ν = 10 – 100 Ghz
  - tunable electronic properties
  - measurement by electronic transport

    ⇒ **use as functional device** (detector)

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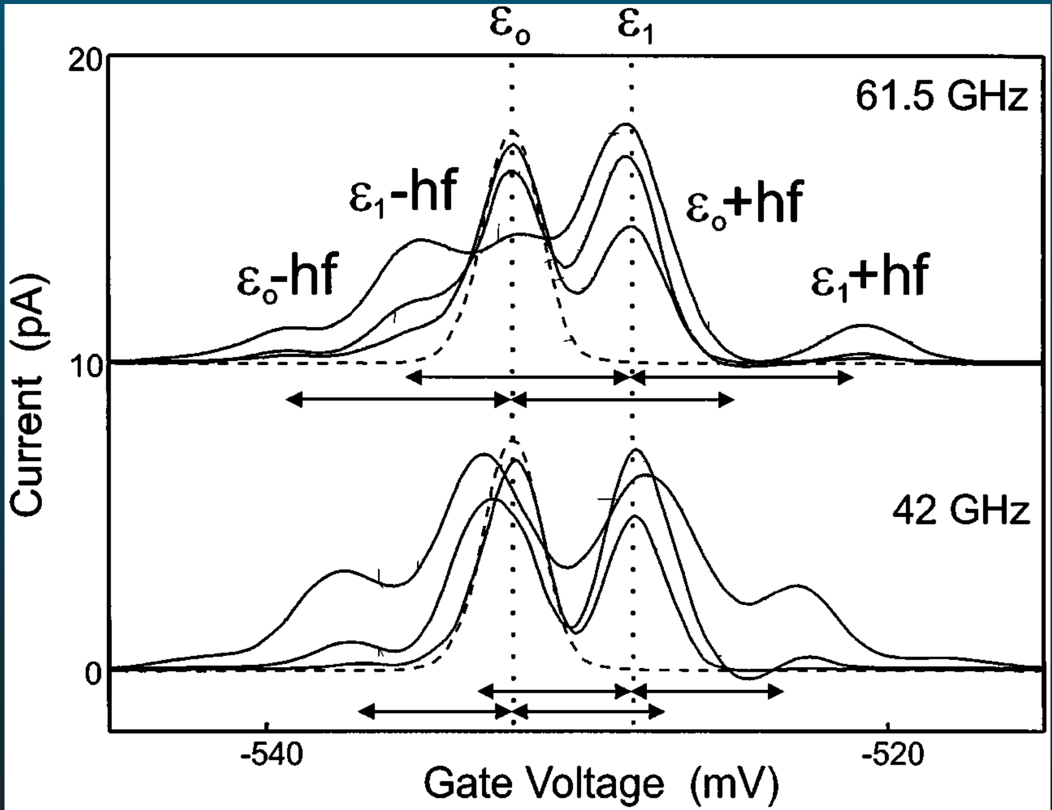
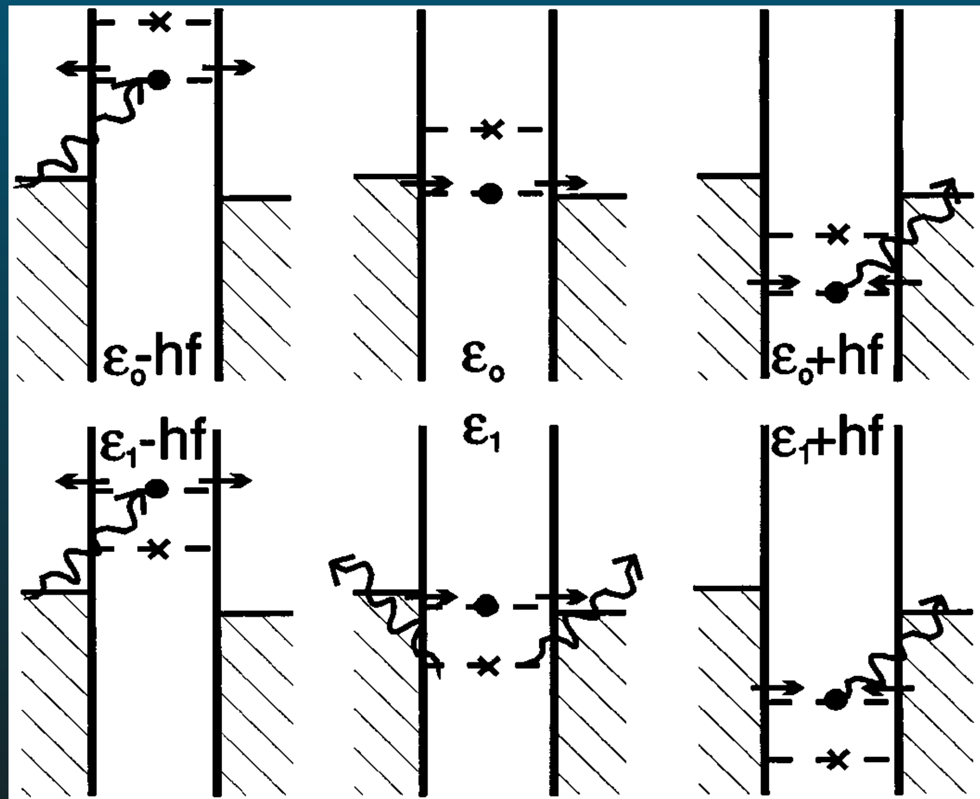
# Photon-assisted tunneling

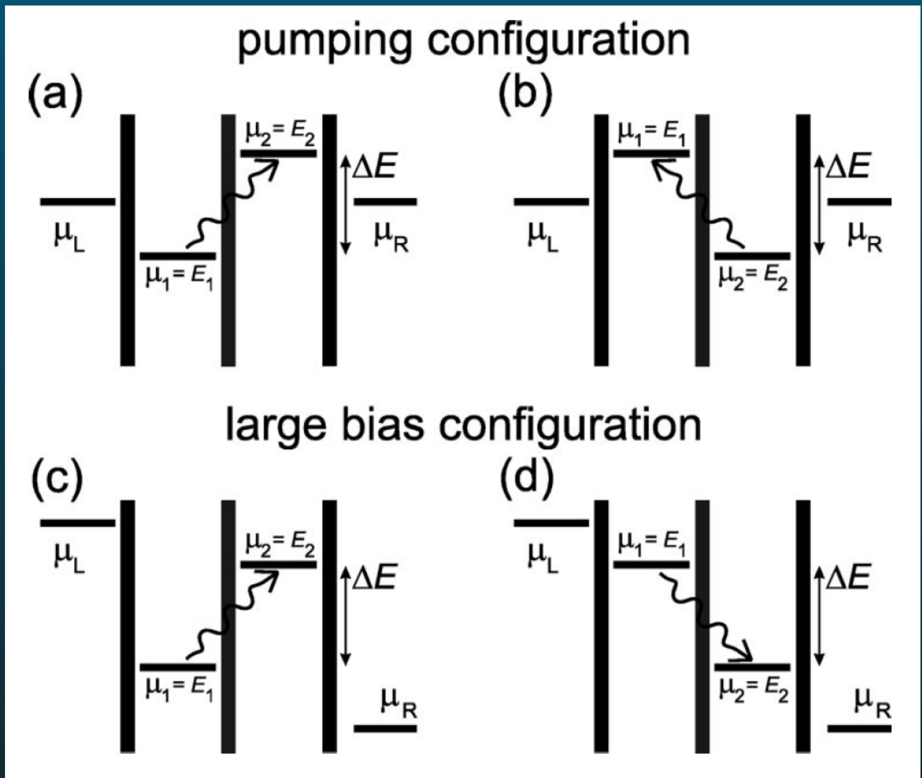
- Single quantum dot T. H. Oosterkamp et al., PRL 78, 1536 (1997)
  - side-bands due to photon-assited tunneling to the leads

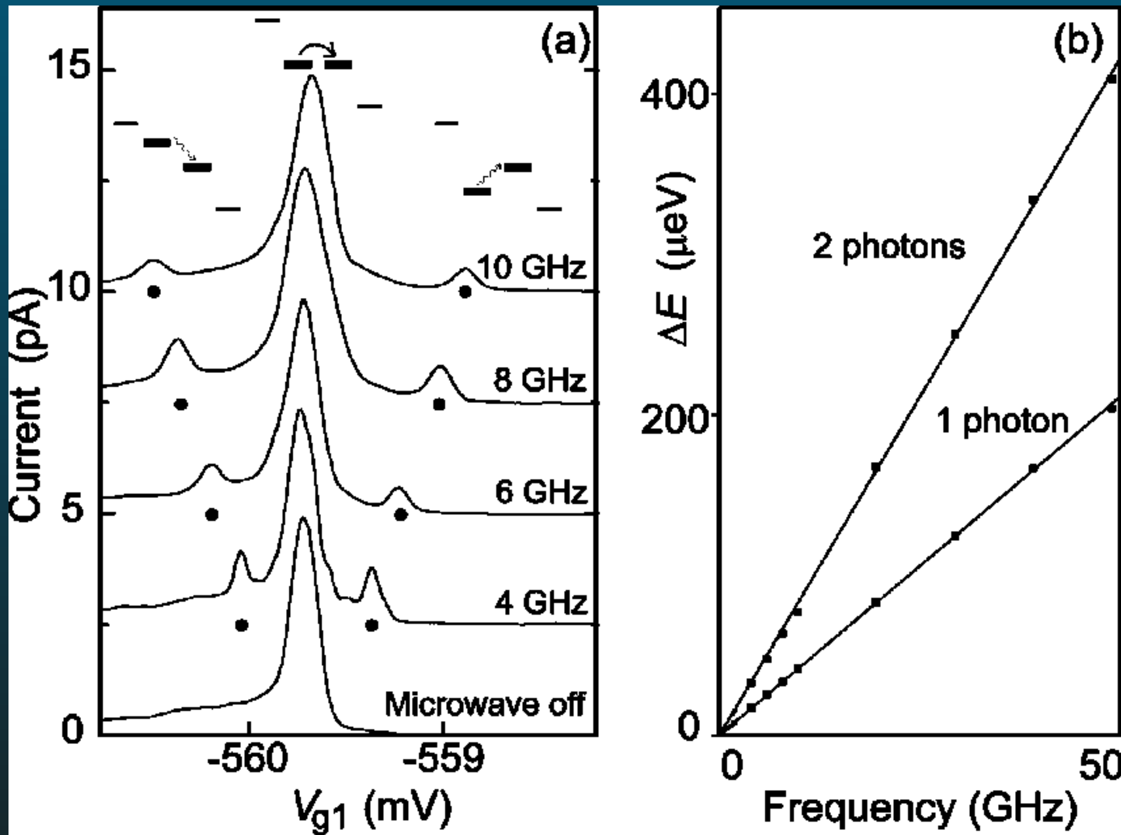


# Photon-assisted tunneling

- Double quantum dot
  - probing internal transitions

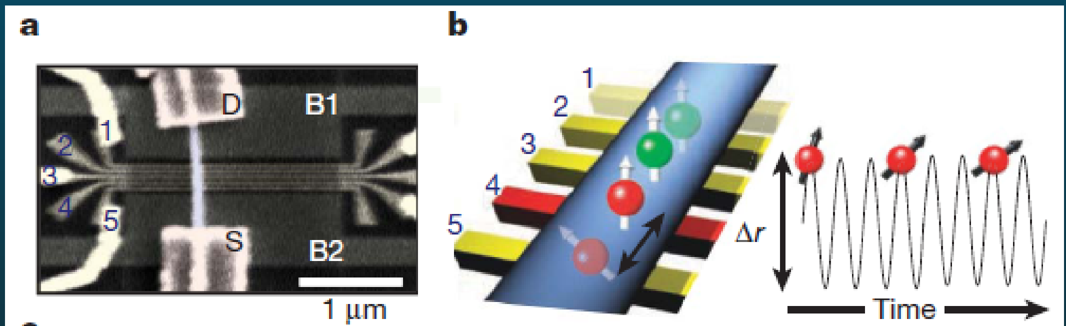


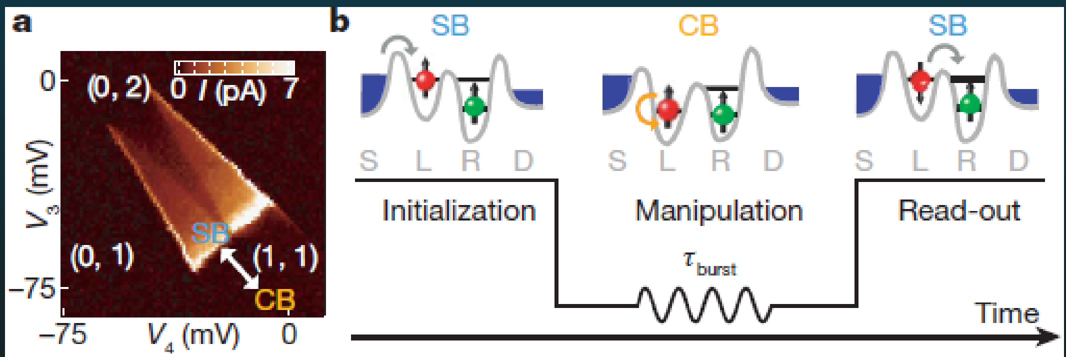



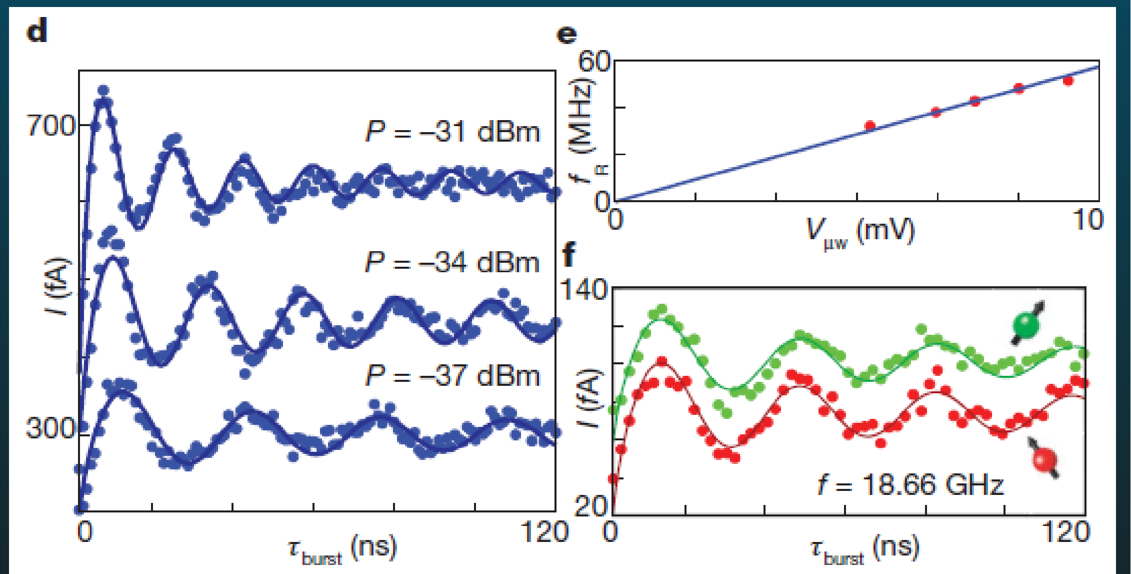

W.G. Van der Wiel *et al.*, RMP **75**, 1 (2003)

# Coherent single spin manipulation

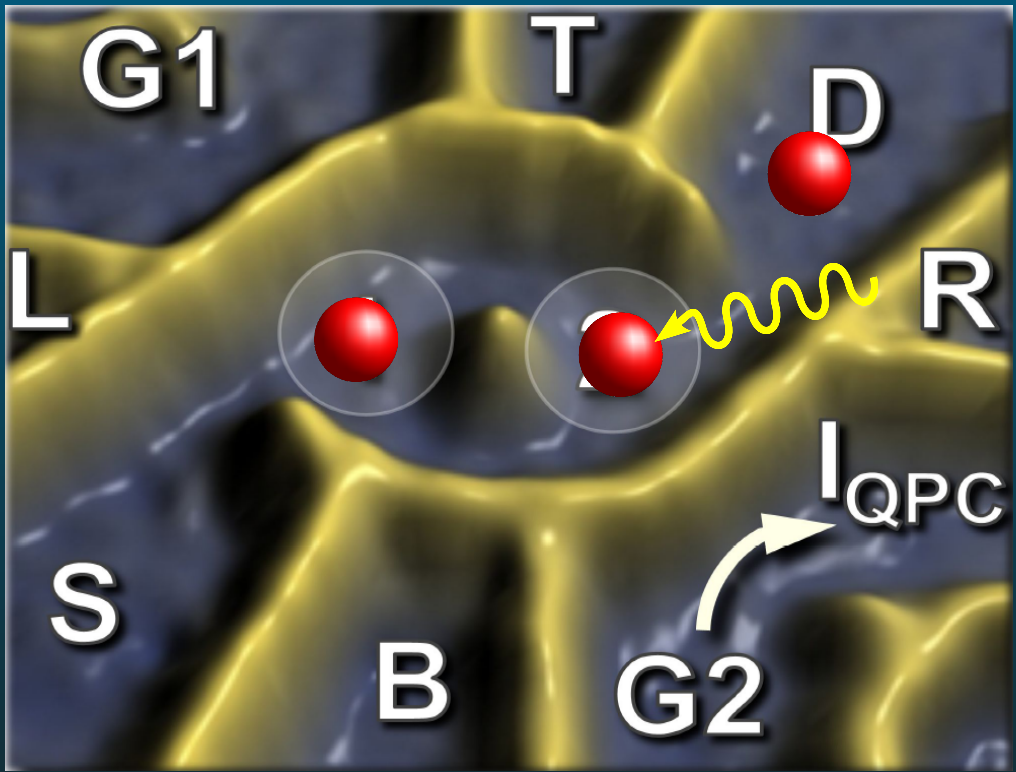
- Electron spin resonance with a single spin
  - first with GaAs quantum dots and high frequency magnetic field
    F. H. L. Koppens *et al.*, Nature **442**, 766 (2006)
  - with InAs nanowire QDs using the spin-orbit interaction
    S. Nadj-Perge *et al.*, Nature **468**, 1084 (2010)

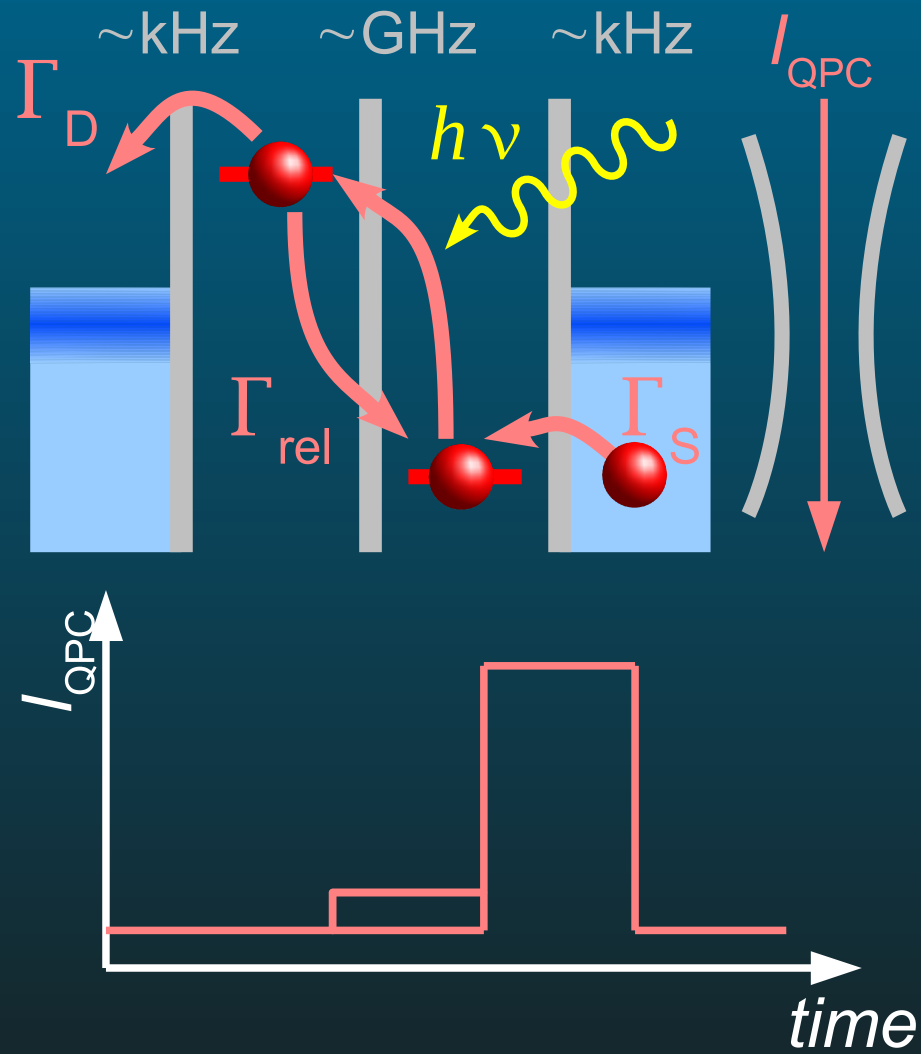


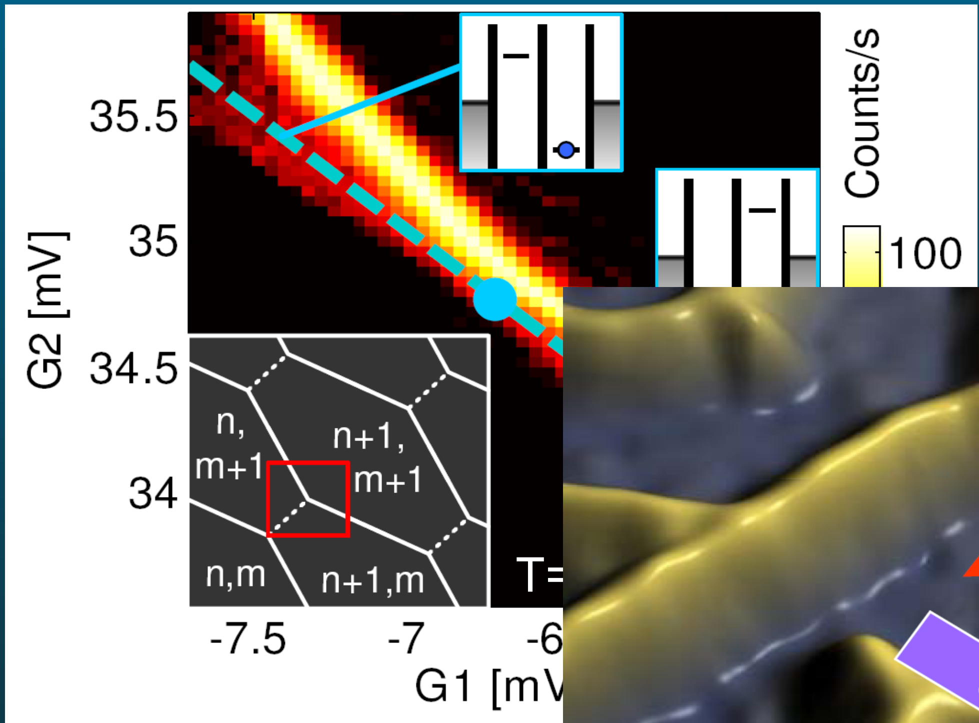



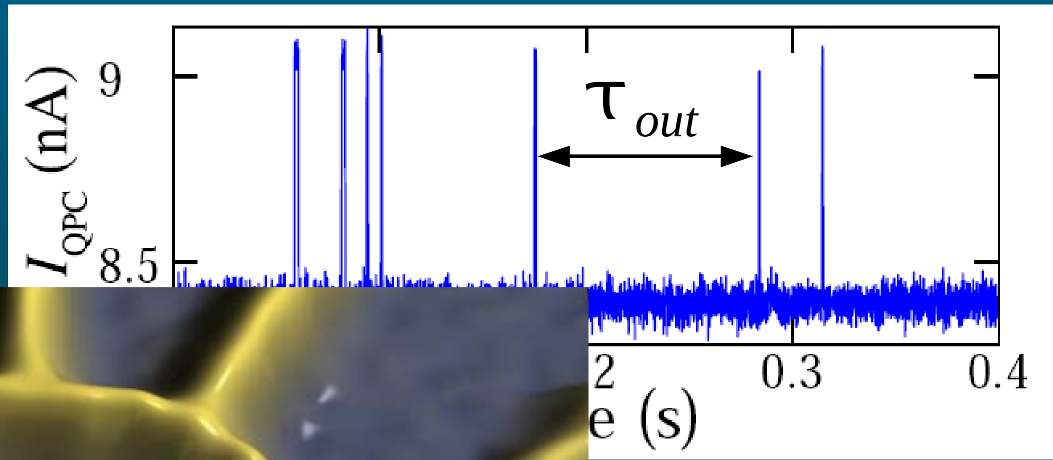
# Time-resolved detection of photon-assisted tunneling

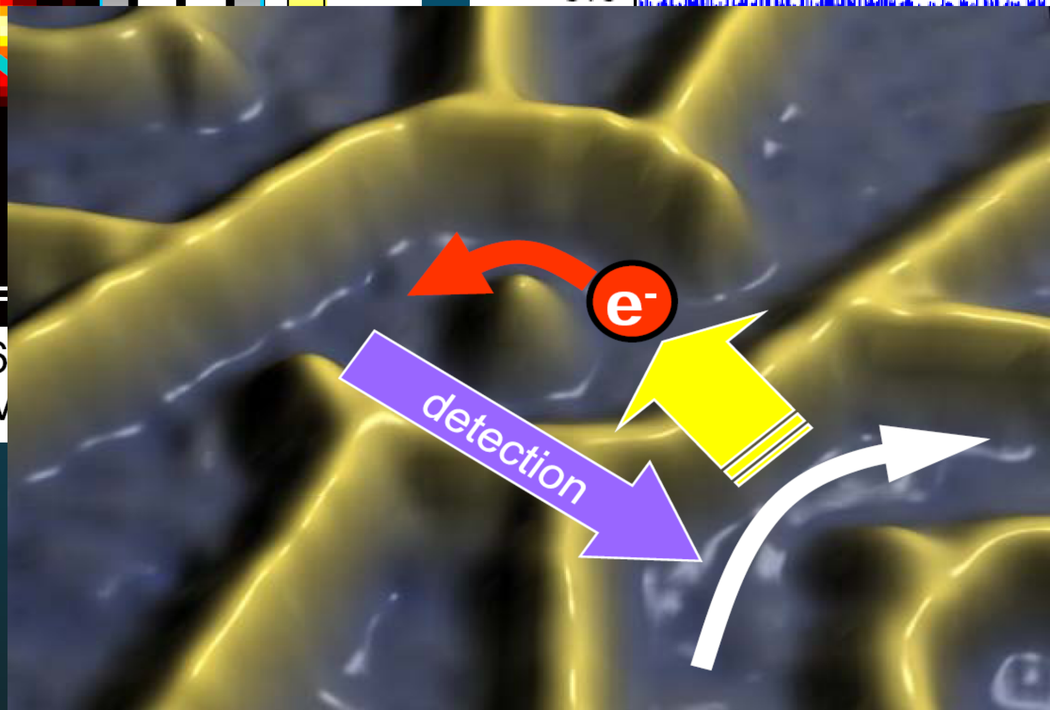


# Time-resolved detection of photon-assisted tunneling




$$\frac{\Gamma_D}{\Gamma_{rel}} \Gamma_{ph}$$

the single-photon conversion

High frequency noise of the quantum point contact!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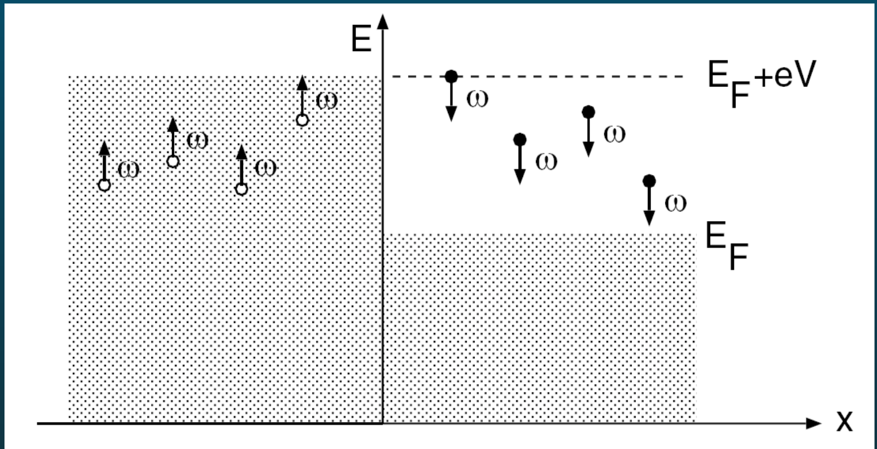
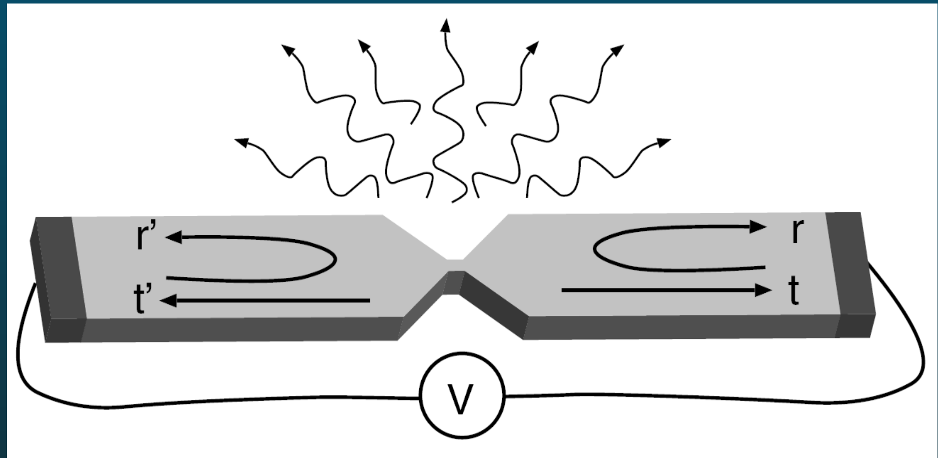
# High-frequency shot noise of a quantum point contact

- Shot noise at high frequency
  G. B. Lesovik, JETP Lett. **49**, 592 (1989); S. E. Yang, Solid State Comm. **81**, 375 (1992); M. Büttiker, PRB **45**, 3807 (1992); R. Aguado & L. P. Kouwenhoven, PRL **83**, 1986 (2000)

  ⇒ emission of microwave photons



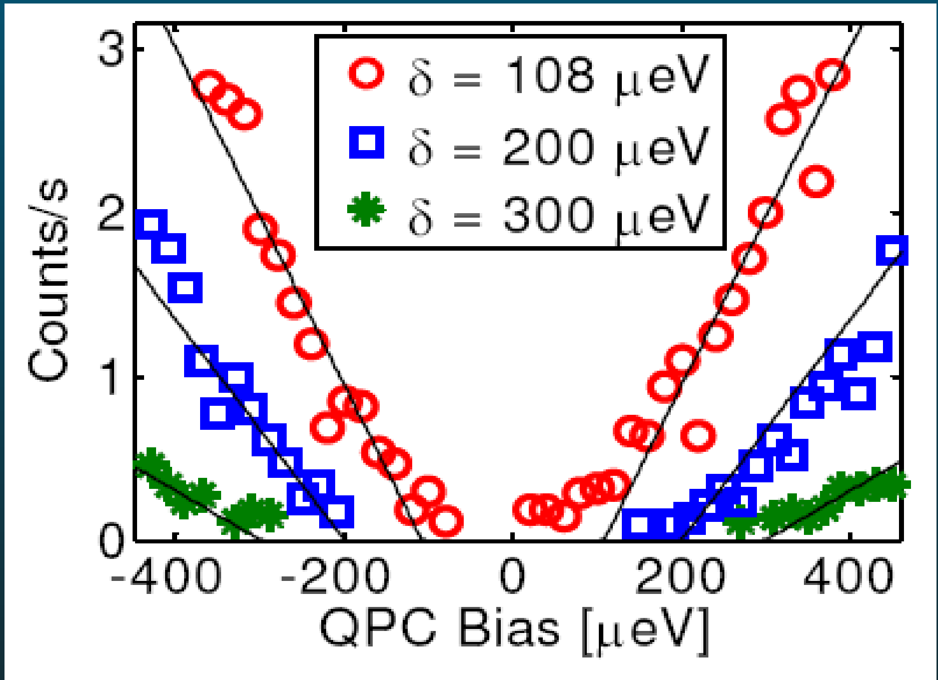

C. W. J. Beenakker & H. Schomerus, PRL **86**, 700 (2001)

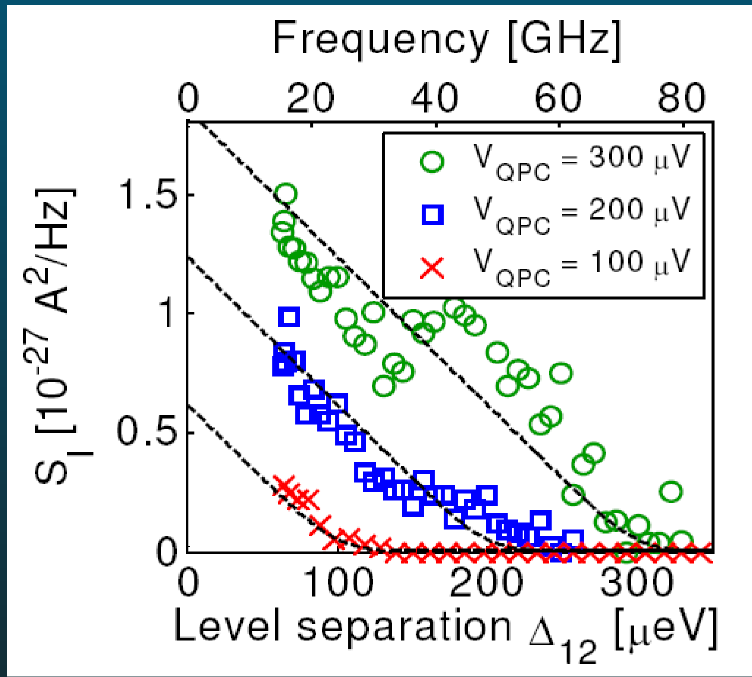
# Detection of the high-frequency noise of the quantum point contact

S. Gustavsson *et al.*, PRL **99**, 206804 (2007)

- Shot noise of the quantum point contact at high frequency

$$\Gamma_{ph} \propto S_I = \frac{2e^2}{h} T(1-T)(e|V_{QPC}| - h|\nu|)$$




see also: E. Onac *et al.*, PRL **96**, 176601 (2006) – on chip with a single quantum dot
E. Zakka-Bajjani *et al.*, PRL **99**, 236803 (2007) – direct detection

# Take-away message (5)

Photon-assisted tunneling for investigating the internal quantum structure of a quantum dot

optical spectroscopy at microwave frequency

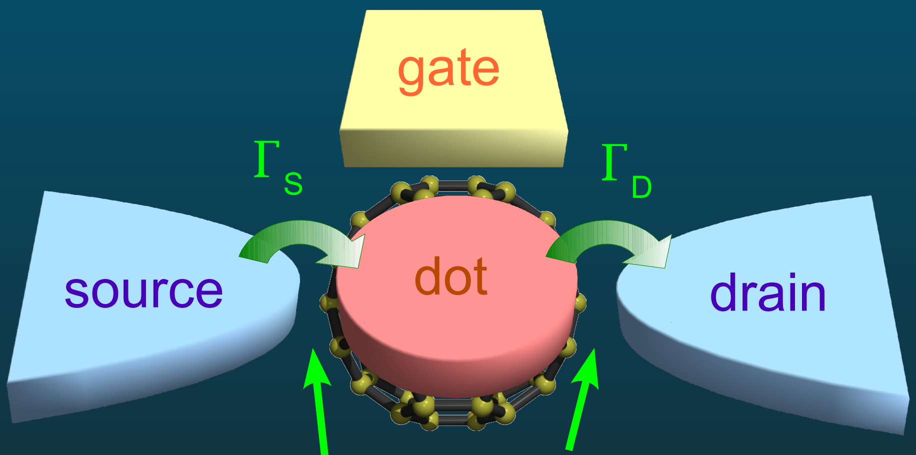
single state manipulation

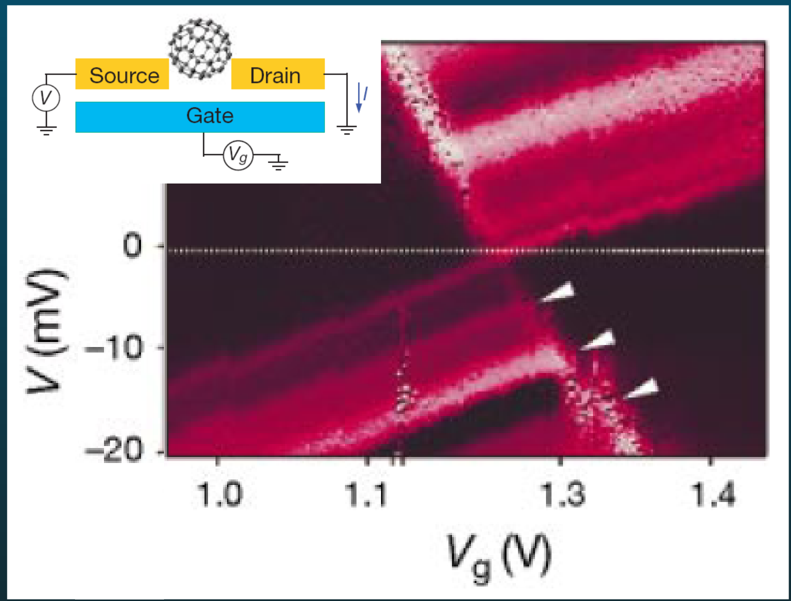
next: Role of the phonons in the energy transfer?

V. S. Khrapai *et al.*, PRL **97**, 176803 (2006)

# 5. Interaction with phonons

- Quantum dots ⇔ artificial atoms and molecules
  - already seen for shell filling and electronic transitions
  - what about vibrational transitions?

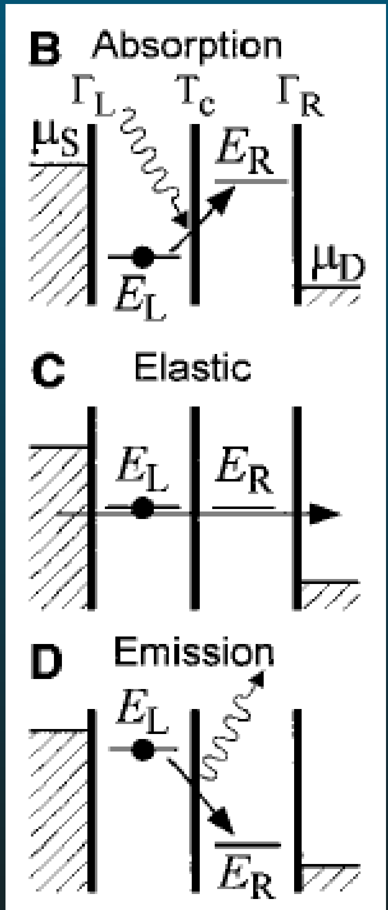


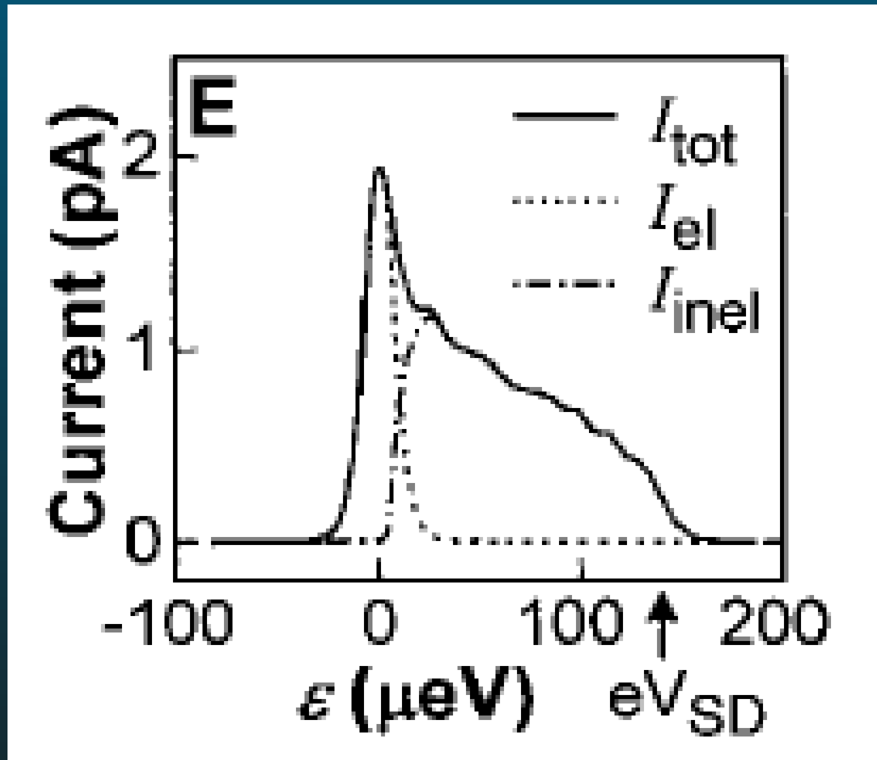




H. Park *et al.*, Nature **407**, 57 (2000)

# Relaxation due to electron-phonon coupling

- Coupling to bulk phonons
  - relaxation mediated by the electron-phonon coupling



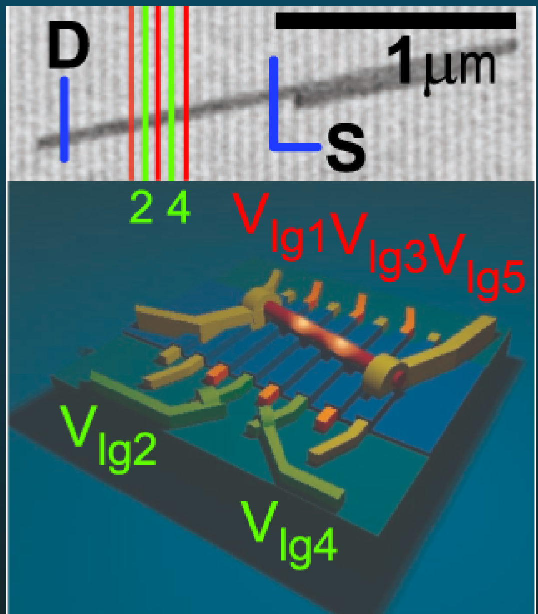



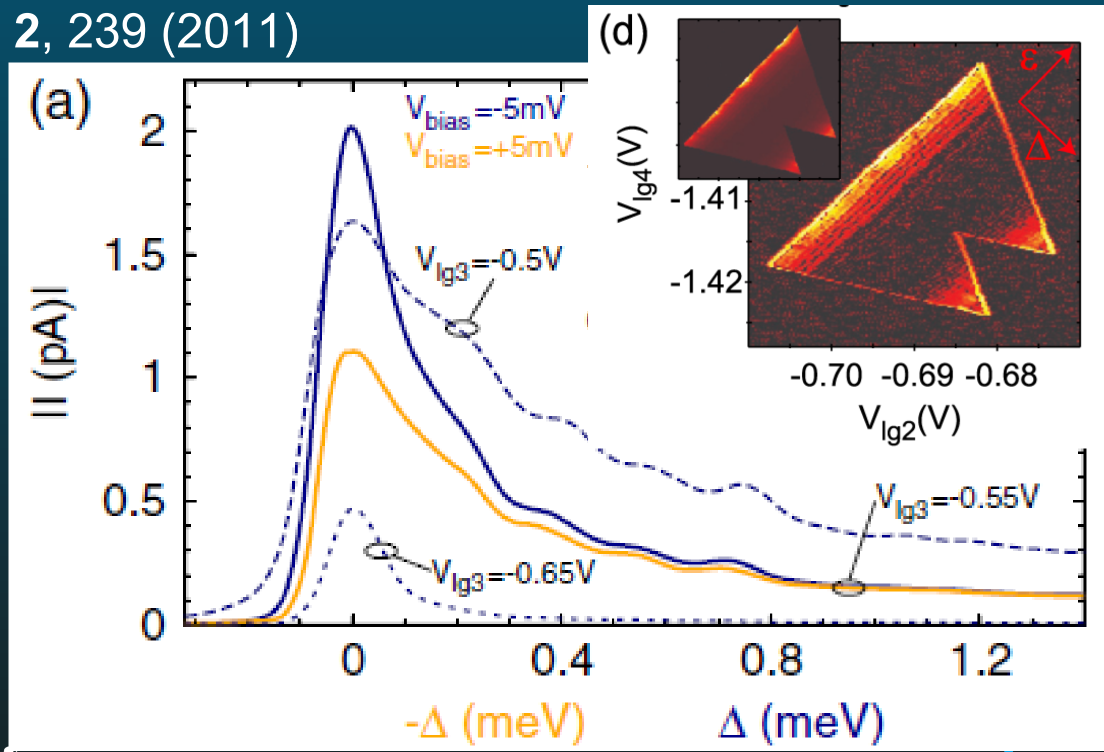

T. Fujisawa *et al.*, Science **282**, 932 (1998)

# Relaxation due to electron-phonon coupling

- Coupling to confined phonons in a nanowire double quantum dot
  - phonons confined in the diameter of the nanowire

C. Weber *et al.*, PRL **104**, 036801 (2010)
P. Roulleau *et al.*, Nature Comm. **2**, 239 (2011)



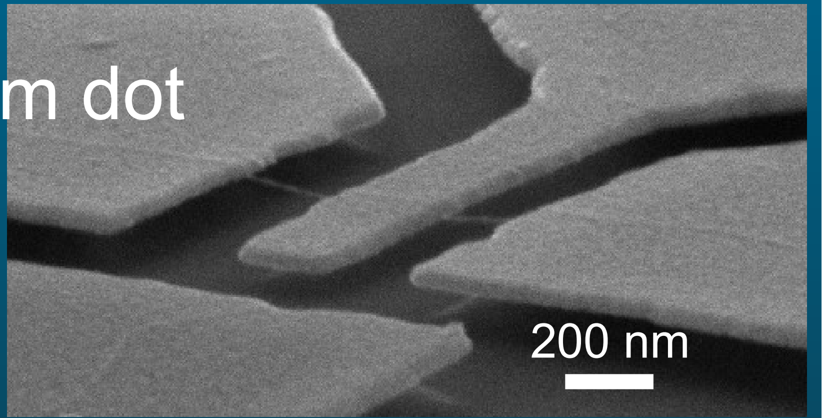



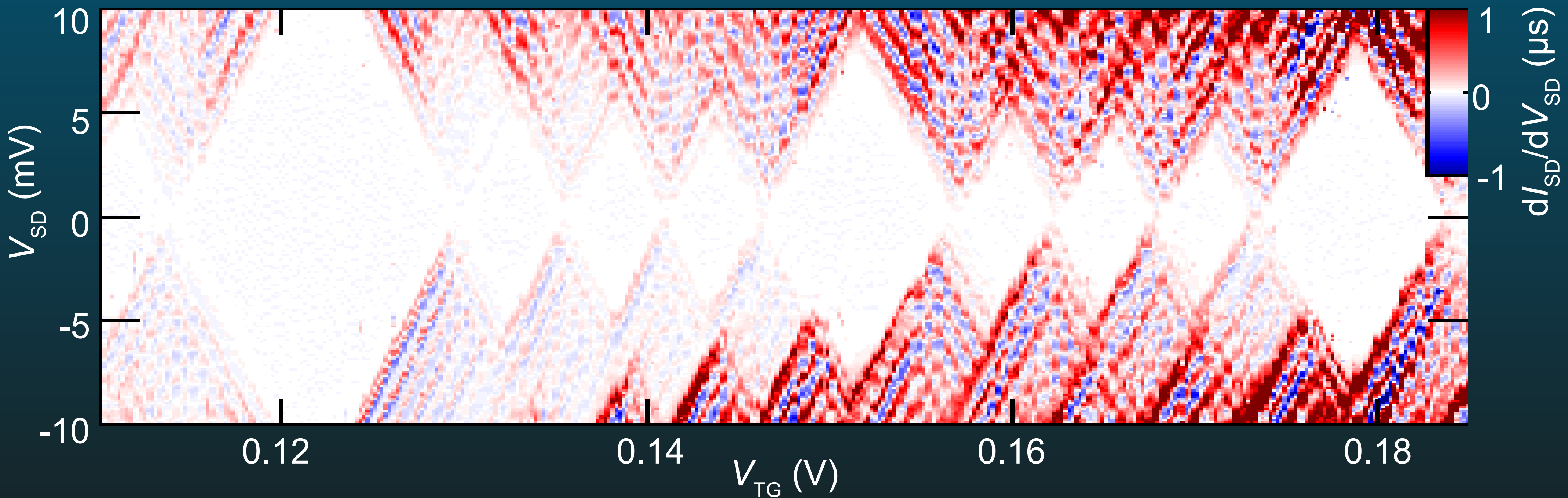
# Franck-Condon blockade in a suspended quantum dot

- Suspended carbon nanotube quantum dot
  - vibronic excited states $\Delta E_{vib} \approx 0.8$ meV

    S. Sapmaz et al., PRL 96, 026801 (2006)

    R. Leturcq *et al.*, Nature Phys. **5**, 327 (2009)

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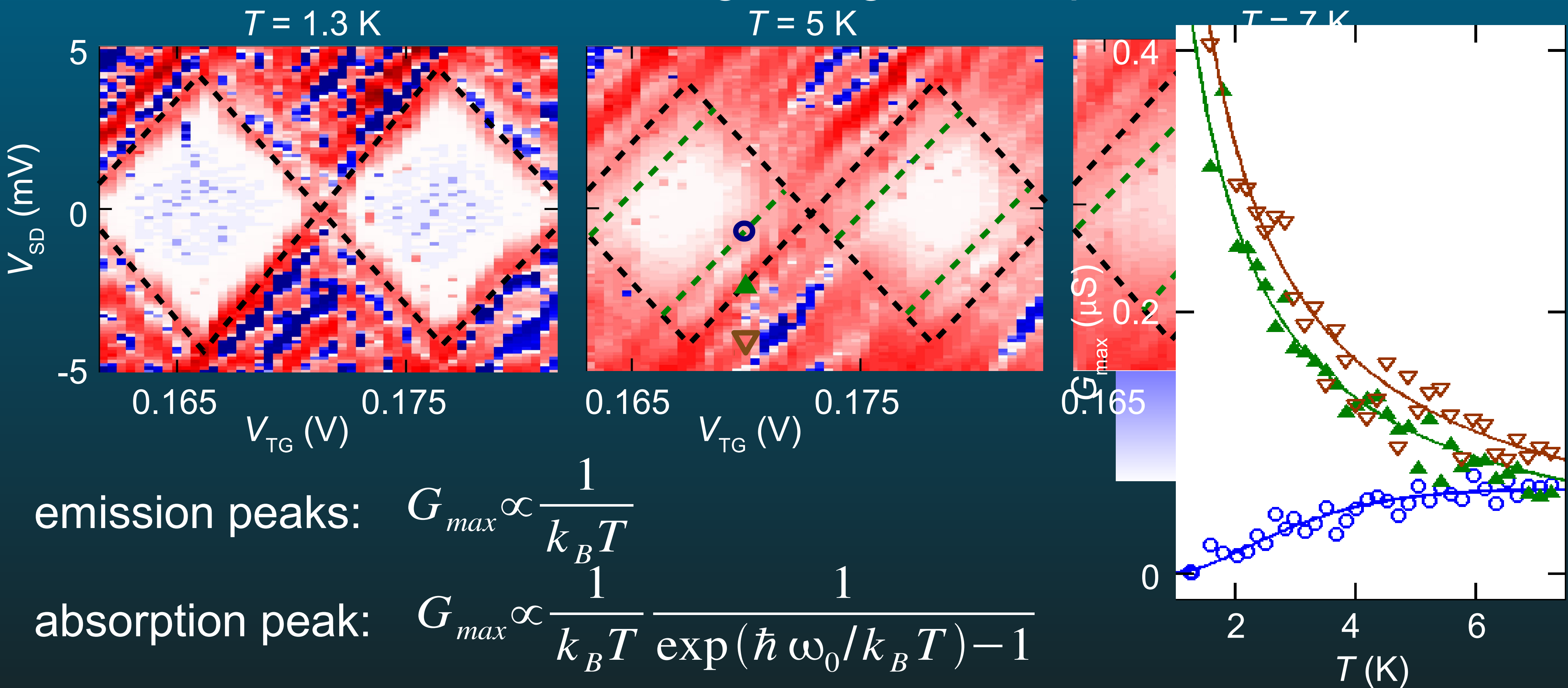
# Franck-Condon blockade in a suspended quantum dot

R. Leturcq, C. Stampfer *et al.*, Nature Phys. **5**, 327 (2009)

- Vibron-assisted tunneling at higher temperature




emission peaks: $G_{max} \propto \frac{1}{k_B T}$

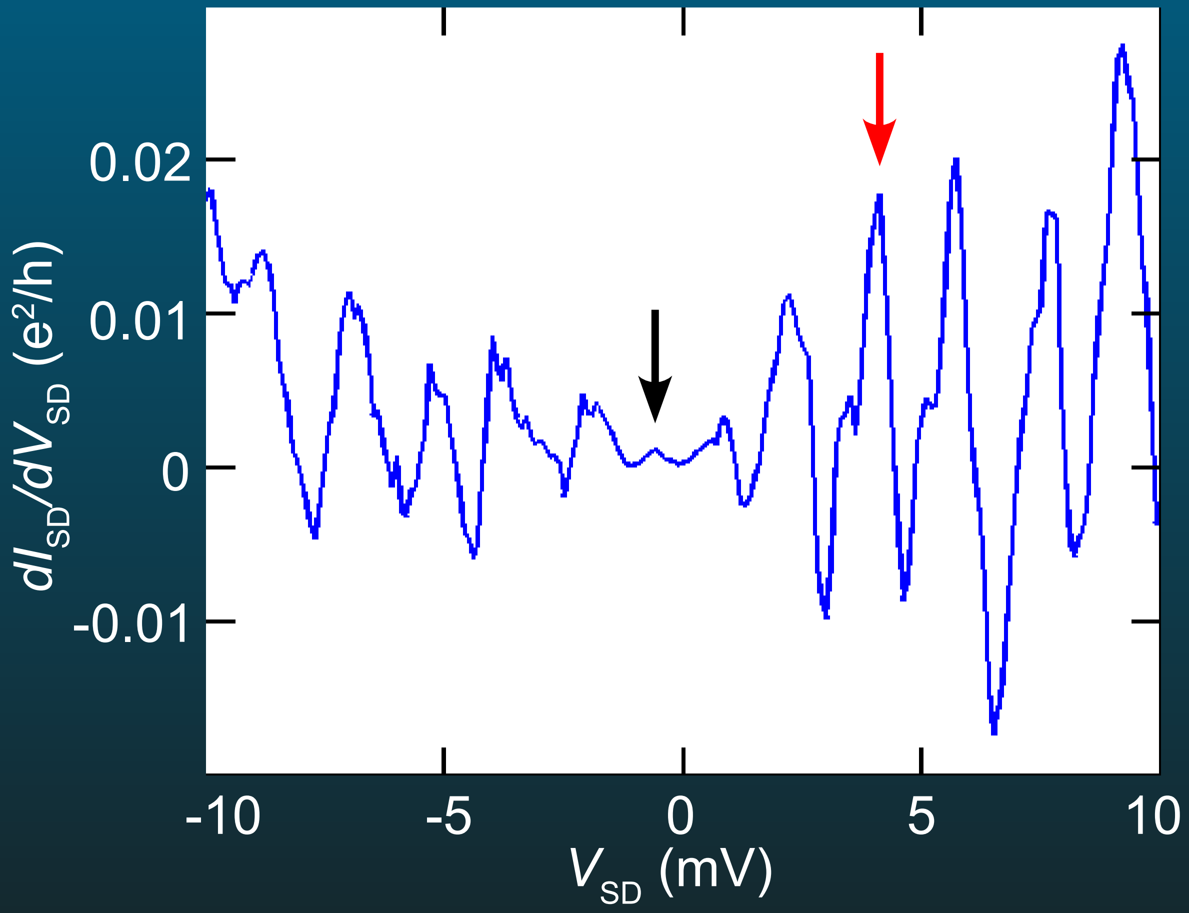
absorption peak: $G_{max} \propto \frac{1}{k_B T} \frac{1}{\exp(\hbar\omega_0 / k_B T) - 1}$

$\hbar\omega_0 = 0.96 \pm 0.08$ meV $\approx \Delta E_{vib}$

# Franck-Condon blockade in a suspended quantum dot

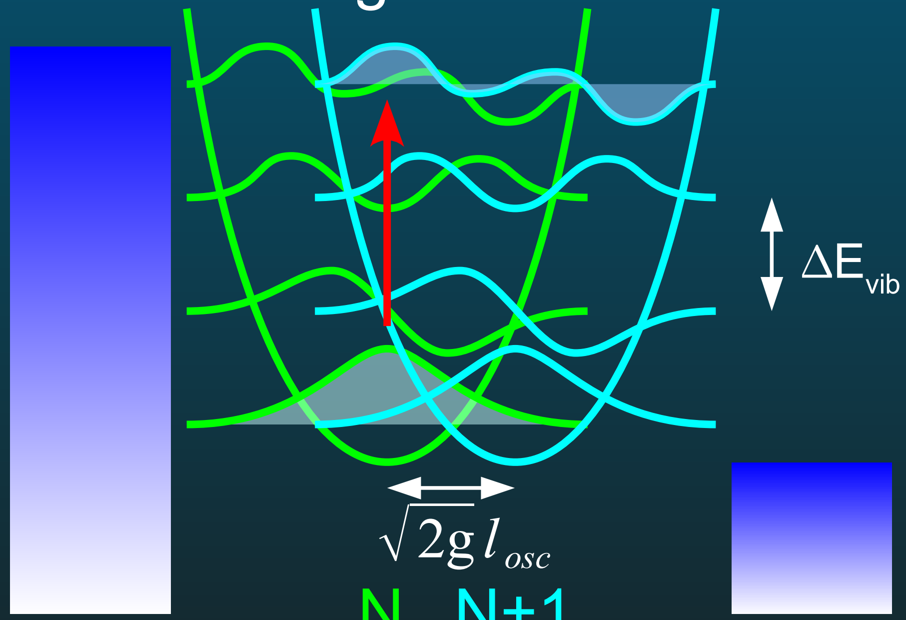
R. Leturcq, C. Stampfer *et al.*, Nature Phys. **5**, 327 (2009)

- Suppression of current at zero bias voltage




strong electron-vibron coupling

g > 1



# take-away message (6)

Investigation of the electron-phonon coupling on the single particle level

weak coupling: relaxation

strong coupling: Franck-Condon blockade

# Conclusion – part I

- Transport in quantum dots
  - wide tunability
  - time-resolved measurement and manipulation
  - interaction of single quantum states with the environment

- Single impurity coupled to Fermi leads ⇔ Kondo physics
  - How the tools available in quantum dots allow to study the Kondo effect on the single impurity level
  - Show what has been done experimentally with quantum dots, discuss what can(not) be done

# Kondo physics in quantum dots

- Single impurity coupled to Fermi leads ⇔ Kondo problem
  L. I. Glazman & M. E. Raikh, JETP Lett. 47, 452 (1988)
  T. K. Ng & P. A. Lee, PRL 61, 1768 (1988)
  - due to on-site Coulomb interaction in the quantum dot

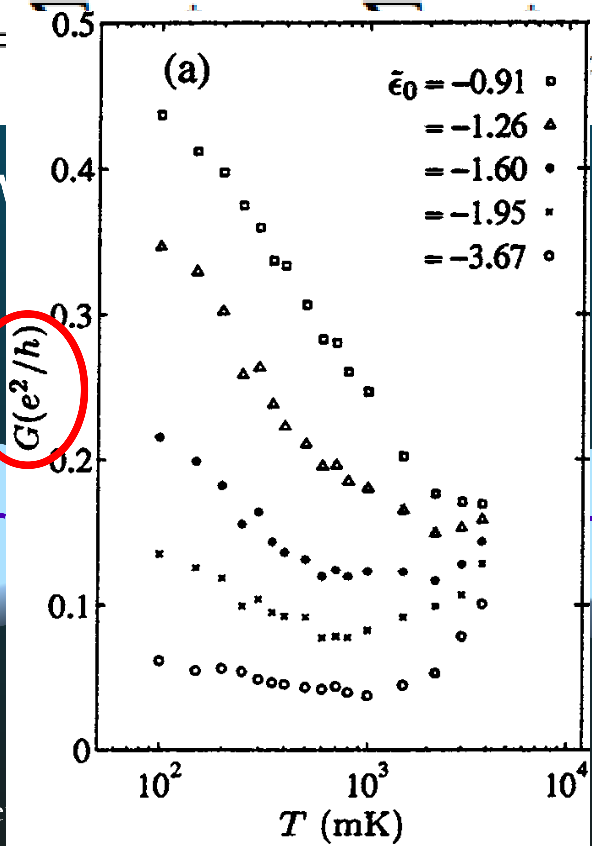
$$H = \ldots + U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow + \sum_{k,\sigma} \left( v_k \ldots \right)$$

  - w... effect ($U$, $\varepsilon_0$, $\nu_k$... $T_K$ ...

conductance

source

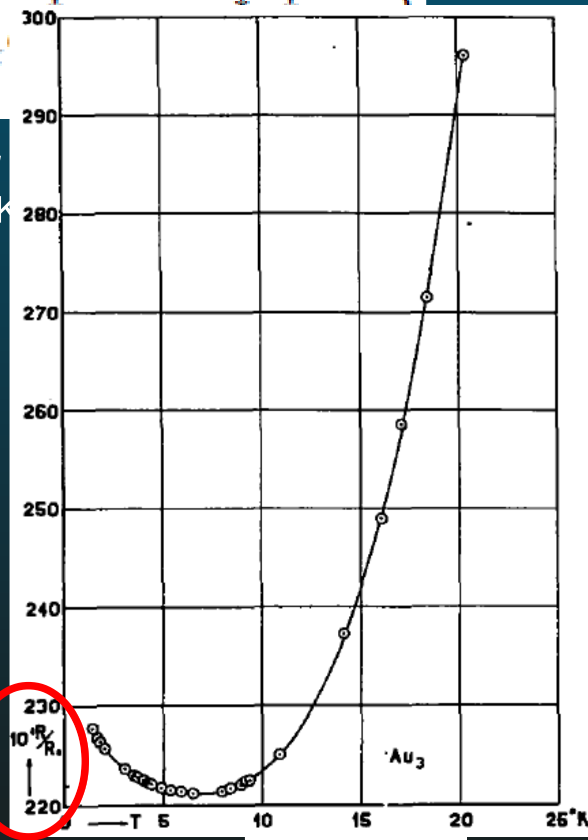
drain




D. Goldhaber-Gordon
PRL 81, 5225 (1998)

⇔

resistance




W. J. De Haas & G. J. Van Den Berg,
Physica 3, 440 (1936)

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# Single electron transport mechanisms

- Sequential tunneling model
  - master equation approach for a single level at energy $\epsilon$
    Beenakker, Phys. Rev. B **44**, 1646 (1991)

$$I = -e\Gamma_L \left[ p_0 f_L - p_1 (1 - f_L) \right] \qquad p_0 + p_1 = 1$$

$$f_L = f(E_L - E_F) \qquad f_R = f(E_R - E_F)$$

$$E_L = \epsilon + \eta eV \qquad E_R = \epsilon - (1-\eta) eV$$

$$\frac{d}{dt}\begin{pmatrix} p_0 \\ p_1 \end{pmatrix} = \begin{pmatrix} -\Gamma_L f_L - \Gamma_R f_R & \Gamma_L (1 - f_L) + \Gamma_R (1 - f_R) \\ \Gamma_L f_L + \Gamma_R f_R & -\Gamma_L (1 - f_L) - \Gamma_R (1 - f_R) \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \end{pmatrix}$$

stationarity: $\frac{d\,p_0}{d\,t} = \frac{d\,p_1}{d\,t} = 0$ $\qquad eV \gg k_B T \Rightarrow I = -e \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$

# Electron transport through a double quantum dot

- Large bias voltage: spectroscopy