Switching in Complex Networks of States: A New Paradigm for Natural Computation

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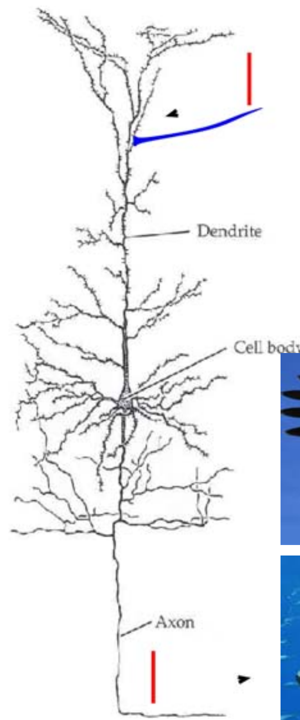
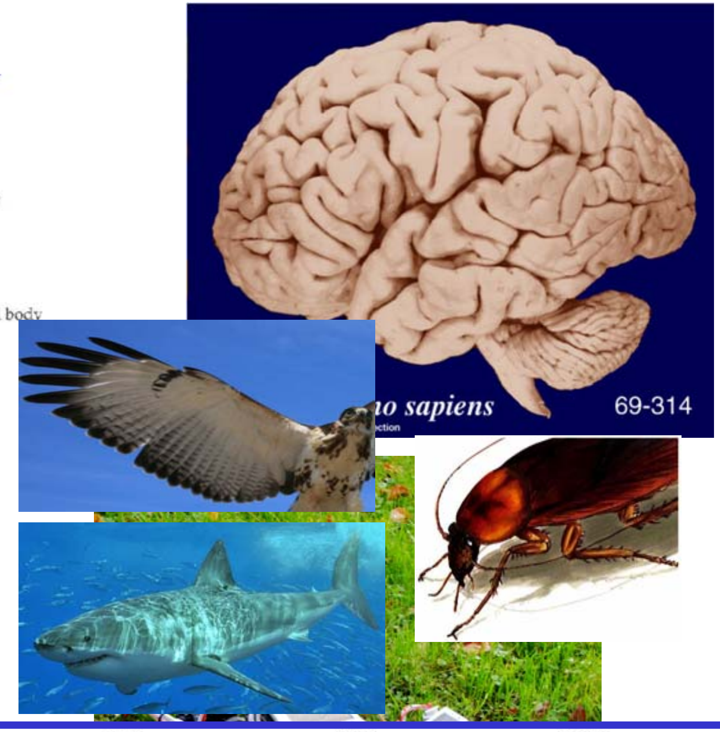
Biological and bio-inspired computation

Biological Networks

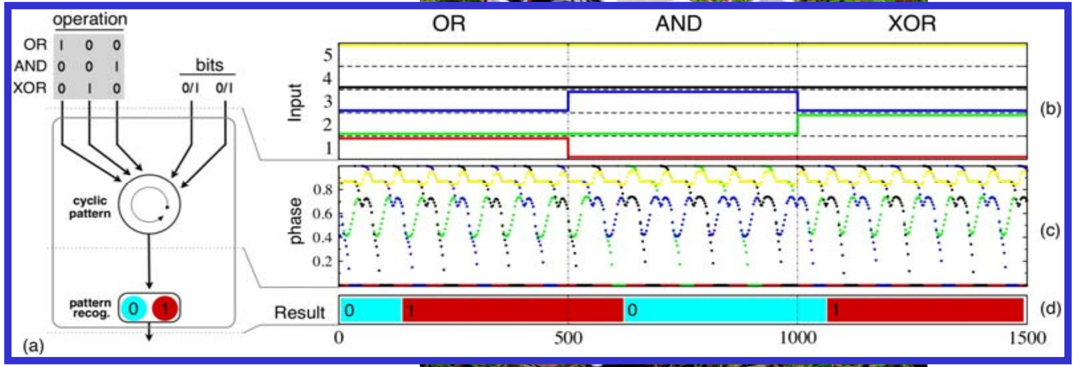
- Neural circuits
  (computation & learning)
- „Tree“ of life
  (evolution)





Bio-inspired networks

- Autonomous robots
- Natural **computing** devices



Towards Natural Computation

Biological Processes:

- are **nonlinear**
- exploit **self-organized, emerging** collective states
- based on learning, adaptation, evolution

Technical computing and behaving (robotic) systems:

- may be realized in a **neuro-analogous** way
  (bio-inspired development & possible explanation of biol. phenomena)
- require understanding
  of collective nonlinear dynamics & self-organization

How to build a natural computer?

Outline

<u>Model:</u> Networks of **symmetrically** pulse-coupled oscillators

<u>Phenomenon:</u> Periodic orbit **attractors** (in the sense of Milnor) …
… that are **unstable**

<u>Analytically Tractable Example:</u> Unstable modes
Switching among attractors
System-independence

<u>Asymmetries</u>: Switching
Selection of complex periodic orbits

<u>Universal Computation:</u> k-winner takes all, binary & **n-ary logics**
N=5 versatility; N=100 & **expon. scaling**

<u>Robots:</u> phototaxis & obstacle avoidance

Neural Model and Phase Description

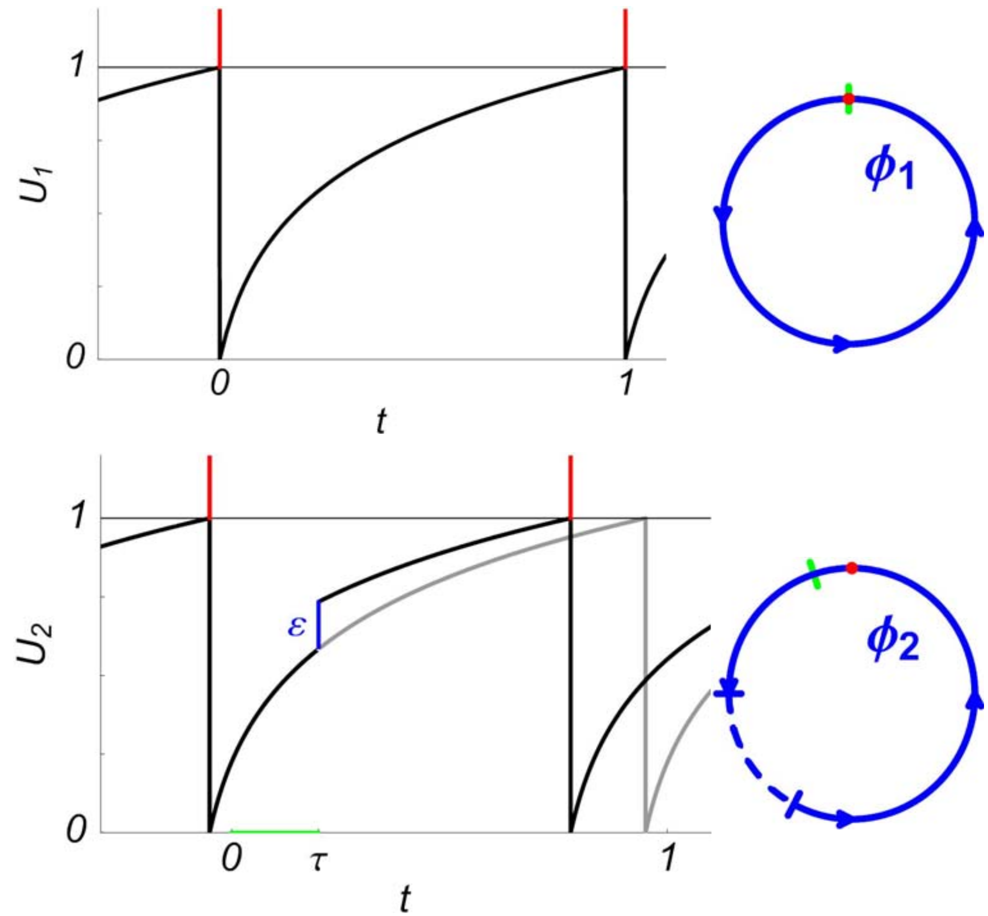
networks of neurons
- oscillatory if driven by current

- uncoupled neurons i
  have increasing (concave) potential $U_i(t)$

- spike sent at $\theta = 1$ threshold

- received after delay time τ

- coupling strength $\propto \varepsilon$



original model: R.E. Mirollo, S.H. Strogatz; *SIAM J. Appl. Math*. 50:1645 (1990)
model with delay: U. Ernst, K. Pawelzik, T. Geisel; *Phys. Rev. Lett.* 74:1570 (1995)

Neural Model and Phase Description

Membrane potential dynamics

$$\frac{dV_i}{dt} = f(V_i) + W_i(t) + \Delta_i$$

Pulse interactions: spike sending $V_j(t_{j,m}^-) \geq 1$

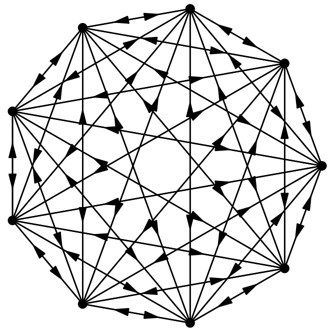
and reset $V_j(t_{j,m}) := 0$

Received after delay time τ

$$W_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \sum_{m \in \mathbb{Z}} \epsilon \delta \left(t - \tau - t_{j,m} \right)$$

$$U(\phi) = \tilde{V}(\phi T) \qquad \dot{\tilde{V}} = f(V); \qquad \tilde{V}(0) = 0, \quad \tilde{V}(T) = 1$$

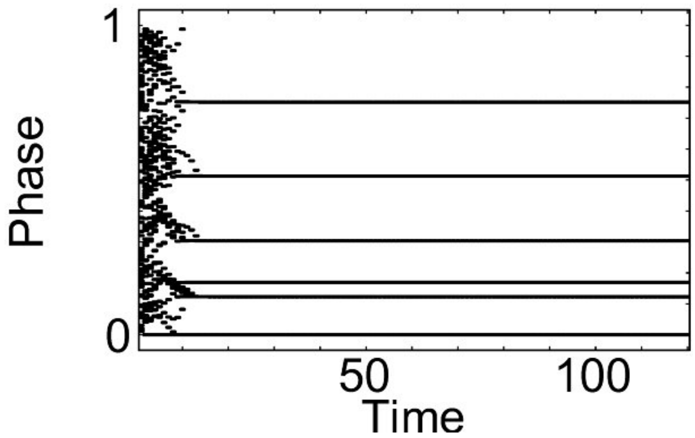
All-to-all Connectivity: Partial Synchrony and Switching



$\varepsilon_{ij} =$ const > 0

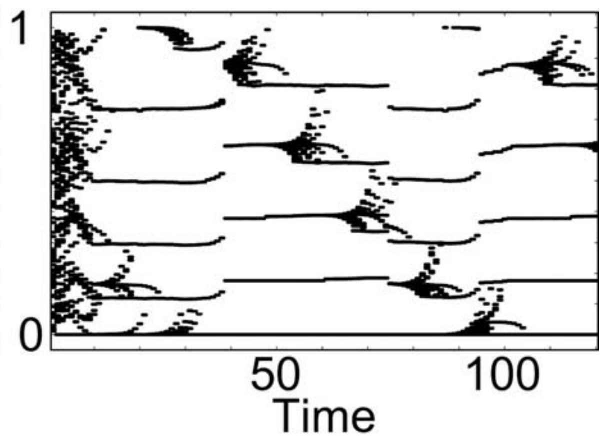
$\tau > 0$

<u>deterministic:</u>
units synchronize into groups (clusters)



→ attractor

<u>weak noise $\eta = 10^{-3}$:</u>
clusters decay



→ switching

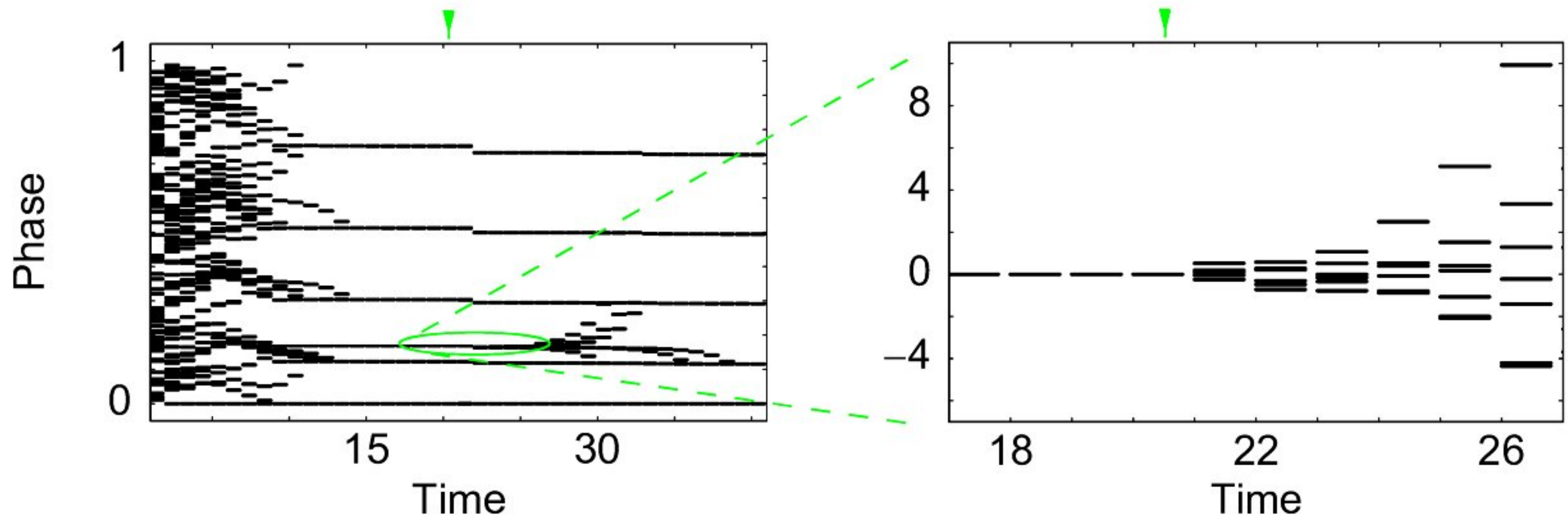
U. Ernst et al., *Phys. Rev. Lett.* 74:1570 (1995)

Switching persists for small noise strengths $\eta = 10^{-22}$

Origin of switching dynamics?

Attracting and yet unstable?

one single random perturbation ($\sigma = 10^{-4}$)

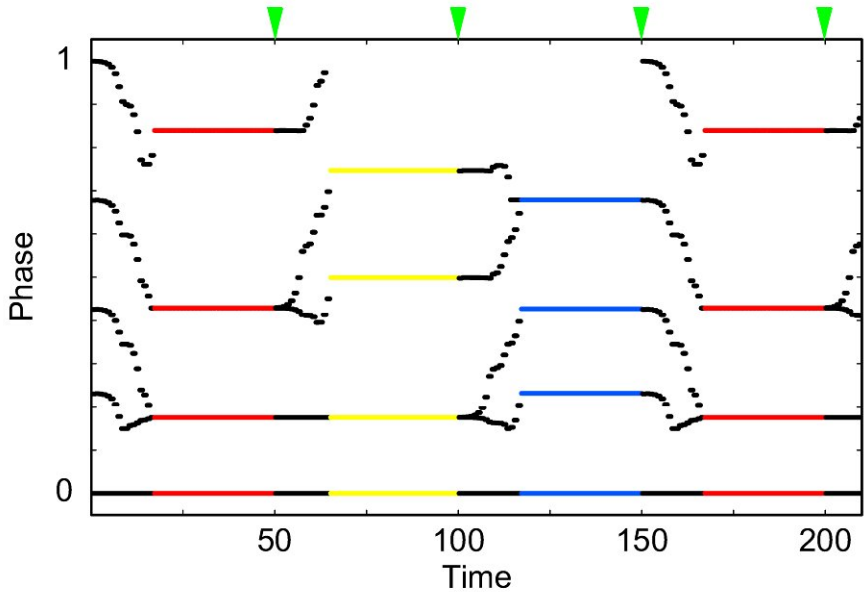


switching towards another attractor

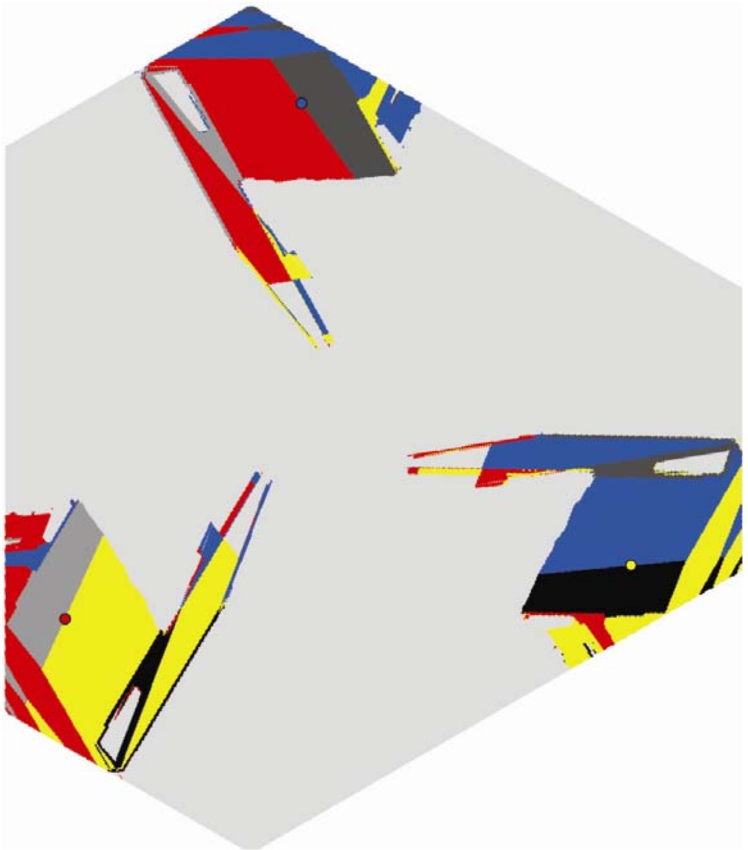
decay also occurs for very small pertubations ($\sigma = 10^{-22}$)

New Kind of Invariant Set: Unstable Attractor

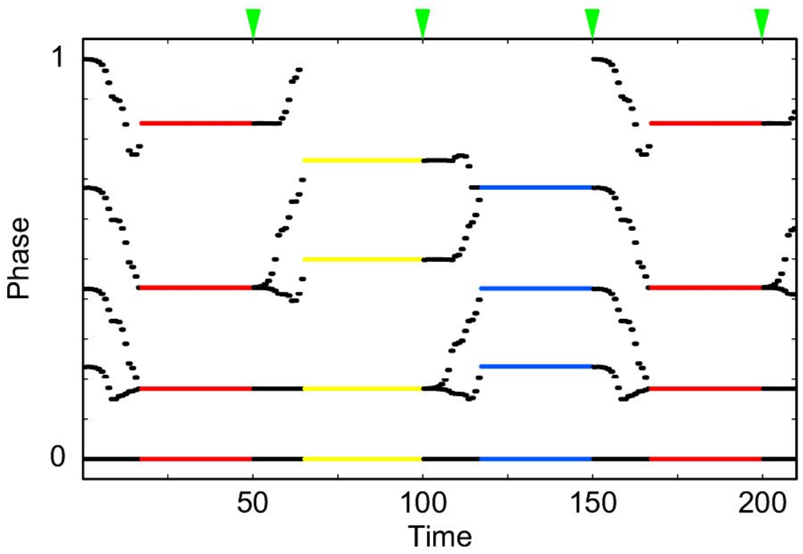
perturbations induce switching



Basin of attraction
(2D section through state space)



Analysis Confirms: Unstable & Attracting



$$\delta_{n+1} = \mathbf{F}(\delta_n) \doteq M\delta_n$$

$$\Downarrow$$

$$\lambda_1 = \ldots = \lambda_4 = 0$$

$$\lambda_5 = \frac{[2U'(c_0) - U'(a_1)]U'(c_1)U'(c_2)U'(c_3)}{U'(a_1)U'(a_2)U'(a_3)U'(a_4)} > 1$$

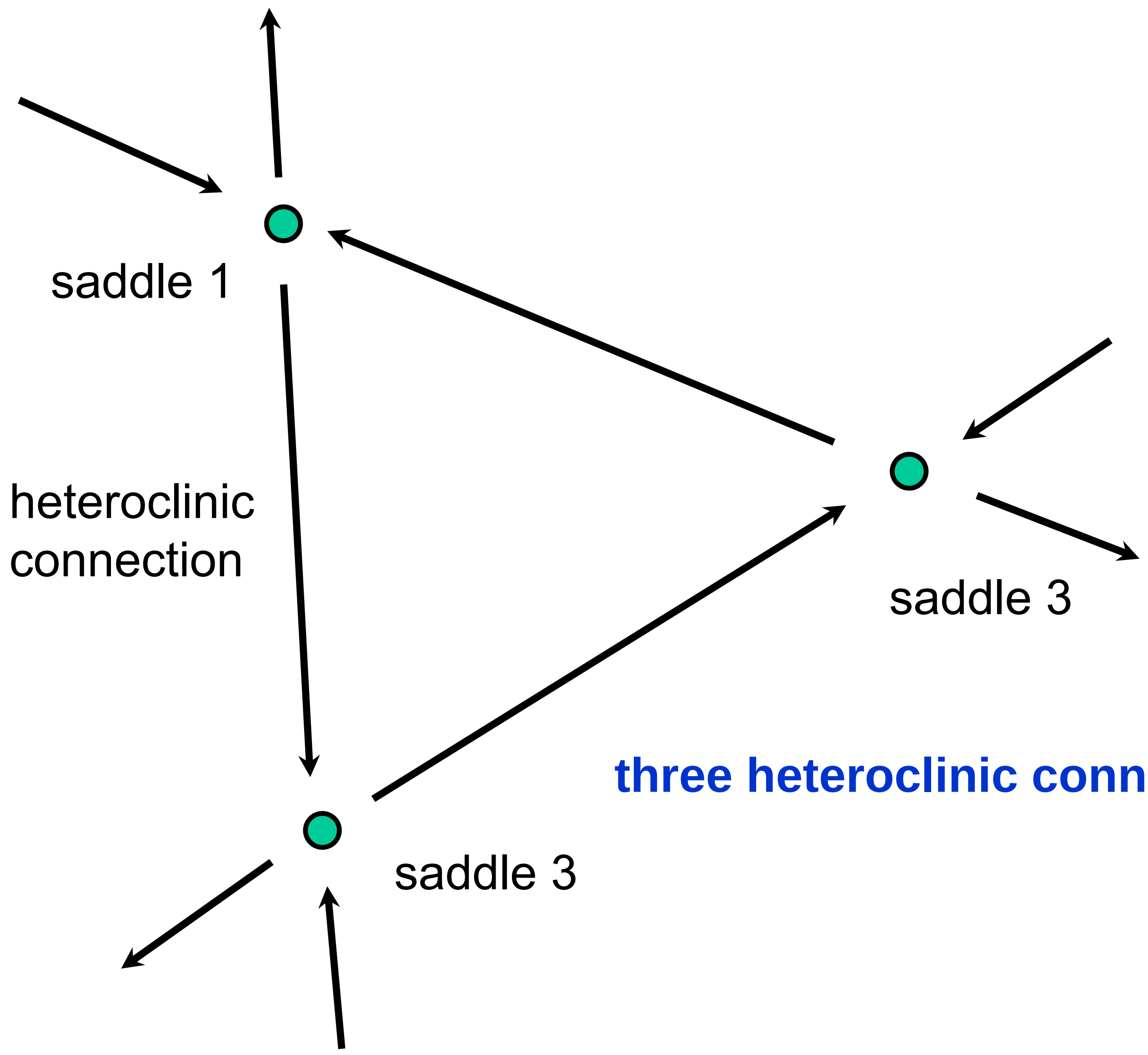
$$c_i > a_i > c_{i-1}$$

→ Locally unstable although attracting
(saddle periodic orbit with positive measure basin)

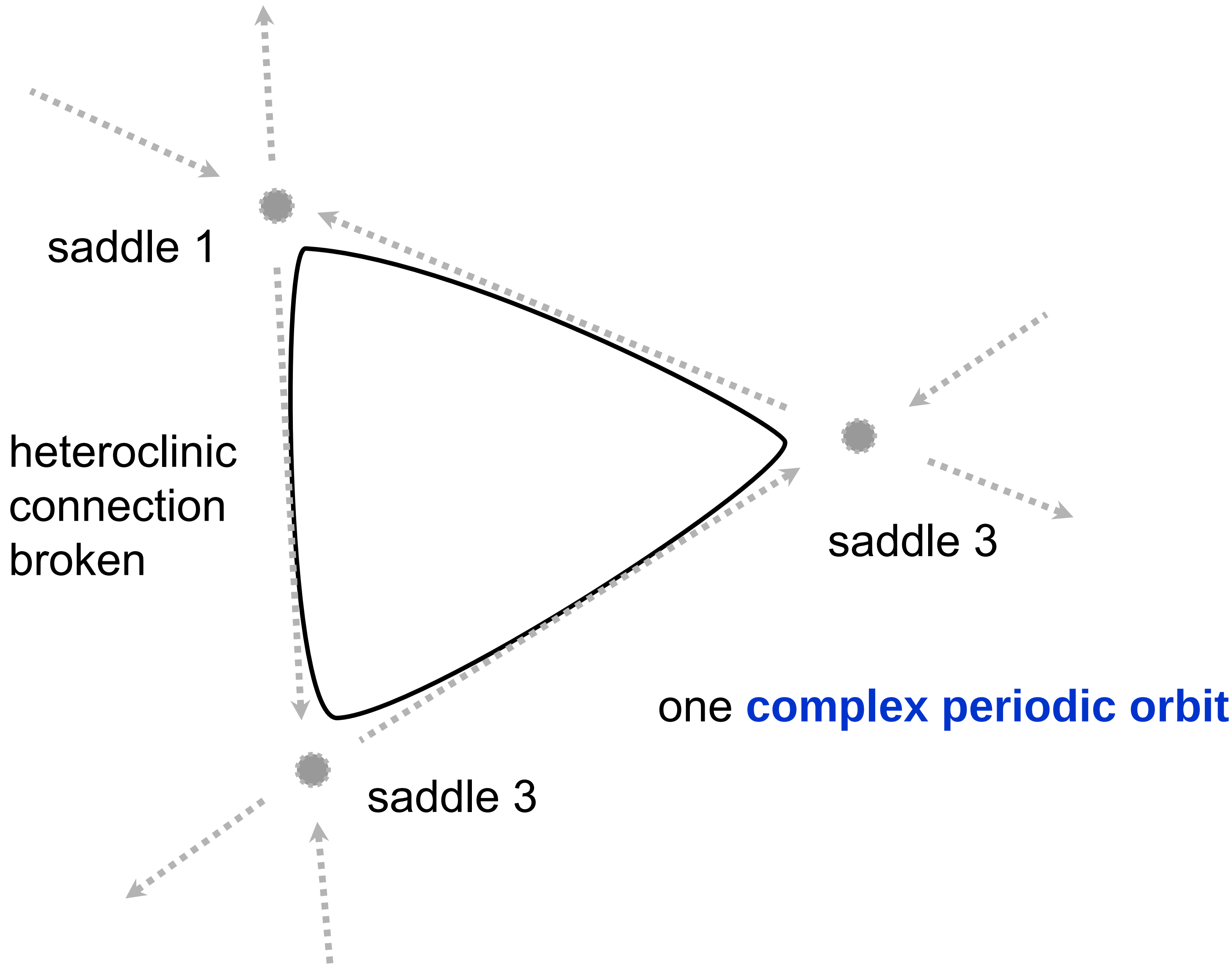
→ new kind of (Milnor) attractor: **unstable attractors**

First identification and analysis: M.T. et al.; *Phys. Rev. Lett.* 89:154105 (2002a)
Large networks: M.T. et al.; *Chaos* 13:377 (2003)
Rigorous results: P. Ashwin and M.T.; *Nonlinearity* 18:2053 (2005)
Functional relevance of switching: P. Ashwin and M.T., *Nature* 436:36 (2005)
Bifurcation: C. Kirst and M.T., *Phys. Rev. E (R)* (2008).

Cartoon of Heteroclinic Cycle in Symmetric Oscillator Systems



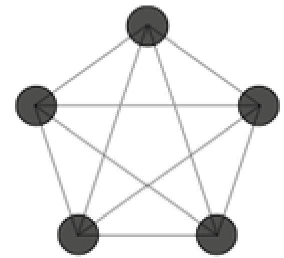
Breaking the Symmetry ➔ Periodic Orbit Close to Heteroclinic Cycle



Full symmetry in a network of N oscillators

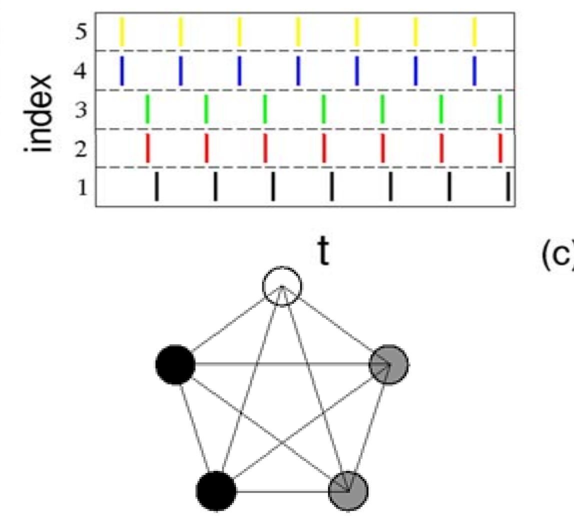
only three parameters: I , ε , τ.

(independent of N)



N=5:
V=(V1,V2,V3,V4,V5).

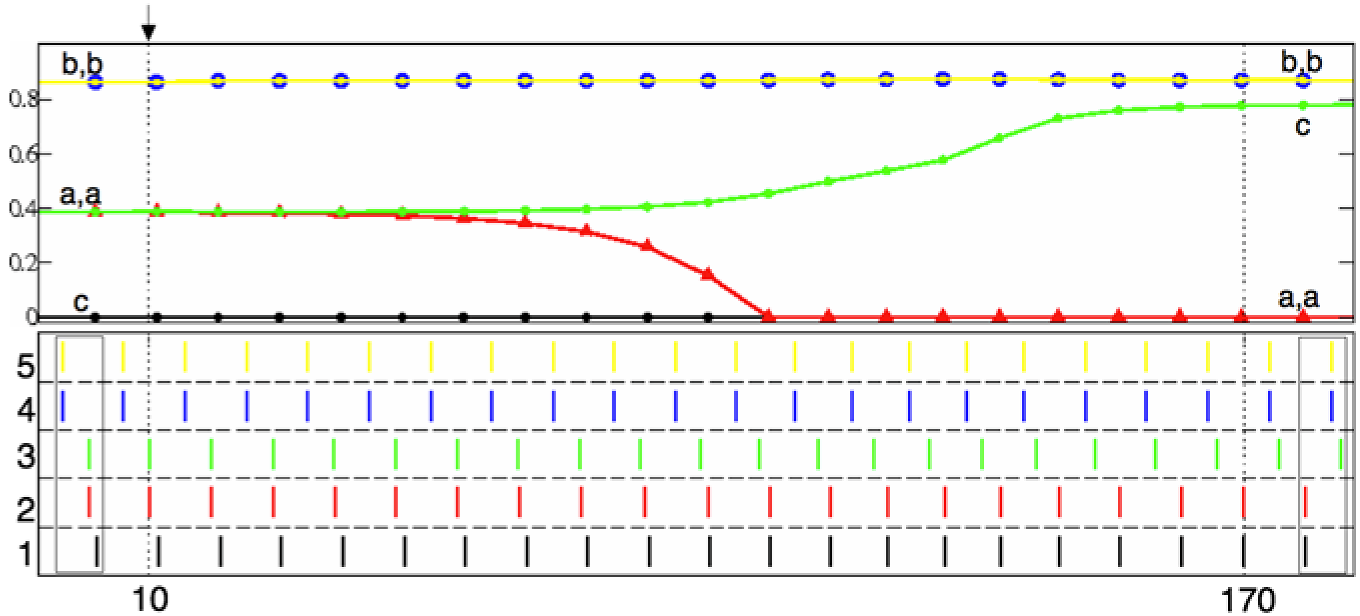
N=5: cluster states of different symmetries:



V=(a,a,b,b,c)

5!/(2!2!) = 30 saddle states

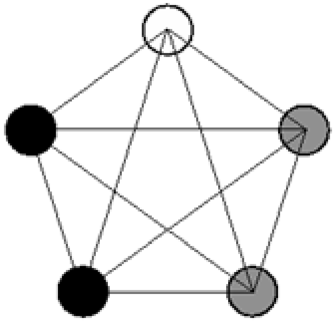
Saddle Instabilities and Heteroclinic Switching



(c,a,a+Δ,b,b) ➔ (b,b,c,a,a)

arbitrarily small perturbation induces **controlled switching**

Two ways to switch: **network of states**

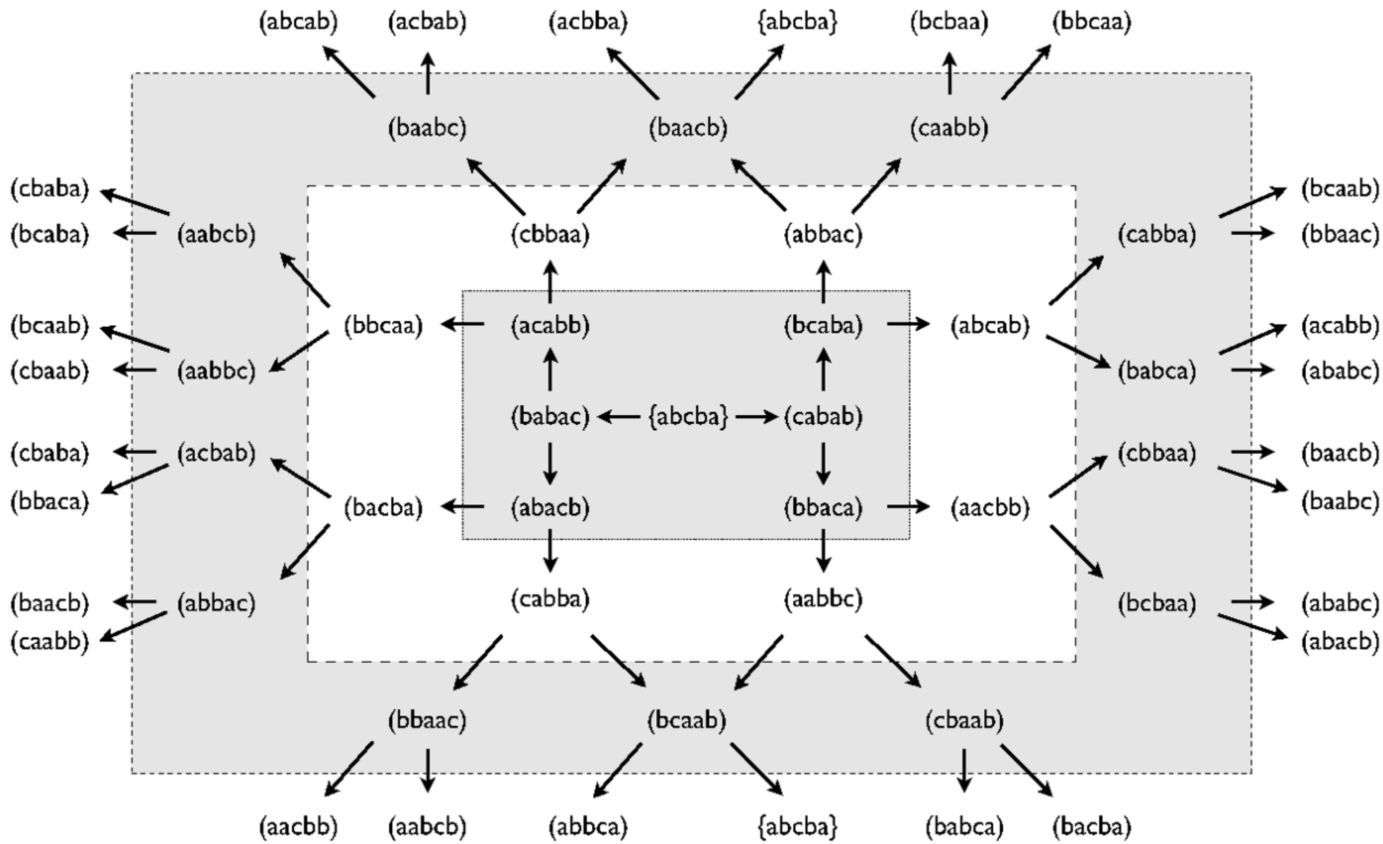


V=(a,a,c,b,b);

gray ‘a’ unstable, black ‘b’ and ‘c’ stable,

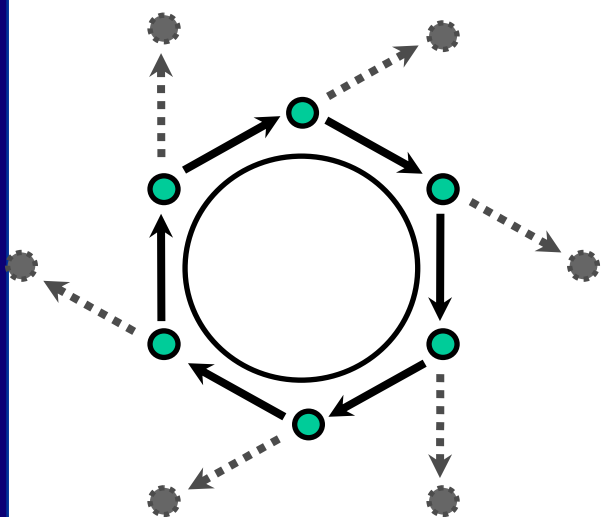
(a+Δ,a,b,b,c) ➔ (c,b,a,a,b)

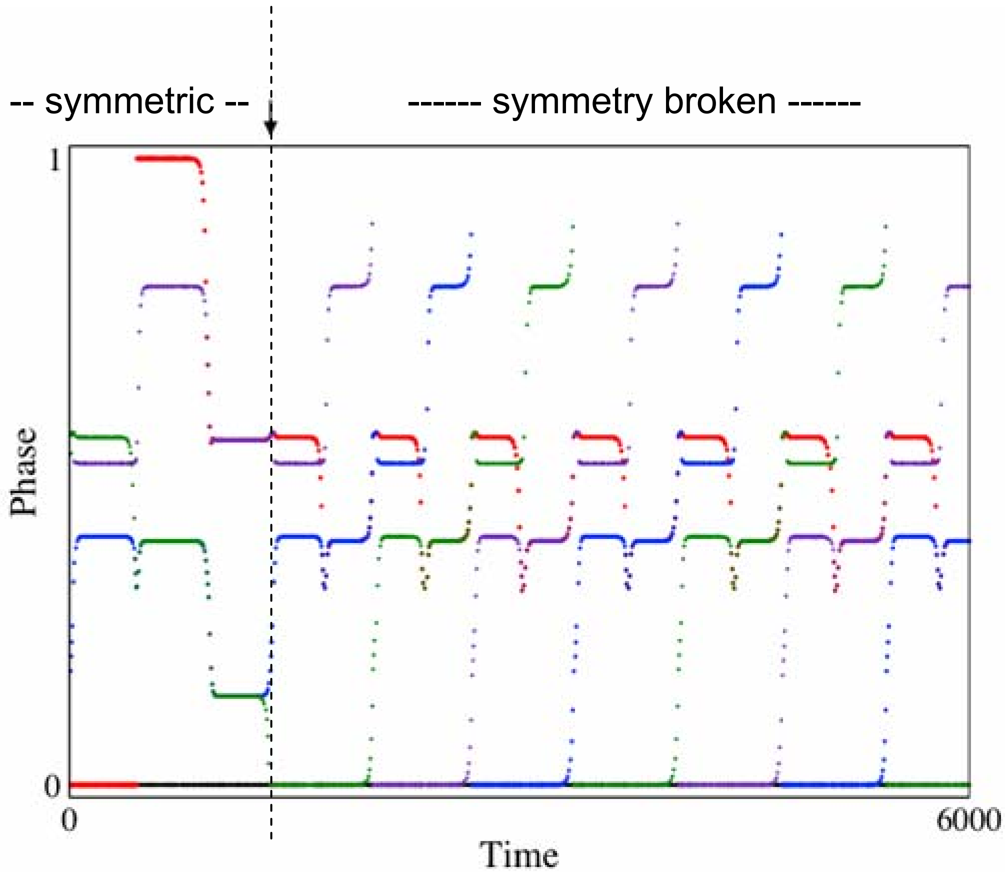
(a,a+Δ,b,b,c) ➔ (b,c,a,a,b)



Symmetry breaking induces **cyclic switching**

Symmetry breaking input currents: $I_1>I_2>I_3>I_4>I_5$

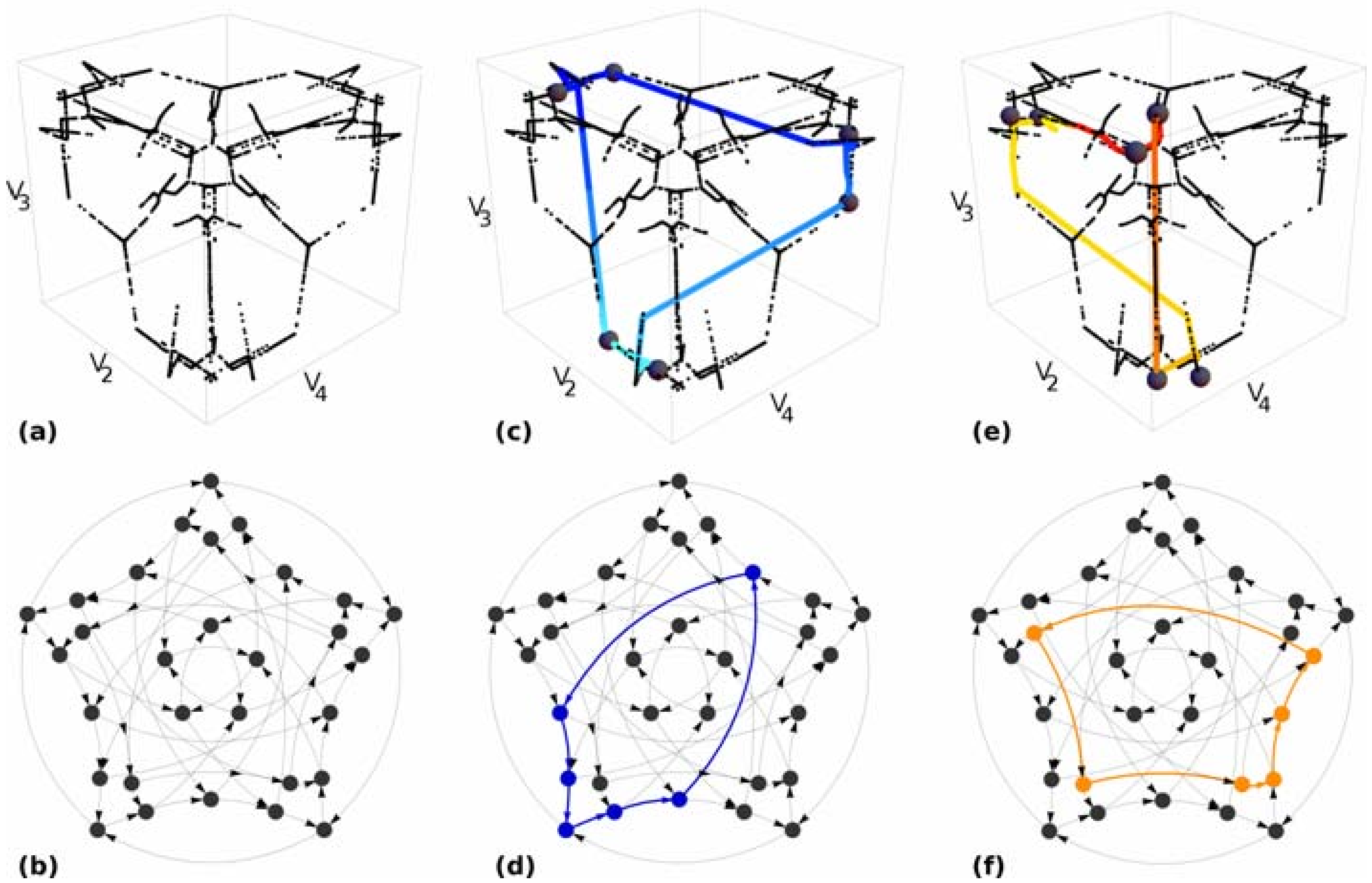




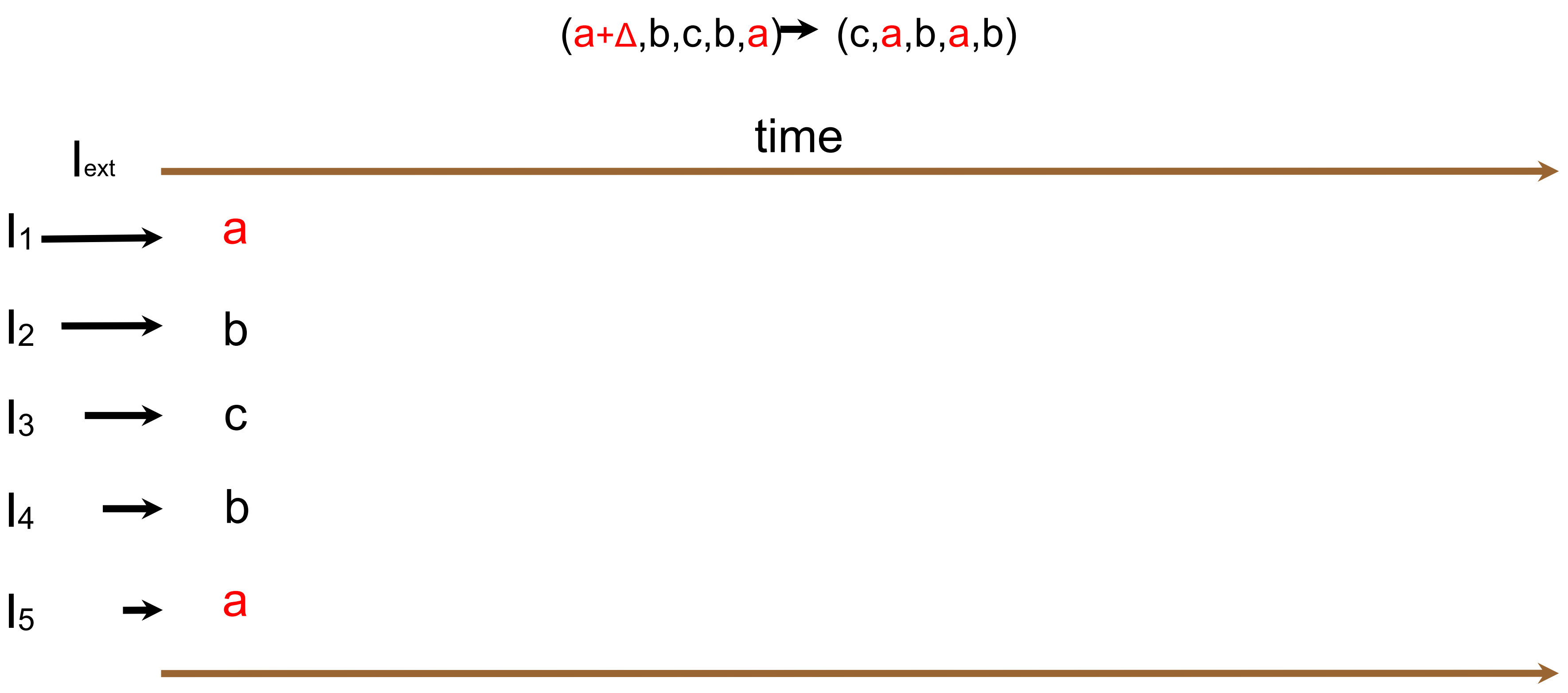
Cyclic switching along **complex periodic orbit**

Complex Network of Saddle States

Noise + asymmetry $\Delta \mathbf{I} = (4,3,2,1,0)$ $\Delta \mathbf{I} = (1,4,3,2,0)$

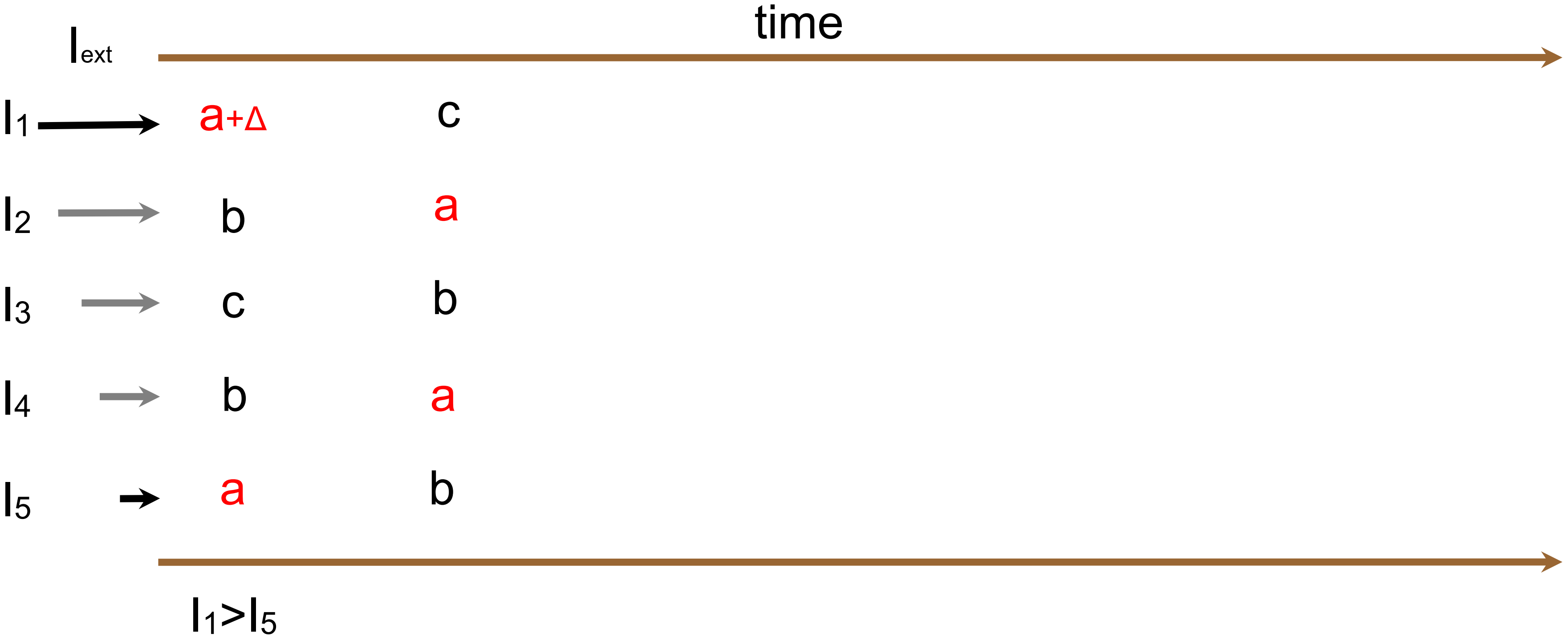


Symmetry breaking ➔ classification



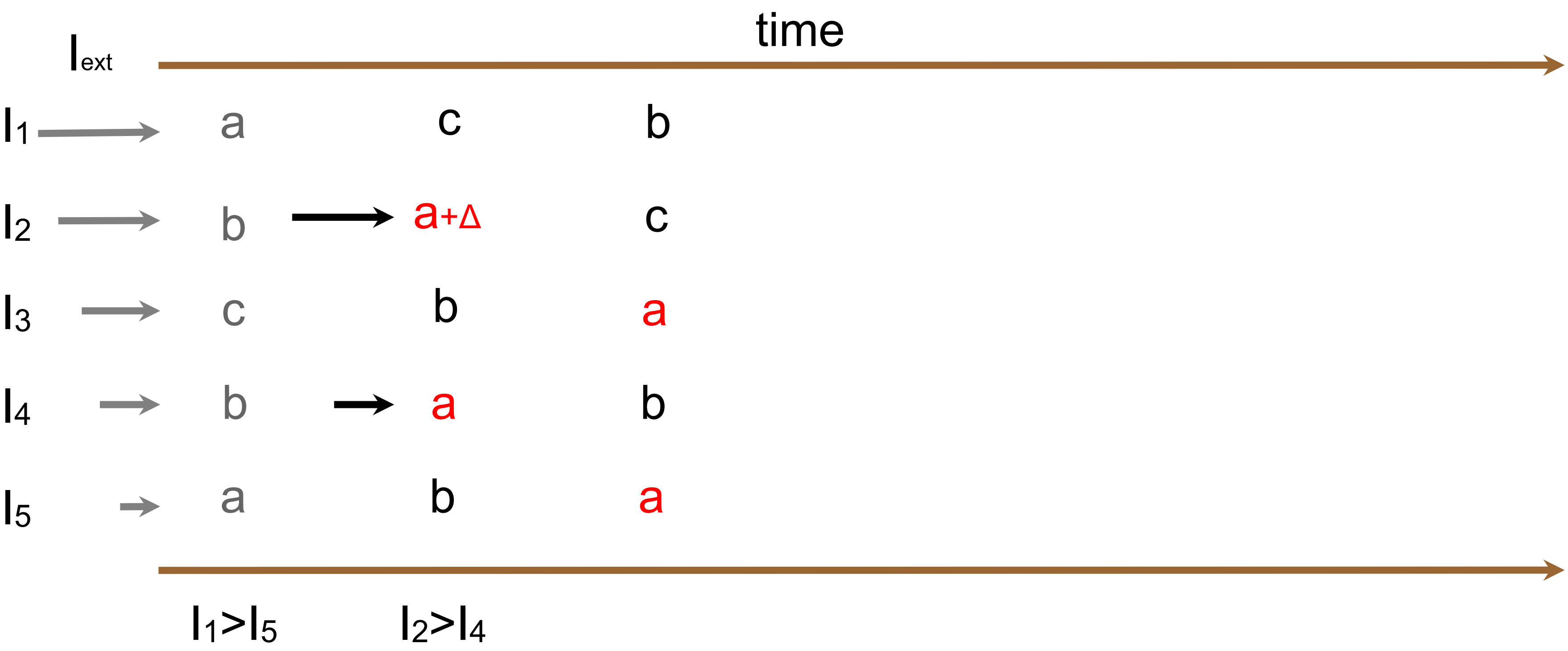
Symmetry breaking → classification

(a+Δ,b,c,b,a) → (c,a,b,a,b)



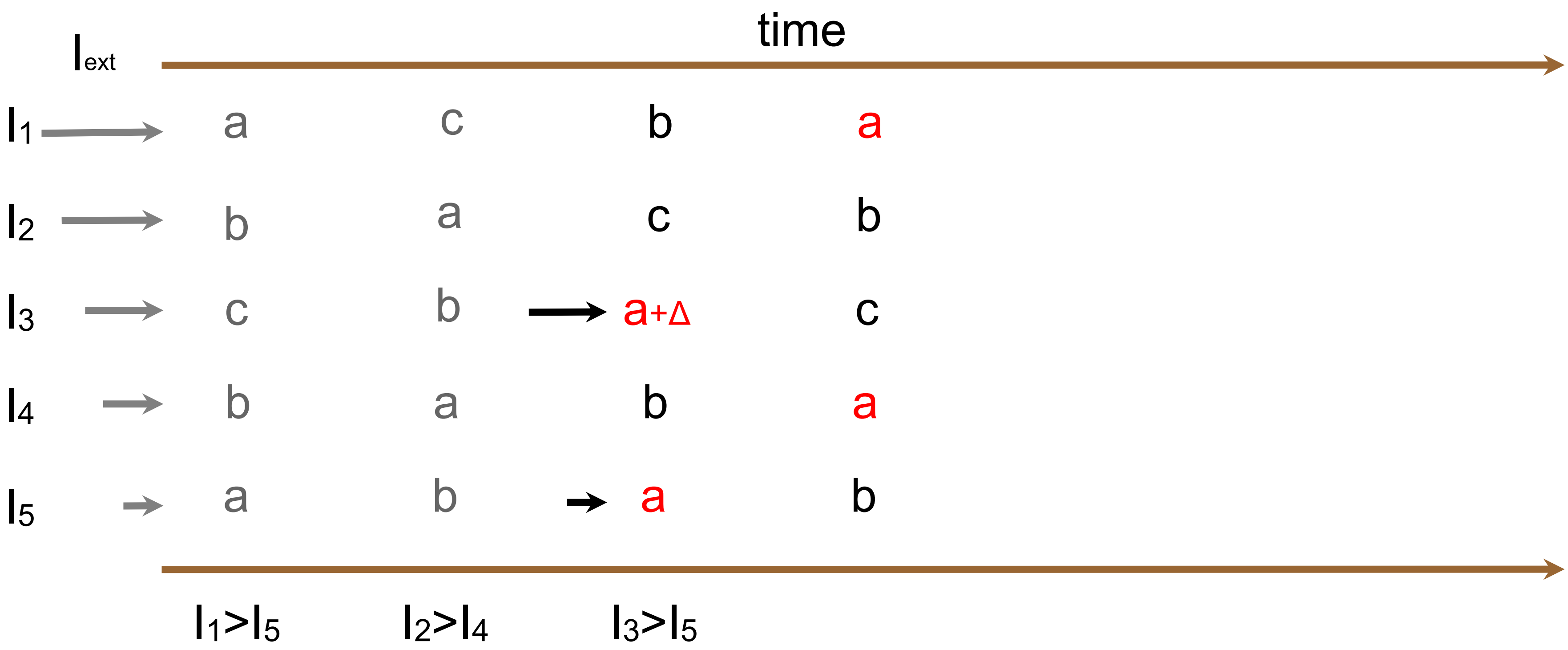
Symmetry breaking → **classification**

$(c, a+\Delta, b, a, b) \rightarrow (b, c, a, b, a)$



Symmetry breaking → classification

$(b,c,a+\Delta,b,a) \rightarrow (a,b,c,a,b)$



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Symmetry breaking → classification

$(a+\Delta,b,c,a,b) \rightarrow (c,a,b,b,a)$

time

I_{ext}					
I_1	a	c	b	$a+\Delta$	c
I_2	b	a	c	b	a
I_3	c	b	a	c	b
I_4	b	a	b	a	b
I_5	a	b	a	b	a
	$I_1>I_5$	$I_2>I_4$	$I_3>I_5$	$I_1>I_4$	

Symmetry breaking → classification

(c,a+Δ,b,b,a) → (b,c,a,a,b)

time

I_{ext}						
I_1 →	a	c	b	a	c	b
I_2 →	b	a	c	b →	a+Δ	c
I_3 →	c	b	a	c	b	a
I_4 →	b	a	b	a	b	a
I_5 →	a	b	a	b →	a	b
	$I_1>I_5$	$I_2>I_4$	$I_3>I_5$	$I_1>I_4$	$I_2>I_5$	

Symmetry breaking → classification

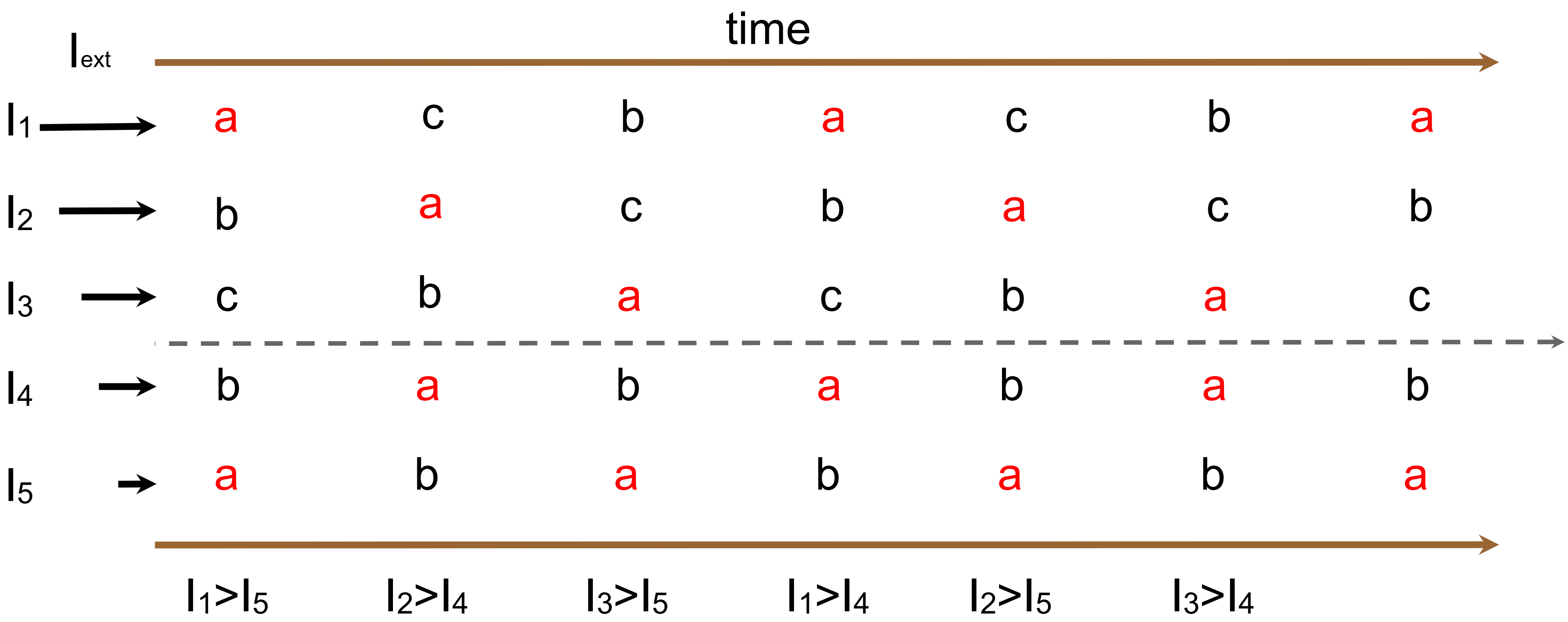
(b,c,a+Δ,a,b) → (a,b,c,b,a)

time

I_{ext}							
I_1 →	a	c	b	a	c	b	a
I_2 →	b	a	c	b	a	c	b
I_3 →	c	b	a	c	b	→ a+Δ	c
I_4 →	b	a	b	a	b	→ a	b
I_5 →	a	b	a	b	a	b	a
	$I_1>I_5$	$I_2>I_4$	$I_3>I_5$	$I_1>I_4$	$I_2>I_5$	$I_3>I_4$	

Symmetry breaking → classification

$(a+\Delta, a, b, b, c) \rightarrow (c, b, a, a, b)$



result: $\{I_1, I_2, I_3\} > \{I_4, I_5\}$

Symmetry breaking → classification

$(a+\Delta, a, b, b, c) \rightarrow (c, b, a, a, b)$

time

I_{ext}								
I_1 →	a	c	b	a	c	b	a	■
I_2 →	b	a	c	b	a	c	b	■
I_3 →	c	b	a	c	b	a	c	■
I_4 →	b	a	b	a	b	a	b	□
I_5 →	a	b	a	b	a	b	a	□
	$I_1>I_5$	$I_2>I_4$	$I_3>I_5$	$I_1>I_4$	$I_2>I_5$	$I_3>I_4$		

result: $\{I_1, I_2, I_3\} > \{I_4, I_5\}$

From Digital Analog Conversion to Classification

Encode 10 different classes by 5 neurons.

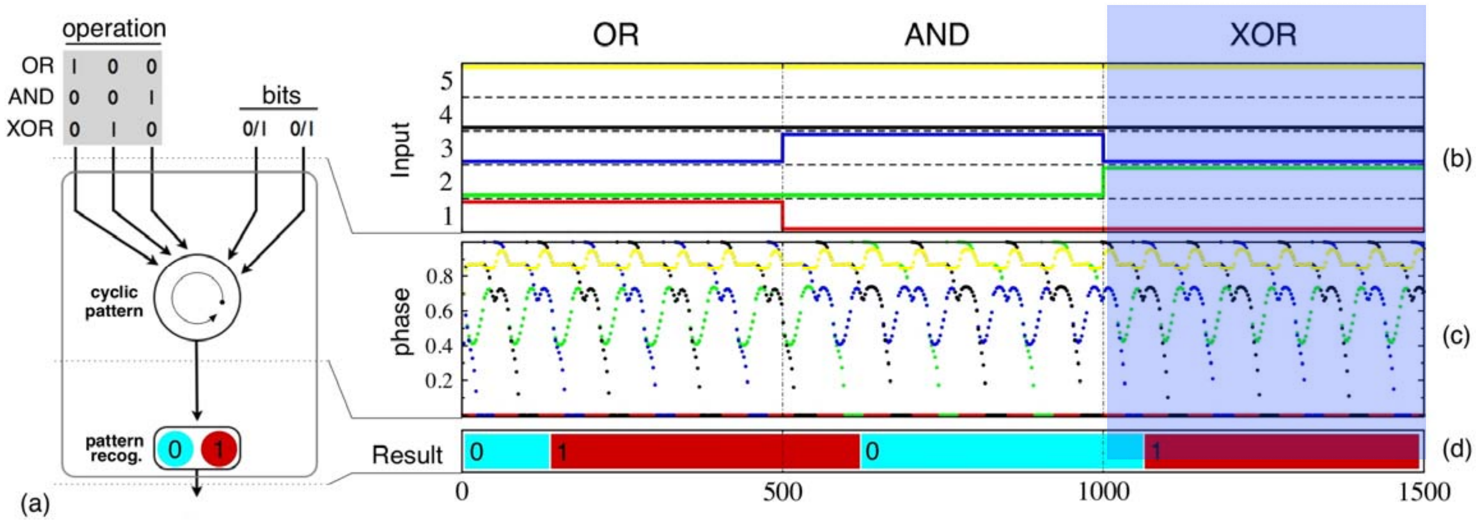
Neuron	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	■	■	■	■	■	□	□	■	□	□
2	■	■	■	□	□	■	■	□	■	□
3	■	□	□	■	■	■	■	□	□	■
4	□	■	□	■	□	■	□	■	■	■
5	□	□	■	□	■	□	■	■	■	■

class

#classes grows exponentially with number N of neurons
e.g. N=100, 10^8 classes

➔ **classification provides basis for computation**

Arbitrary Binary Computation

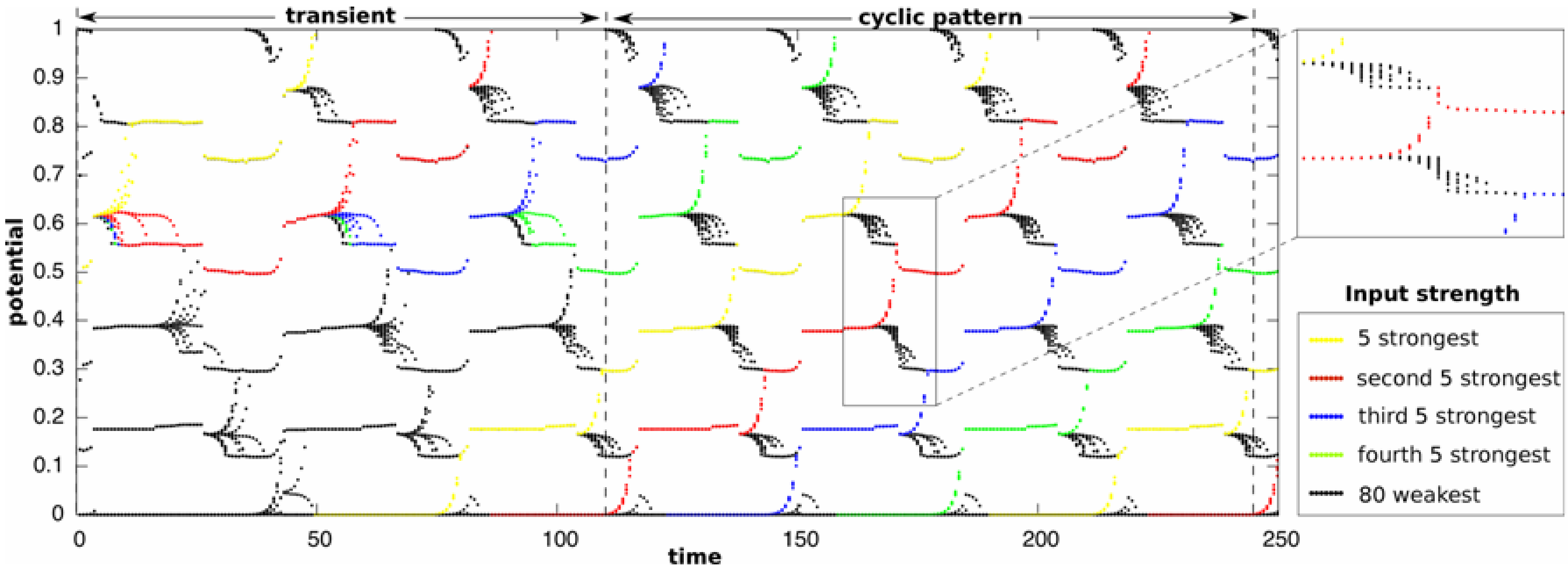


Operation table: XOR (0,1)

Input signal		0	1	0	**0**	**1**
Input weights ($\times 10^{-4}$)	×	2.5	2.5	2.5	5	5
effective input ($\times 10^{-4}$)		0	2.5	0	0	5
base asymmetry ($\times 10^{-4}$)	+	4	3	2	1	0
total asymmetry ($\times 10^{-4}$)		4	5.5	2	1	5
result category		■	■	□	□	■

Operation	Input signal	out. category	
AND	(0,0,1,**0,0**)	■■■□□	0
	(0,0,1,**0,1**)	■□■□■	0
	(0,0,1,**1,0**)	■□■■□	0
	(0,0,1,**1,1**)	□□■■■	1
OR	(1,0,0,**0,0**)	■■■□□	0
	(1,0,0,**0,1**)	■■□□■	1
	(1,0,0,**1,0**)	■■□■□	1
	(1,0,0,**1,1**)	■□□■■	1
XOR	(0,1,0,**0,0**)	■■■□□	0
	(0,1,0,**0,1**)	■■□□■	1
	(0,1,0,**1,0**)	■■□■□	1
	(0,1,0,**1,1**)	□■□■■	0

Larger networks ➔ gigantic no. of options



20-winner-take-all computation;

5 x 10^20 possible outcomes with N=100 neurons
(exponential in N)

Complex Networks of States → New Type of Natural Computer

Versatile with only five identical units:

- **unary** processing, e.g.: NOT
- **single binary** processing
- **multiple binary** processing
  (up to three operations at given parameters)
- **single ternary**
  (because 2^3=8 possible input classes < 10 possible output classes)

Universal computation as generic feature

- **analog-digital converter, classifier**
- **k winner takes all**
- **n-ary logics**
- **scales well** with system size

Future & current work:

- **adaptation**
- **hardware/robot** implementation (in progress)
- transfer of **spatio-temporal patterns** (instead of binary conversion)

Encode 10 different classes by 5 neurons.

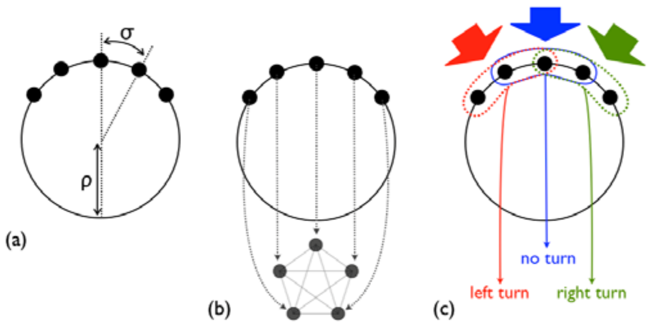
Neuron	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	■	■	■	■	■	□	□	■	□	□
2	■	■	■	□	□	■	■	□	■	□
3	■	□	□	■	■	■	■	□	□	■
4	□	■	□	■	□	■	□	■	■	■
5	□	□	■	□	■	□	■	■	■	■

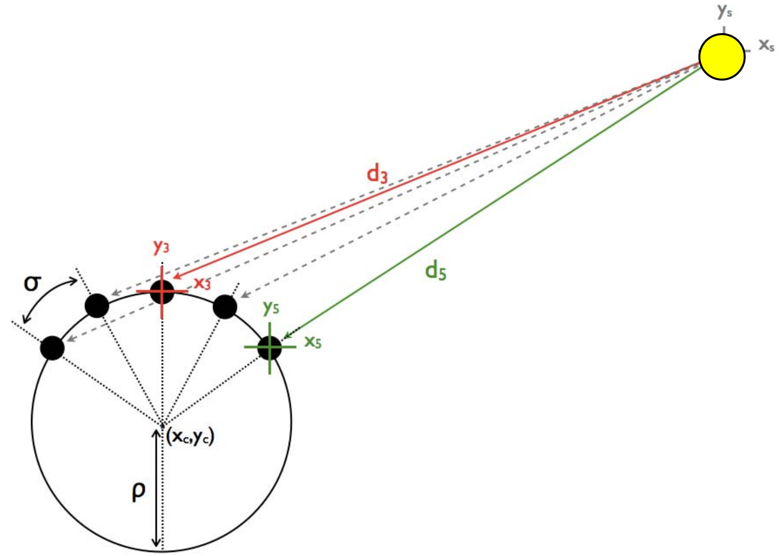
class

Behavioral Autonomies

Phototaxis

finding or following a light (=food) source





Obstacle avoidance & untrapping

Find the way out of a number of „Network Science“books

Behavioral Autonomies

Theoretical Challenges in Network Dynamics: **Adaptation**, **Inference**, **Computation** & **Behavior**

Selected works:

- **Network Dynamics and Information Processing**

Phys. Rev. Lett. 89:258701 (2002c);
Phys. Rev. Lett. 92:074103 (2004a);
Phys. Rev. Lett. 92:074101 (2004b);
Phys. Rev. Lett. 93:074101 (2004c);
Chaos 16:015108 (2006);
Phys. Rev. E 78:065201(R) (2008);
Nonlinearity 21:1579 (2008);
Phys. Rev. Lett. 102:068101 (2009);
Chaos, 21:025113 (2011);
Europhys. Lett. 90:48002 (2010);
SIAM J. Appl. Math. 70:2119 (2010)

- **Network Inference: Design, Reconstruction and Stability**

Phys. Rev. Lett. 97:188101 (2006);
Physica D 224:182 (2006);
Europhys. Lett. 76:367 (2006);
Phys. Rev. Lett. 98:224101 (2007);
Phys. Rev. Lett. 100:048102 (2008);
Frontiers in Comput. Neurosci. 3:13 (2009);
Frontiers Comp. Neurosci. 5:3 (2010);
New J. Physics. 13:013004 (2011)

- **Spatio-temporal patterns, control and computation**

***Phys. Rev. Lett.* 89:154105 (2002a);**
***Chaos* 13:377 (2003);**
***Nonlinearity* 18:20 (2005);**
Neurocomputing 70:2096 (2007);
Neurosci. Res. 61:S280 (2008);
Frontiers in Neurosci. 3:2 (2009);
Discr. Cont. Dyn, Syst. 28:1555 (2010);
Handbook on Biological Networks (Chapter on 'Spike Patterns'), World Scientific (2010);

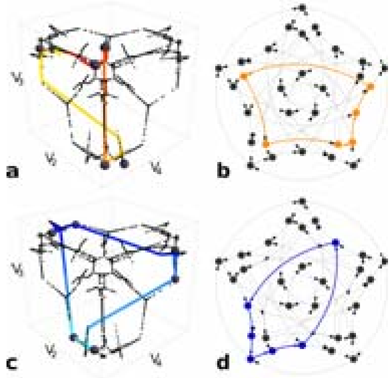
Theoretical Challenges in Network Dynamics:
Adaptation, **Inference**, **Computation** & **Behavior**

- **Adaptation** and **autonomous robots** via nonlinear dynamics

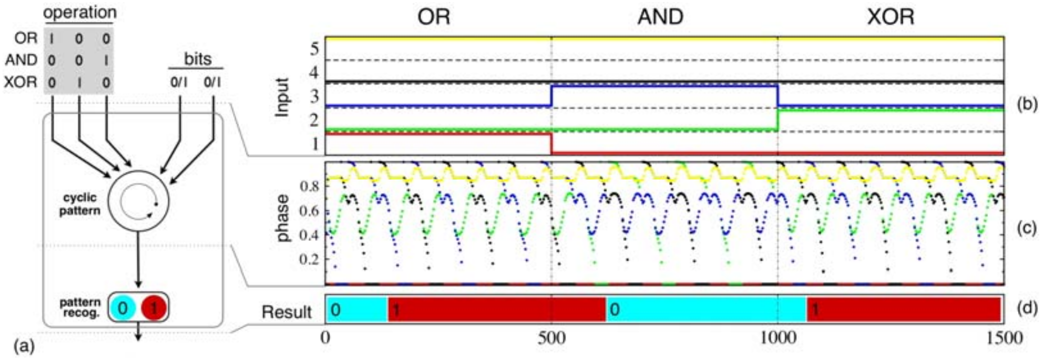
  ***Nature* 436:36 (2005);** ***J. Phys. A: Math. Theor.* 42:345103 (2009);**
  Phys. Rev. Lett., under review (Kielblock et al., 2012)
  ***Nature Phys.* 6:224 (2010);**

- **Intelligent coordination and new computational devices**

  ***Phys. Rev. Lett.* 88:245501 (2002b);** *Cornell Rep.* 1813:1352 (2007);
  New J. Phys. 11:023001 (2009); *J. Phys. A: Math. Theor.*, 43:175002 (2010);
  Nature Phys., 7:265 (2011); ***Phys. Rev. Lett.* in press (Schittler Neves & MT, 2012)**



➔



Thanks to ...

Network Dynamics Group – MPI f. Dynamics & Self-Organization

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Shubham Dipt	Federico Faraci	Britta Feldsmann
Frederik Fix	Carsten Grabow	Sven Jahnke
Hinrich Kielblock	Christoph Kirst	Christoph Kolodziejski
Fabio Schittler-Neves	Martin Rohden	Andreas Sorge
Heike Vester	Gunter Weber	Dirk Witthaut

Raoul-Martin Memmesheimer **Harvard / Nijmegen**

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Fred Wolf, Andre Fiala, all colleagues at MPIDS & BCCN **Göttingen**

Silke Steingrube **Solar Energy Research, Univ. Hannover**

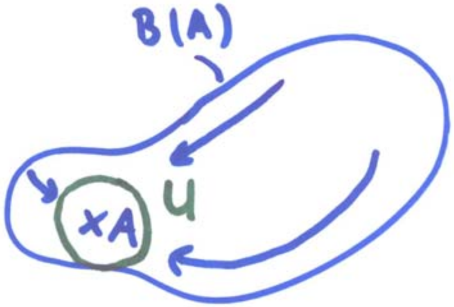
Shuwen Chang, Holger Taschenberger **MPI BPC Göttingen**

YOU all for your attention !

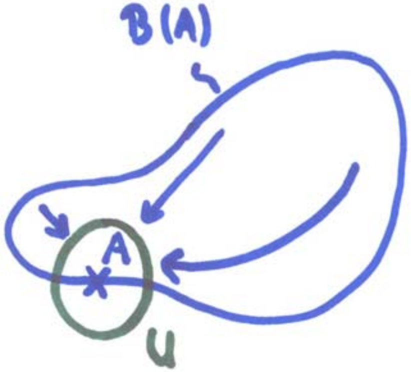
Questions & Comments Welcome!

Different concepts of an attractor

main requirement: attractor *A* has basin of attraction *B(A)* of positive volume



conventionally: contracting neighbourhood *U*
⇒**these attractors are stable**

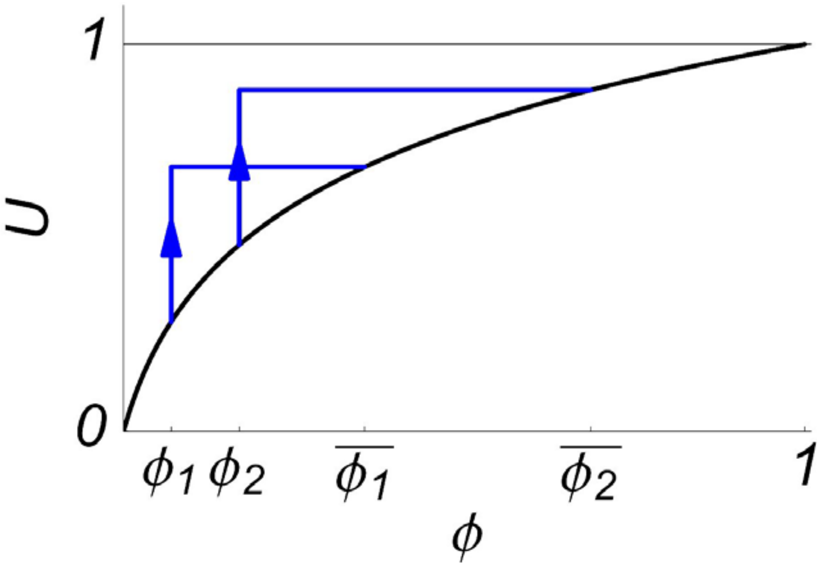


Milnor: **no** contracting neighbourhood *U*
⇒**attractors may be unstable**

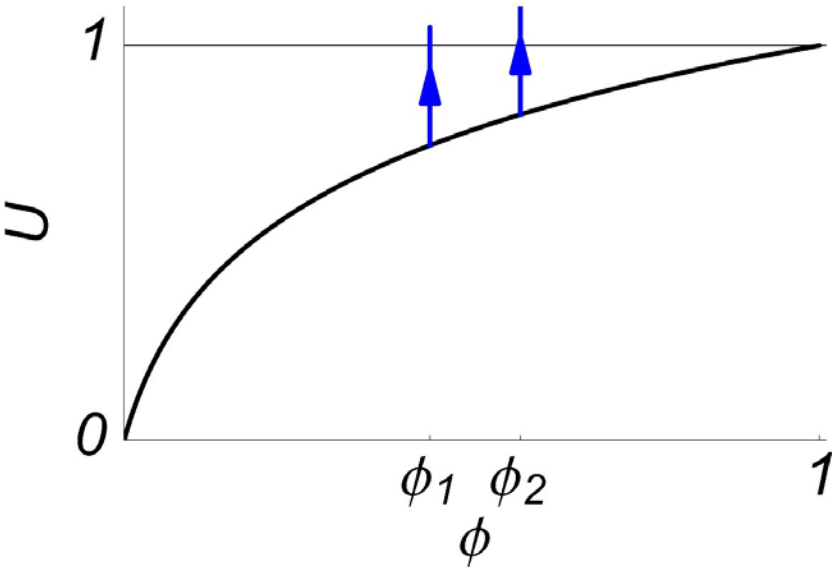
parameter tuning to obtain unstable attractors ?

Instability and attraction

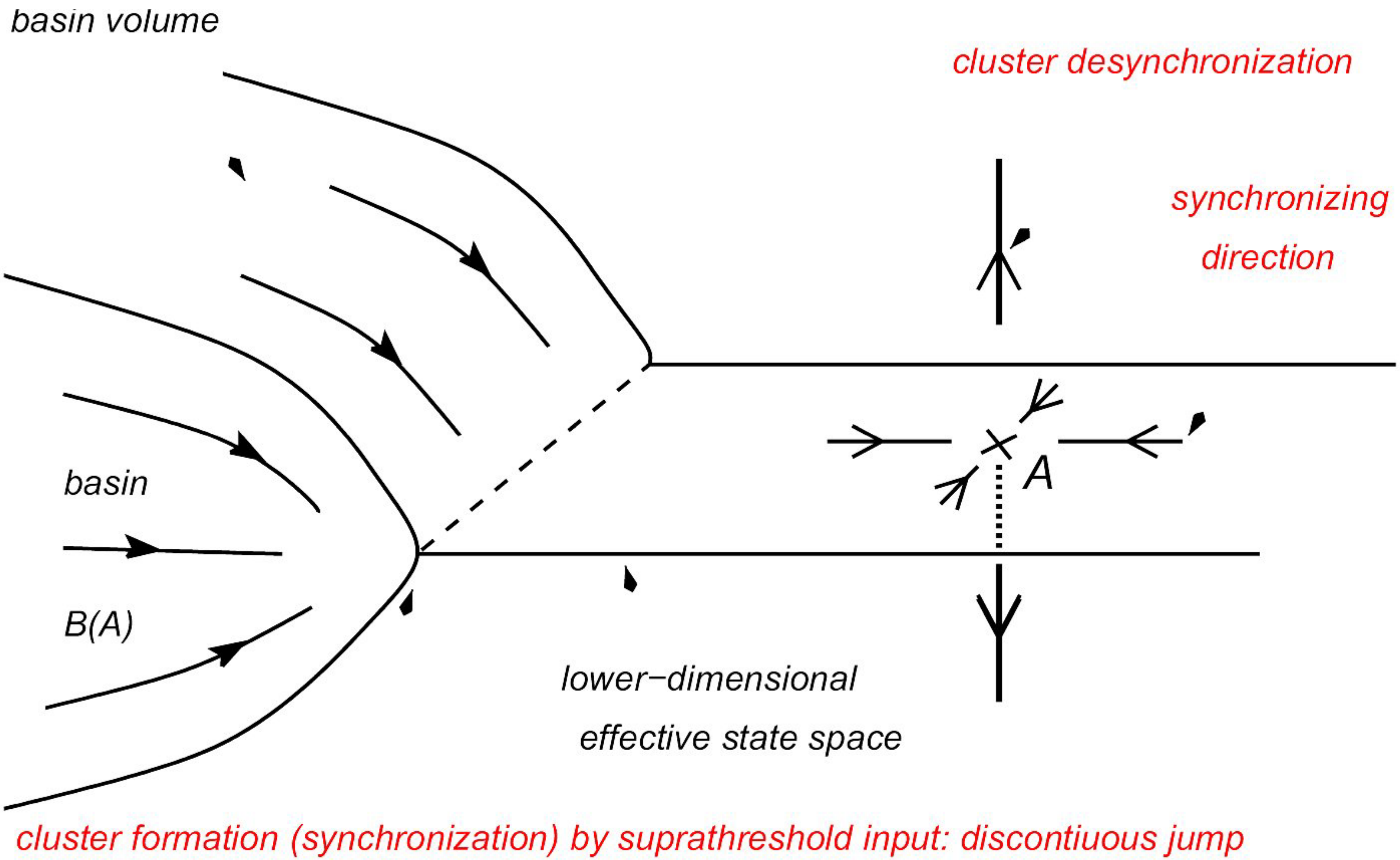
subthreshold input desynchronizes
$\Rightarrow$ cluster states may be unstable



suprathreshold input synchronizes
$\Rightarrow$ cluster states may be attractors

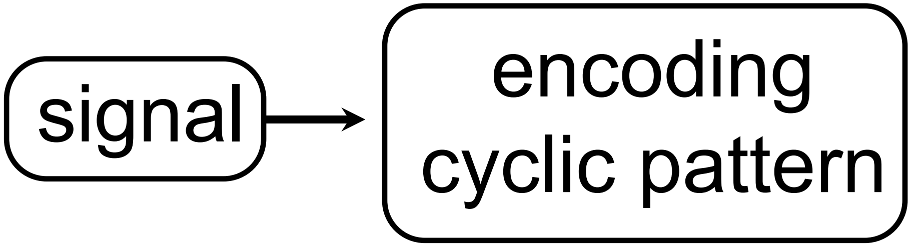


How does an unstable attractor work?



mechanism in large systems: M.T., F. Wolf, T. Geisel, *Chaos* 13:377-387 (2003)

Use of heteroclinic switching so far: **encoding**



D. Hansel, G. Mato, and C. Meunier, *Phys. Rev. E* (1993):

U. Ernst, K. Pawelzik, and T. Geisel, *Phys. Rev. Lett.* (1995); *Phys. Rev. E* (1998).

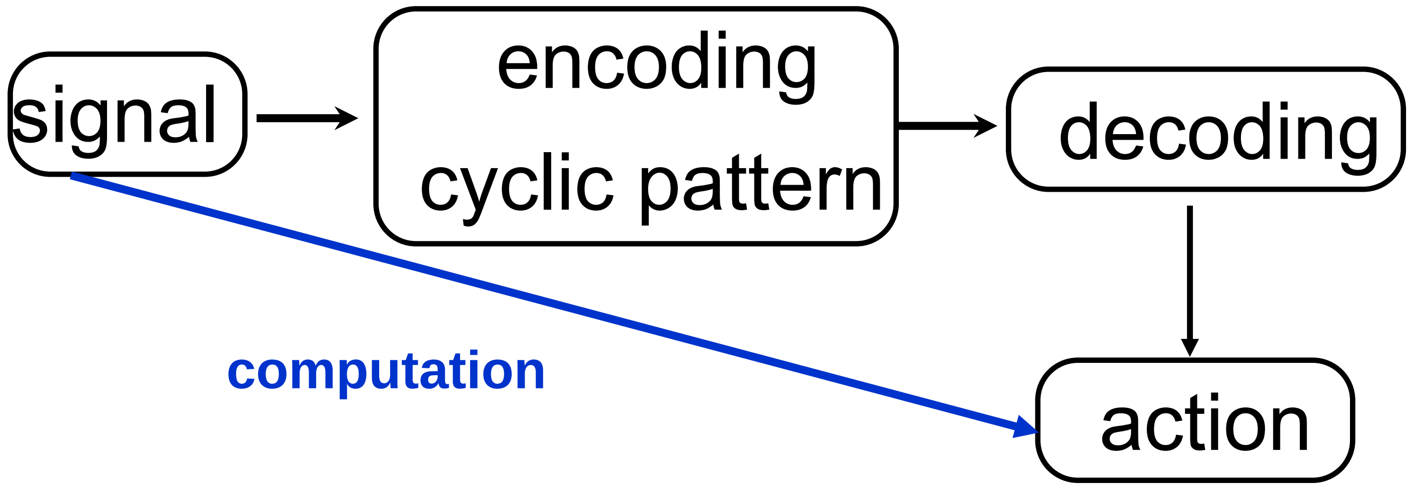
M. Rabinovich et al., *Phys. Rev. Lett.* (2001); T. Nowotny & M. Rabinovich, *Phys. Rev. Lett.* (2008);

M.T., F. Wolf, T. Geisel, *Phys. Rev. Lett.* (2002); *Chaos* (2003)

G. Orosz et al., *Proc. Appl. Math. Mech*. (2007).; J. Borresen & P. Ashwin, *Phys. Rev. E* (2008)

C. Kirst & M.T., *Phys. Rev. E* (2008). F. Schittler Neves & M.T., *J. Phys. A: Math. Theor*., (2009).

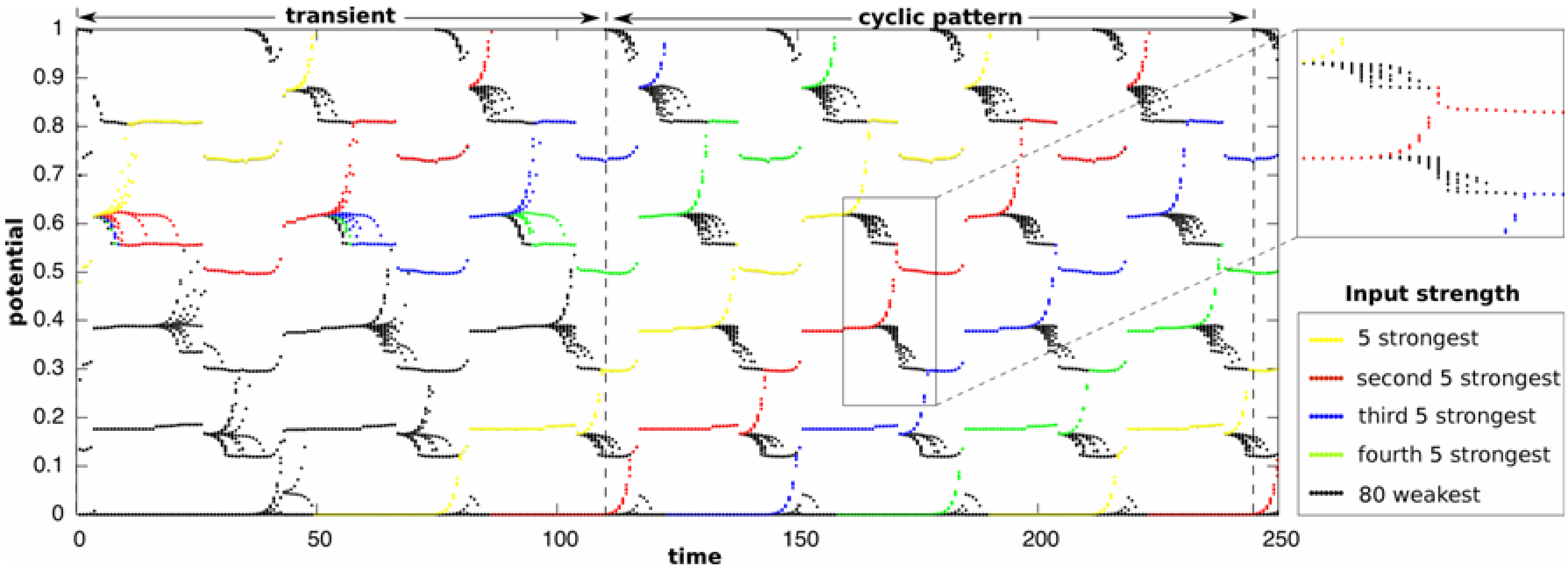
Towards **computation** via heteroclinic switching



P. Ashwin and J. Borresen.
Discrete computation using a perturbed heteroclinic network.
Phys. Lett. A (2005);

(still requires *external* logic device)

Larger Networks, other clustering



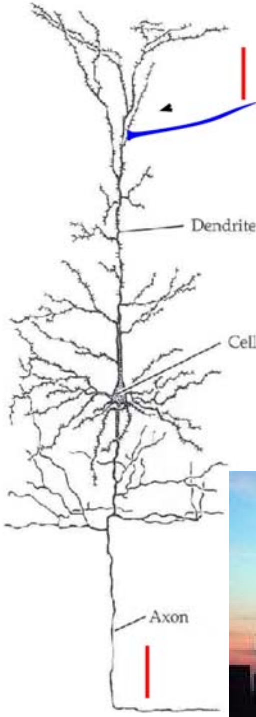
20 winner-take-all computation

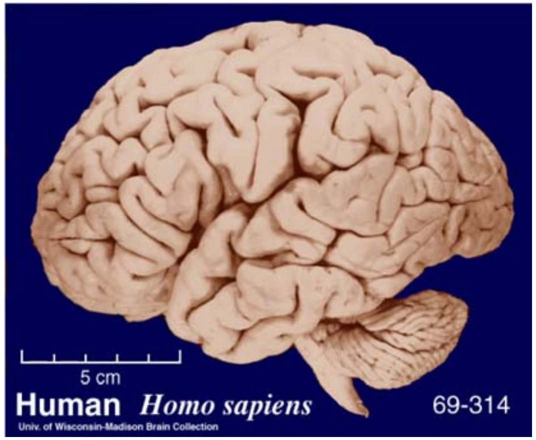
Spatio-Temporal Patterns in Networks of Biology and Physics

Biological Networks

$(10^{-3} - 10^{10} s;\ 10^{-5} - 10^{-1} m)$

- Computation in Neural circuits
- „Tree“ of life
  (speciation in early evolution)





Networks of physical & artificial units

$(10^{-2} - 10^{10} s;\ 10^{-9} - 10^{6} m)$

- Complex disordered media
- Modern power grids (mind the re
- Autonomous robots
- Natural **computing** devices