Role of timescales in evolution of bimodal degree distribution networks

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Abstract

We present an algorithmic scheme for constructing a network of desired degree distribution, typically one with bimodal degree distribution. The procedure adopted is to add nodes to the network with a probability p and delete the links between nodes with probability (1 − p).

We introduce an additional constraint in the process through an immunity score, which controls the dynamics of the growth process based on the feedback value of the last few steps. We find that this then leads to bimodal nature for the degree distribution. We study the standard network characterizers like average path length and clustering coefficient in the context of our growth process.

Objective

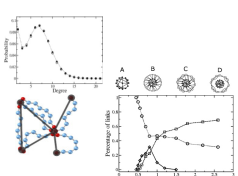
The topology not only helps to classify a network but also is indicative of its robustness and functional organization.

The stability of a network to random node removal (error tolerance) or targeted removal (attack tolerance) is found to depend on its topology [1]. The SF topology is more robust to random node removals than to targeted node removals since the presence of hubs in scale free networks makes them vulnerable to targeted node removals.

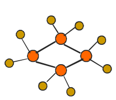
In contrast, bimodal degree distribution networks (BDDN) are found to be robust to both targeted and random strategies of removal of nodes and edges [2,3]. The nodes of this network falls under two modes. For each mode, we denote the local mean degree by ‘k’. Then, one mode involving the nodes whose local mean degree k1 are called as the ‘super-peer’ nodes while the second mode has nodes called ‘peer’ nodes. These modes provides the BDDN with enhanced stability under random or targeted attacks.

How do we generate networks which have bimodal degree distribution?

e.g. Networks with bimodal degree distribution

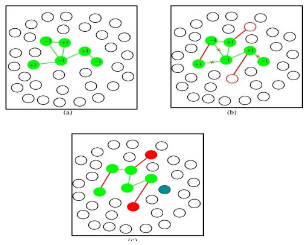


Super peer networks (Gnutella, Fast Track KaZaA, Skype) emerges as most widely used network



ALGORITHM

We need to find a way to segregate nodes based on their degree. To achieve this we propose a simple model as follows:



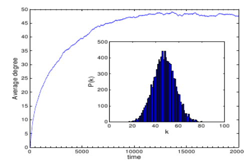


Fig. 1: (Color Online) Average degree $\bar{k}$ vs time for network size 10000 for unbiased case $p = 0.5$. The average degree increases and then saturates due to finite size limit. In the inset, degree distribution $P(k)$ of the network is shown at timestep 20000.

THE MODEL:

Three types of nodes

1) Active: Have at least one link..get a coin toss!!

2) Inactive: No link but may become active when They get a link.

3)Dead: No coin toss for them ever!!

1. Toss a coin for each of active nodes at time t.

2. Each node ‘i’ makes new link to a node A with probability r(i)<=p. This might set an ‘inactive’ node A as ‘active’ for next time step.

3. With probability r(i) > p, the node 'i' breaks one of its link with existing connections with equal probability.

4. Check the degree of all nodes after all coin-tosses, set those nodes as ‘inactive’ with no link to any other node. These ‘inactive’ nodes can again be activated if they gain link from an active node.

5. Repeat step 1 to 4.

Prize for the performers and punishments for the Failures!!

A version of Carrot and stick!!

Immunity Score

1)We increment the immunity score of each node, who has performed BETTER by gaining new links for α consecutive time steps i.e. +1 +1 +1 ..α times.

2) We decrement the immunity score of each node, who has faired POOR (unlucky) by losing links for β consecutive times i.e. -1 -1 -1...β times

Initially no one has a immunity score.

Add **step 6 : Check immunity score for each node, if zero make the node 'dead'.**

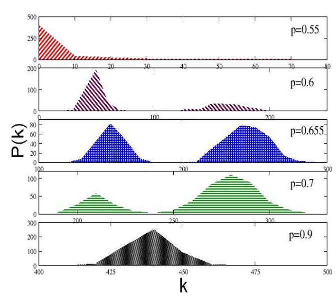
EMERGENCE OF BIMODAL DEGREE DISTRIBUTION

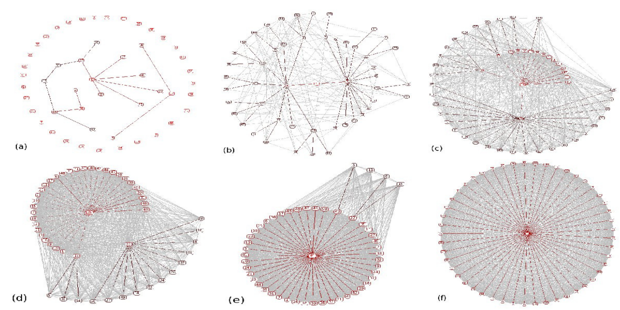
The parameter p decides how biased is your coin.

It is also index of performance by a node. If we bias the coin more …we get a continuous change in the heights of modes of the degree distribution.

There are two means …one for 'peers' and one for “super peers‘ . At high values of p, network turns into a all to all network!

For instance, if the immunity score is ‘f’, the node will participate in dynamics at least f times. Here, consistent ‘good (bad) performance’ is measured in terms of consecutive (+1) or (-1) coin toss outcomes. The algorithm has the machinery of rewards in the form of immunity score increment and punishment in form of decrement. A node with consistent ‘good’ performance gets an opportunity to remain active for longer time and gain more links and thus introduce segregation in degree distribution.





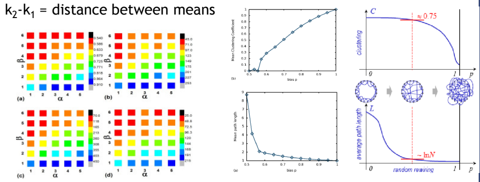
CONCLUSIONS

Design parameters:

p_{bin} = probability of coin toss where two modes are equal,

k_1 = first mean, k_2 = second mean,

k_2-k_1 = distance between means



Average path length and clustering coefficients show that the Bimodal degree distribution network belong to the Small world Family and are also stable to random or Targeted attacks.

1) Starting from small seed, we have stochastically evolved a network with bimodal degree distribution as a result of immunity score given to each node. This is a “consistent lives longer” approach.

2) The timescales associated with each immunity increments/ decrements decide that a network will be a bimodal or not, location of mean, etc

3) Performance based incentive scheme like 'carrot and stick‘ makes two categories of nodes separate from each other....Halos and Horns!!

4) The fittest survive longer, gain more links!!..”fittest is the oldest”!

5) The equal modal heights do not occur at a high p..but at an optimal value p_bin!!

6) Continuous transition from a sparse unimodal to dense unimodal with bimodality in between!!

References

1] Cohen R. and Havlin S., Complex Networks: Structure, Robustness and Functions (Cambridge University Press,UK) 2010

2] Paul G.,Tanizawa T., Havlin S. and Stanley H. E., Euro. Phys. J. B, 38 (2004) 187-191.

3] Tanizawa T., Paul G., Havlin S. and Stanley H. E., Phys. Rev. E, 74 (2006) 016215.

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