Avoiding dark states by tailored initial correlations

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Abstract

We study the transport of excitations on a V-shaped network of three coupled two-level systems. A common feature of these networks is the existence of a dark-state that blocks the transport to the drain. Here we propose a means to avoid this state by a suitable choice of initial correlations, induced by a source that is common to both coupled nodes. These results are also valid if we couple the system to an environment that induces incoherent hopping between the nodes [1].

Excitation dynamics and quantum stochastic walks

Quantum stochastic walks

The quantum stochastic walk [2] is a model to describe excitation dynamics on networks under the influence of interactions with an environment, by using a Lindblad master equation that interpolates between coherent dynamics and the Pauli master equation:

$$\frac{d\rho(t)}{dt} \equiv \mathcal{L}(\rho(t)) = -i(1-\alpha)\,[H_S, \rho(t)] + \alpha \sum_{k,l=1}^{3} \lambda_{kl} \mathcal{D}(L_{kl}, \rho(t)),$$

where $\alpha \in [0,1]$. The Hamiltonian H_S for a V-shaped network is given by:

$$H_S = \sum_{i=1}^{3} E\,|i\rangle\langle i| + V\,(|1\rangle\langle 3| + |2\rangle\langle 3| + \text{h.c.})$$

and the Lindblad terms for the operators $L_{kl} = |k\rangle\langle l|$ are defined by:

$$\mathcal{D}(L_{kl}, \rho(t)) = L_{kl}\rho(t)L_{kl}^{\dagger} - \frac{1}{2}\left\{L_{kl}^{\dagger}L_{kl}, \rho(t)\right\}$$

For $k = l$ we choose the coupling constants $\lambda_{kk} = \lambda$, corresponding to a global dephasing process for each node. For $k \neq l$, we choose $\lambda_{kl} = |H_{kl}|^2$, corresponding to transition rates that are estimated with Fermi's golden rule.

The dark state

Our model exhibits a dark state $\rho = |\psi_D\rangle\langle\psi_D|$, with $|\psi_D\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, that has no overlap with the node connected to the drain. In the purely coherent limit $\alpha \to 0$, this causes a blocking of the transport.

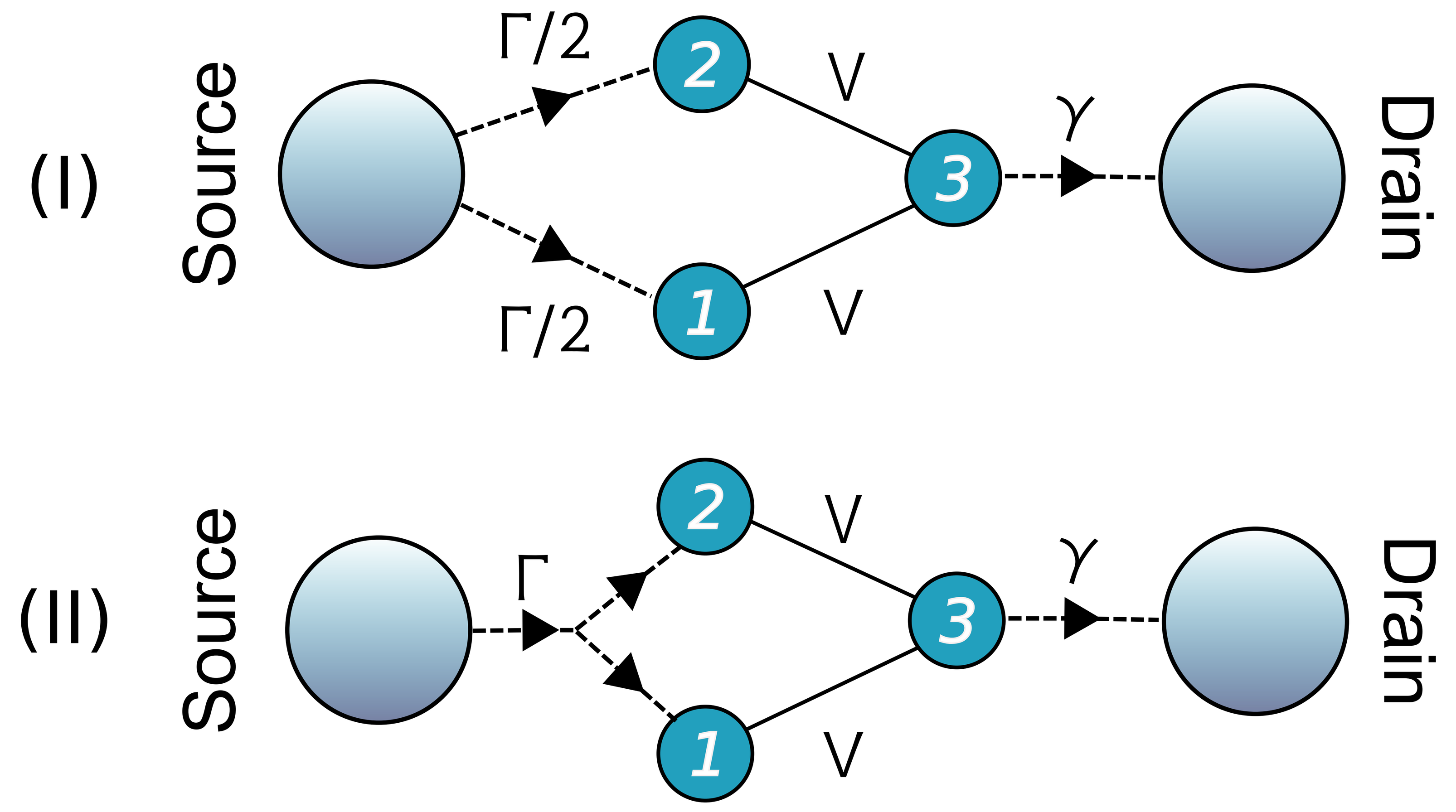
Transport efficiency

A good measure for the transport efficiency is the *expected survival time* (EST) for the excitation in the network [3]:

$$\eta(\alpha) = \int_0^{\infty} dt \sum_{k=0}^{N} \rho_{kk}(t,\alpha) = \int_0^{\infty} dt\,[1 - \rho_{N+1,N+1}(t,\alpha)]$$

A large value of $\eta(\alpha)$ implies a low transport efficiency and visa-versa.

Creating initial correlations with a source



Transitions from the source $|0\rangle$ to a state $|\psi\rangle$ of the network are modelled by the Lindblad operator $L_s = |\psi\rangle\langle 0|$. There are two interesting ways to model these transitions:

- We can consider two independent transitions to nodes $|1\rangle$ and $|2\rangle$:

$$\mathcal{L}^{(1)}_{\text{source}}(\rho) = \frac{\Gamma}{2}\mathcal{D}(|1\rangle\langle 0|, \rho) + \frac{\Gamma}{2}\mathcal{D}(|2\rangle\langle 0|, \rho)$$

- We can consider a transition that induces initial correlations:

$$\mathcal{L}^{(2)}_{\text{source}}(\rho) = \Gamma\mathcal{D}(|\psi_\phi\rangle\langle 0|, \rho) \quad \text{with} \quad |\psi_\phi\rangle = (|1\rangle + e^{i\phi}|2\rangle)/\sqrt{2}$$

The full master equation (including the drain $|d\rangle$) now takes the form:

$$\frac{d\rho}{dt} = \mathcal{L}(\rho(t)) + \gamma\mathcal{D}(|d\rangle\langle 3|, \rho(t)) + \mathcal{L}_{\text{source}}(\rho(t))$$

Analytical results for the EST

Assuming $\gamma = V = 1$, we find the following analytical solutions for the EST $\eta(\alpha)$:

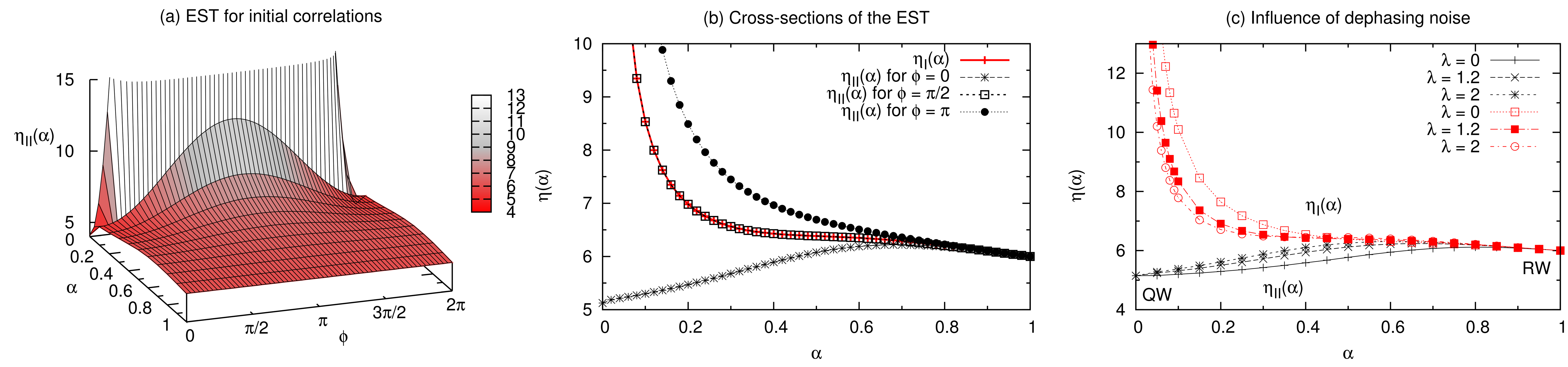
$$\boxed{\eta_I(\alpha) = 1/\Gamma + f(\alpha)/g(\alpha), \qquad \eta_{II}(\alpha) = \eta_I(\alpha) - h(\alpha)\cos\phi/g(\alpha)}$$

with $h(\alpha) = 4(1-\alpha)^2$. The functions $f(\alpha)$ and $g(\alpha)$ are monotonically increasing with α.

Conclusions and outlook

- It is possible to overcome the transport inhibiting effects of the dark state with a source that induces specific initial correlations.
- The efficient transport in the presence of certain initial correlations is robust under dephasing noise.
- We expect the results obtained here also hold for larger networks that exhibit invariant subspaces.

Numerical results



Numerical parameters: $E = 1, V = 1, \gamma = 1$ and $\Gamma = 1$. For Fig (a) and (b) we use $\lambda = 1$ and for Fig (c) we use $\phi = 0$.

References

[1] P. Schijven and O. Mülken, arXiV:1204.0954
[2] J. Whitfield et. al. Phys. Rev. A **81**, 022323 (Feb 2010)
[3] J. Cao and R. Silbey, Journal of Phys. Chem. A **113**, 13825 (Nov. 2009)

Acknowledgments

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