Noise induced coherent quantum transport in spin network

MAPCON12

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We consider the problem of coherent quantum state transfer between two sites of a spin network. It is known that the perfect state transfer could achieved only for specific forms of the Hamiltonian interaction governing the network dynamics. Here we show that the addition of a Non-Hamiltonian, dissipative or dephasing, term can increase the fidelity of the coherent state transfer. Eventually, perfect state transfer can be reached in the highly noisy limit, governed by quantum Zeno dynamics.

Introduction. In view of application, quantum networks defined as a collection of interacting qubits (spins) play the fundamental role for realization of multi users quantum networks idea[1] . Recently though, it was realized the presence of the quantum coherence in biological system[2]where the study of quantum network is worthwhile in this context[3]. The quantum networks have been considered to state transfer underlying the quantum communication. The important obstacles to obtain the perfect state transfer(PST) between two nodes of the network are the dispersion effects and destructive quantum interference. Although, there are some ways based on engineering the coupling and lifting the encoding qubits into the multiparticle states[4], the PST can be obtained for only some special network topologies without tuning the couplings or altering the network topology[5]. In particular, for the fully connected network the PST has been obtained just in the presence of optimal energy shift to desired sites or missing the link between the input and output vertices(*I* /*O*)[6]. It is well known that, noise does not always hinder the efficiency of an information process[7] and has beneficial effects in some bio systems[3, 8]. In this perspective, we motivated to investigate the role of the addition noise to the fully connected networks and find the way to reach the PST in these systems.

The Model. We consider the fully connected (spin) network with N vertices. The Hamiltonian that describes the interaction of qubits is given by

$$H = \omega \sum_{i \neq i} (\sigma_i \sigma_j^\dagger + \sigma_i^\dagger \sigma_j),$$

To consider a state transfer between two nodes, we employ white noise as a external stochastic non-Hamiltonian terms to any edge, except those linked contain the input/output node,

$$C_{k,k'} = \xi(t)(\sigma_k \sigma_{k'}^\dagger + \sigma_k^\dagger \sigma_{k'})$$

where *is a Gaussian white noise term having*

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2\eta\delta(t-t').$$

In this case, our system is an instance of a general dynamical system described by a master equation of the form

$$\dot{\rho}(t) = -i[H, \rho(t)] + \mathcal{D}(\rho(t)) \, ,$$

where *H* is a time-independent Hamiltonian *and D is a* non-Hamiltonian term. Let us now restrict our attention to the single excitation subspace *H, i.e.*, the subspace of dimension n spanned by the vectors *{|1⟩; …; |n⟩}*. The *|j⟩* indicates the state of the network with one excitation on the j-th spin, with all the other spins being in the ground states.
The state transfer fidelity between nodes i and j defines *as* $\langle i | \rho_{i,j}(t) | j \rangle$, where $\rho_{i,j}(t)$ *is* the reduced density matrix of vertices *i and j*. We assume the network is in the ground state and there is not any excitation at first. The state transfer fidelity of encoding excitation (*|1⟩*) between two nodes (nodes 1 and 2) for the network with N = 4 has been obtained analytically. When the noise acts on the link does not contains I /O (link 3-4), the corresponding fidelity is

$$F_{12} = \frac{1}{8}\{3 + e^{-\eta t/2}(Cosh(\frac{ts_1}{2}) + \frac{\eta}{s_1} Sinh(\frac{ts_1}{2}) + \frac{2\eta^2(s_2-4)}{s_2 s_5} Sinh(\frac{ts_3}{2}) - 2Cosh(\frac{ts_3}{2}) - 2Cosh(\frac{ts_4}{2}) + \frac{2(s_2-4)}{s_2 s_5} Sinh(\frac{ts_4}{2}))\},$$

Where $s_1 = \sqrt{\eta^2 - 64}$, $s_2 = \sqrt{16 - \eta^2}$, $s_3 = \sqrt{\eta^2 - 32 + 8s_2}$, $s_4 = \sqrt{\eta^2 - 8(s_2 + 4)}$ and $s_5 = \sqrt{\eta^2 + 8(s_2 - 4)}$.

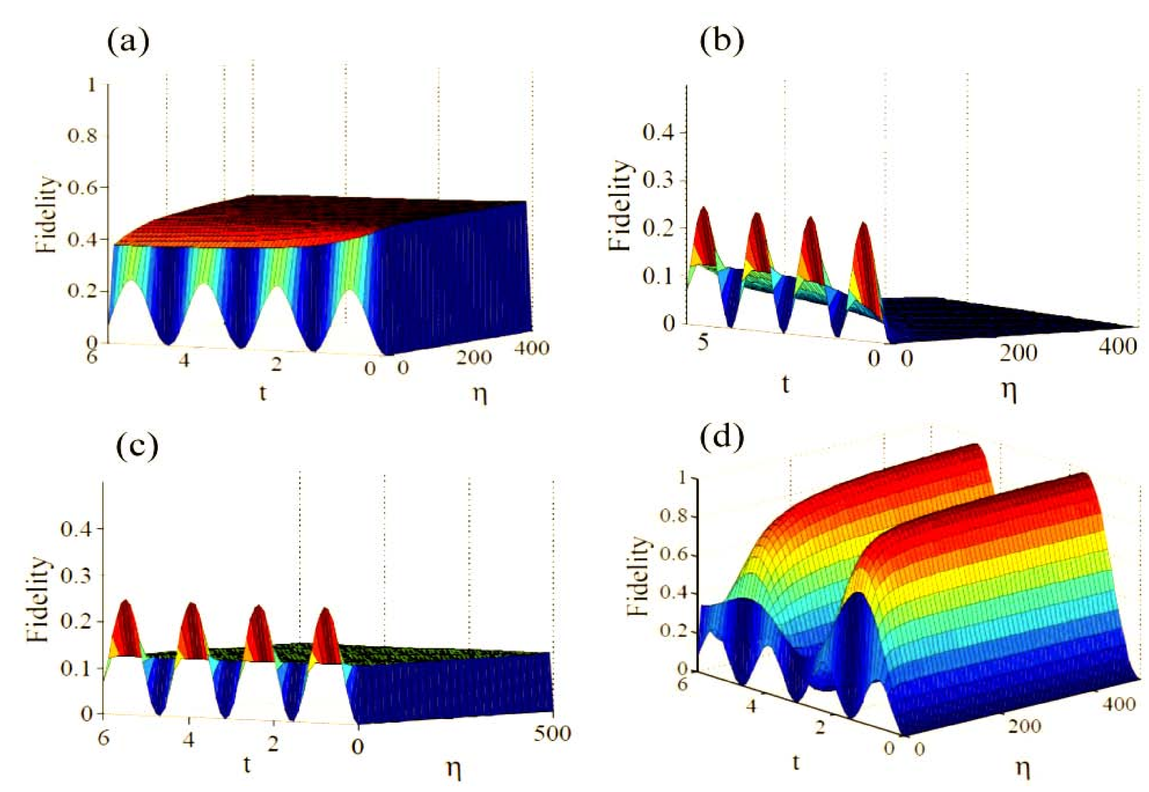

FIG. 1. (Color online) The variation of state transfer fidelity between nodes 1 and 2 of the network with N=4 versus the time and strength of the noise the noise acts on the link (a) contain input-output nodes(link 1-2). (b) contain one of the input-output nodes(link 1-3, 1-4, 2-3 or 2-4). (c) contain all the nodes. (d)contain input-output nodes (link 3-4).

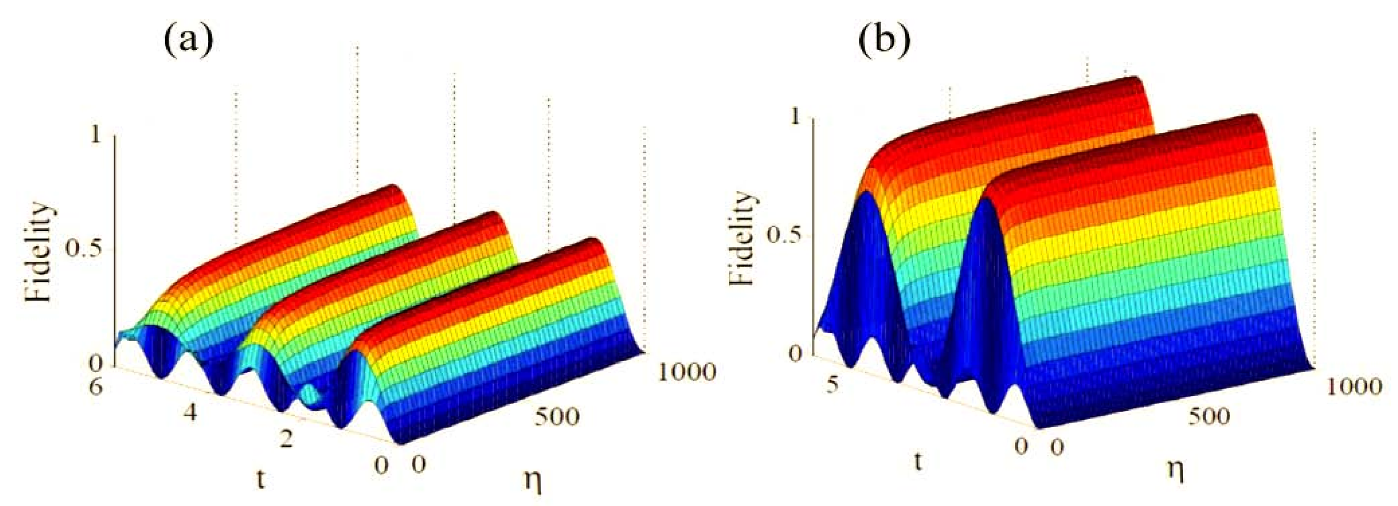

FIG. 2. (Color online) The variation of state transfer fidelity between nodes 1 and 2 of the network with N=5 versus the time and strength of the noise the noises act on the (a)one link does not contain input-output nodes(link 3-4). (b) all links do not contain input-output nodes (links 3-4, 3-5 and 4-5)

In the limit of very strong noise, $\eta/\omega \gg 1$, the dynamics of the non-Hamiltonian term is much faster than the Hamiltonian part. In this limit the adiabatic approximation can be assumed. It implies that, on the time scale defined by the Hamiltonian term, the system is instantaneously in a steady state. Working in the interaction picture, that means:

$$\mathcal{D}_t(\rho_I^s(t)) = 0 \, .$$

Equivalently, we can say that the very strong non-Hamiltonian interaction corresponds to a continuous measurement of the system. According to the quantum Zeno effect, this implies that the system is projected into the corresponding subspace.
To fix the ideas, we restrict to the case of a fully connected network of 4 qubits, with a non-Hamiltonian term coupling (say) qubits number 3 and 4. The Hamiltonian reads:

$$H = \omega \left[|1\rangle \left(\langle 2| + \langle 3| + \langle 4| \right) + |2\rangle \left(\langle 3| + \langle 4| \right) + |3\rangle\langle 4| \right] + \text{h.c.}$$

The non-Hamiltonian term is chosen of the following form:

$$\mathcal{D}(\rho) = \eta \left(2c\rho c^\dagger - c^\dagger c \rho - \rho c^\dagger c \right) \, ,$$

Where

$$c = c^\dagger = |3\rangle\langle 4| + |4\rangle\langle 3| \, .$$

The first step is to compute the steady states of D, that is the solutions of D(ρ) = 0. We restrict to the subspace of states with at most one excitation (the number of excitations is preserved in this model). Then we have two steady states:

$$|+\rangle = \frac{|3\rangle + |4\rangle}{\sqrt{2}} \, ,$$

$$|-\rangle = \frac{|3\rangle - |4\rangle}{\sqrt{2}} \, ,$$

and the steady subspace:

$$\mathcal{H} = \text{span}\{|0\rangle, |1\rangle, |2\rangle\} \, .$$

A generic steady state (with up to one excitation) is hence of the form

$$\rho = p_1 |+\rangle\langle +| + p_2 |-\rangle\langle -| + p_3 \rho_{\mathcal{H}} \, ,$$

Where $\sum_{j=1}^{3} p_j = 1$ and $\rho_{\mathcal{H}}$ is a density operator with support in H.

Adiabatic dynamics. The Adiabatic approximation tells us that the system is instantaneously in a steady state. We are interested in the case in which the initial state is |1⟩ (for excitation transfer), or $\alpha|0\rangle + \beta|1\rangle$ (for the quantum information transfer). In both the cases the initial state belongs to H, hence it will evolve inside the instantaneous steady space H_t = span{|0⟩; $|1\rangle_t$; $|2\rangle_t$}. Denoting P(t) the instantaneous projector into H_t, it is well known that the adiabatic dynamics is governed by the equation (in the interaction picture)

$$\frac{d}{dt}|\psi_I(t)\rangle = iA(t)|\psi_I(t)\rangle \, ,$$

Where

$$iA(t) = [\frac{P(t)}{dt}, P(t)] = \frac{P(t)}{dt}P(t) - P(t)\frac{P(t)}{dt} \, .$$

We have

$$\frac{P(t)}{dt} = i[H, P(t)] \, ,$$

And

$$iA = \frac{P(t)}{dt} = i[[H, P(t)], P(t)] = i\{HP(t) + P(t)H - 2P(t)HP(t)\} \, .$$

Finally, coming back to the Schrodinger picture, we have:

$$\frac{d}{dt}|\psi(t)\rangle = -i\{H - HP(0) - P(0)H + 2P(0)HP(0)\}|\psi(t)\rangle \, .$$

Noticing that, for all initial state $|\psi(0)\rangle \in \mathcal{H}$, the latter equation is equivalent to

$$\frac{d}{dt}|\psi(t)\rangle = -i\omega\{|1\rangle\langle 2| + |2\rangle\langle 1|\}|\psi(t)\rangle \, .$$

It is well known that this equation allows the perfect excitation and quantum information transfer between site 1 and 2. The generalization to bigger network and more general noise is straightforward: the introduction of a noisy term |a⟩⟨b| + |b⟩⟨a| corresponds (in the adiabatic limit) to ”cancel” the corresponding network sites. Clearly, the elimination of two or more sites in the network make the transfer more favorable.

Conclusions. We have evaluated analytically and numerically the fidelity of state transfer between two nodes of the fully network connected in the presence of the external stochastic Gaussian white noises. We showed that, noise can be extremely helpful for this purpose. Indeed, the addition of these noises to any spin-spin coupling (edge), except those link contain one of the input/output node, enhances the fidelity of state transfer. Moreover, such enhancement depends on both the strength of the noise and the number of edges affected by noise. Our results showed that PST obtain in the very strong limit of noises which acts on all links except such links contain input-output nodes. Furthermore, in this process, it does not need to tuning the couplings or altering the network topology to reach the PST for fully connected networks.

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