Part I:

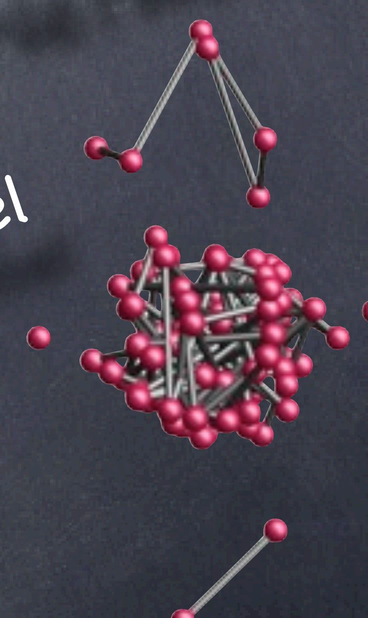
De vil`s stair case in compe titive percolation

in collaboration with
Tyge Tiessen & Harold W. Gutch

Jan Nagler
MPI for Dynamics + Self-Organization,
SPICE group, Göttingen

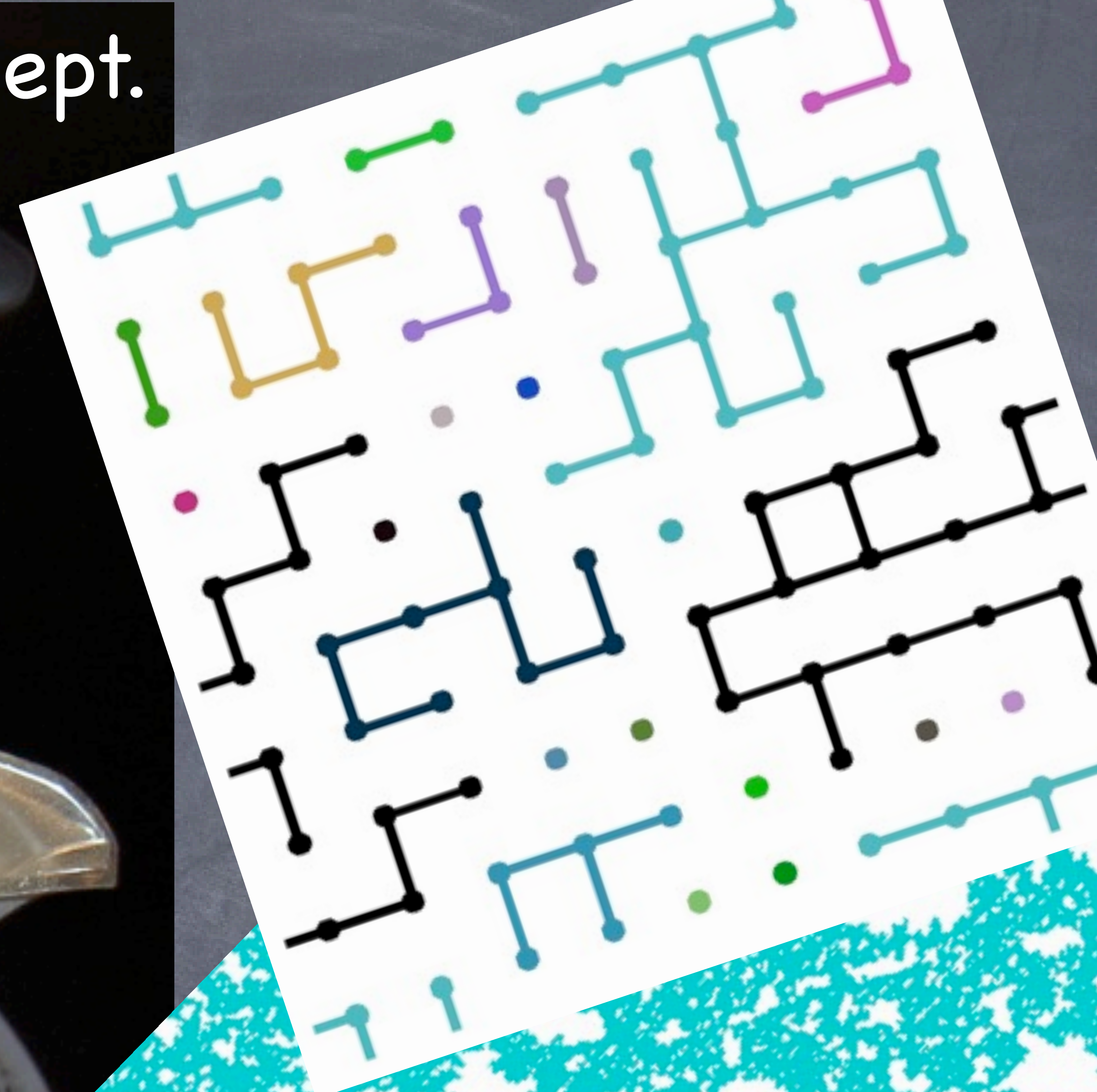
Part II: Complete Reconstruction of Correlation Networks

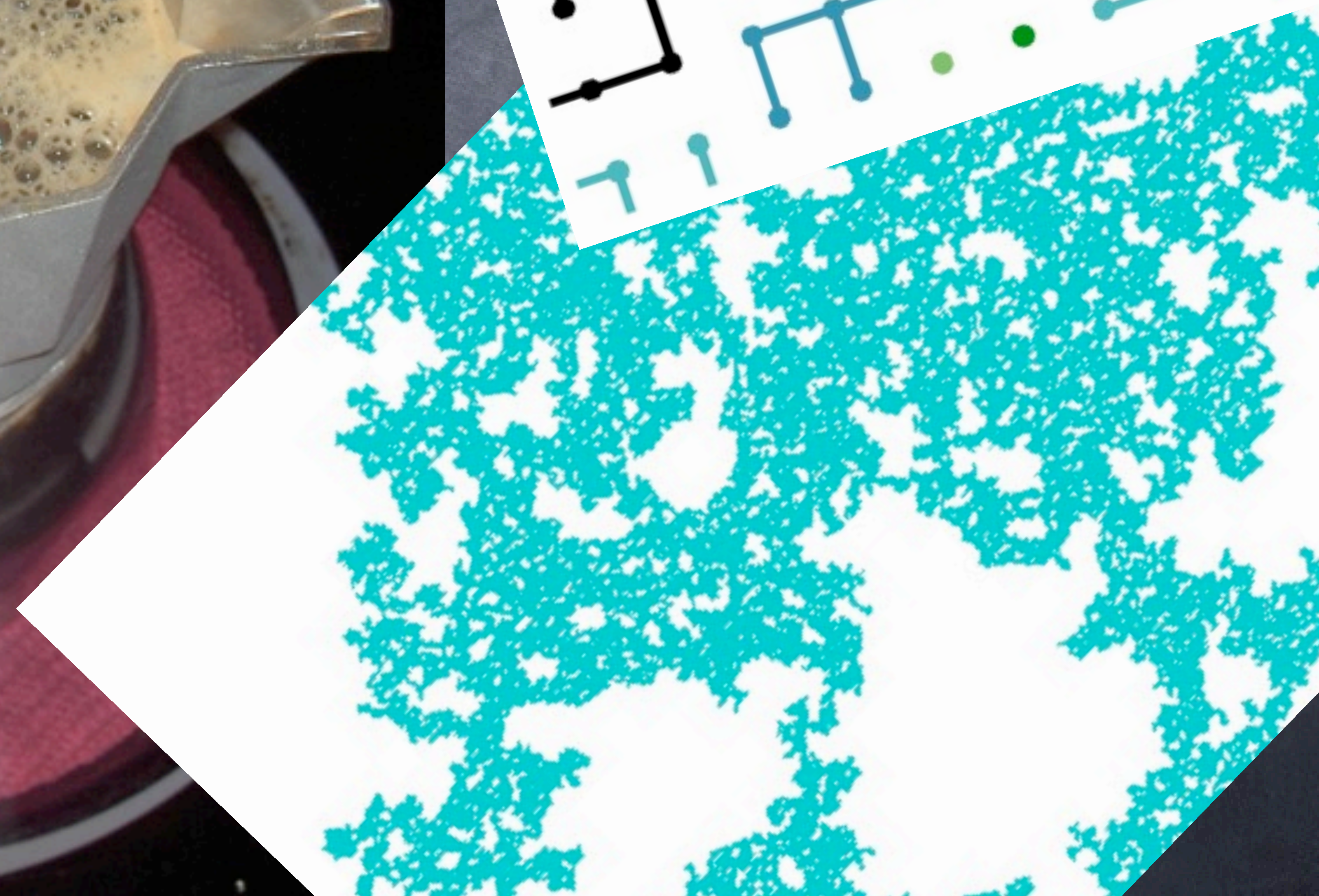
in collaboration with
Magdalena Kersting, Annette Witt & Theo Geisel

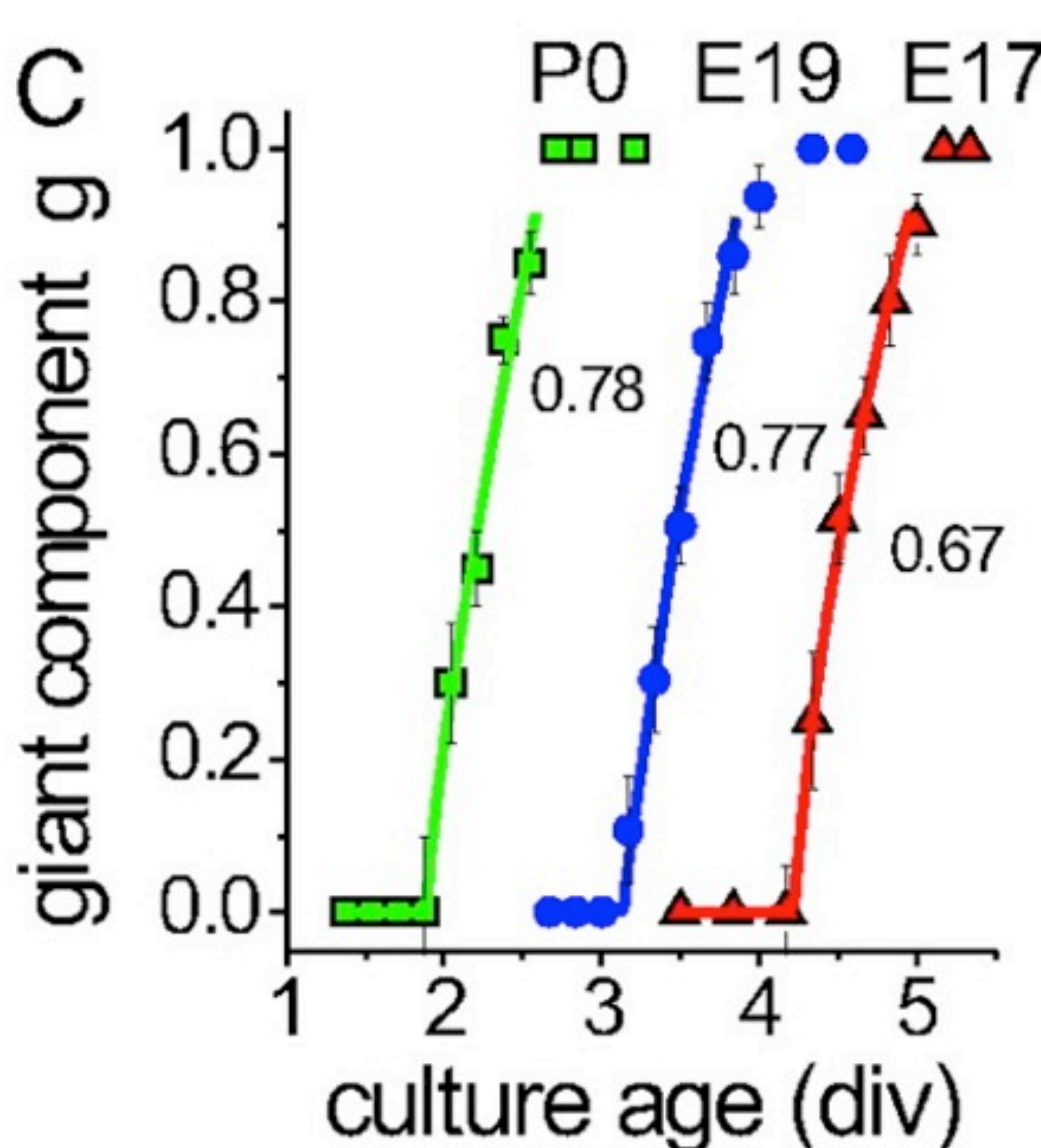


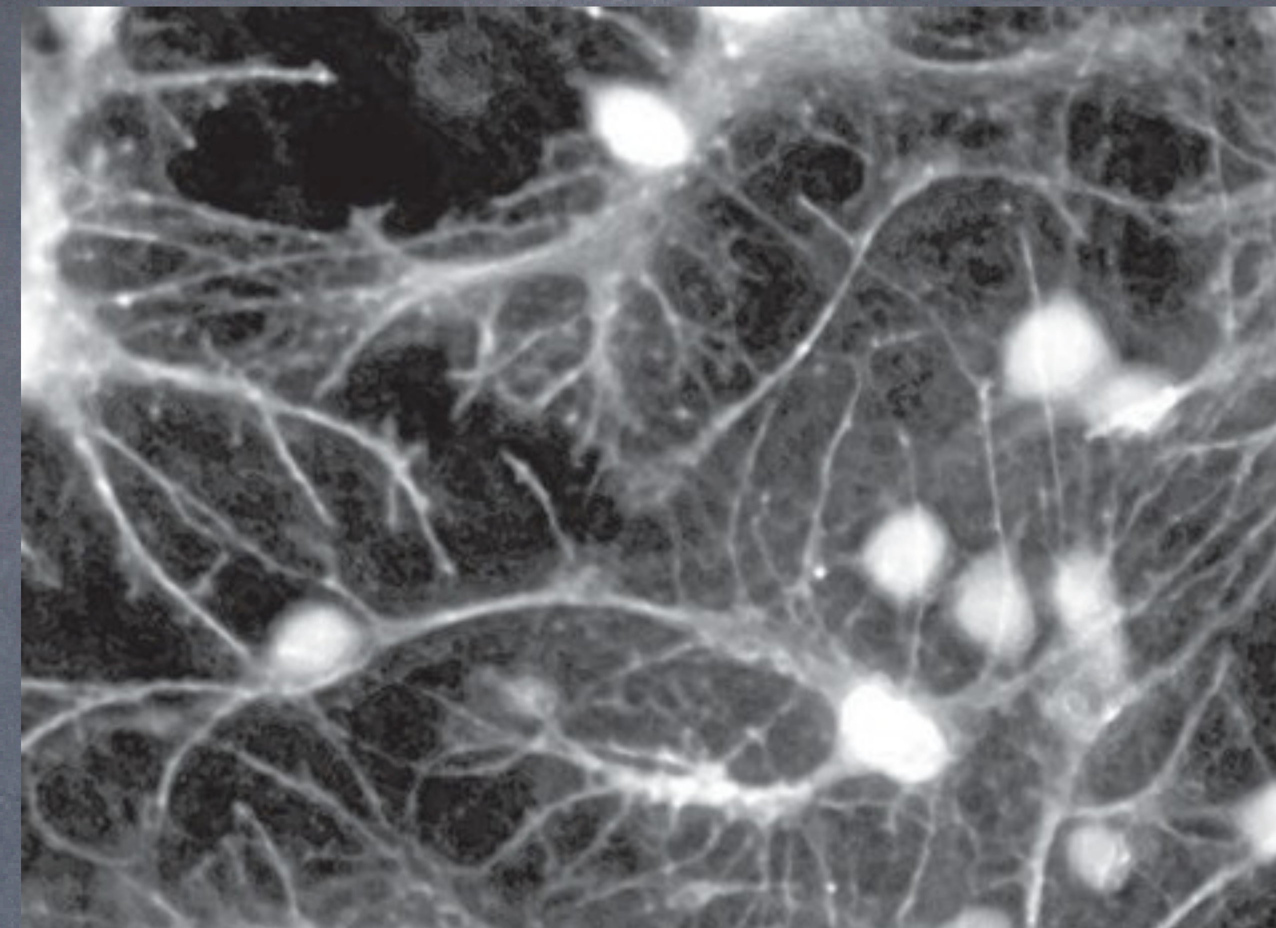
MAPCON 2012, MPI PKS Dresden

Percolation is a pervaisive concept.









Percolation in living neural networks, Breskin et al., PRL 2006;
Soriano et al., Development of input connections in neural cultures, PNAS 2008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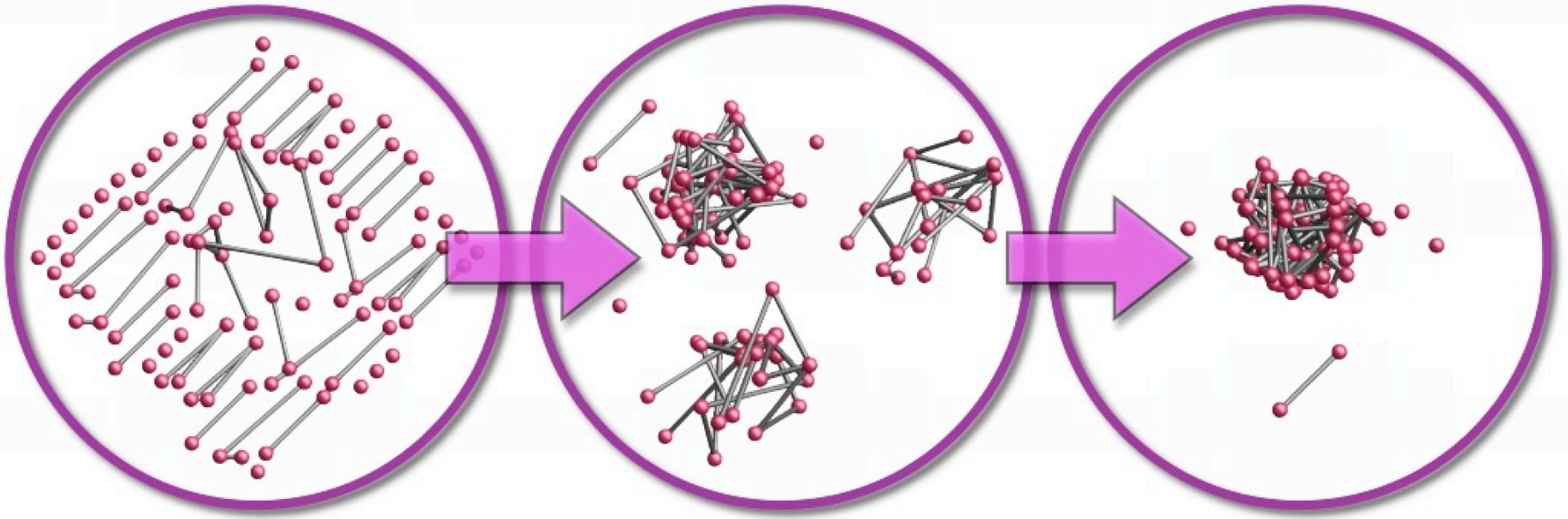
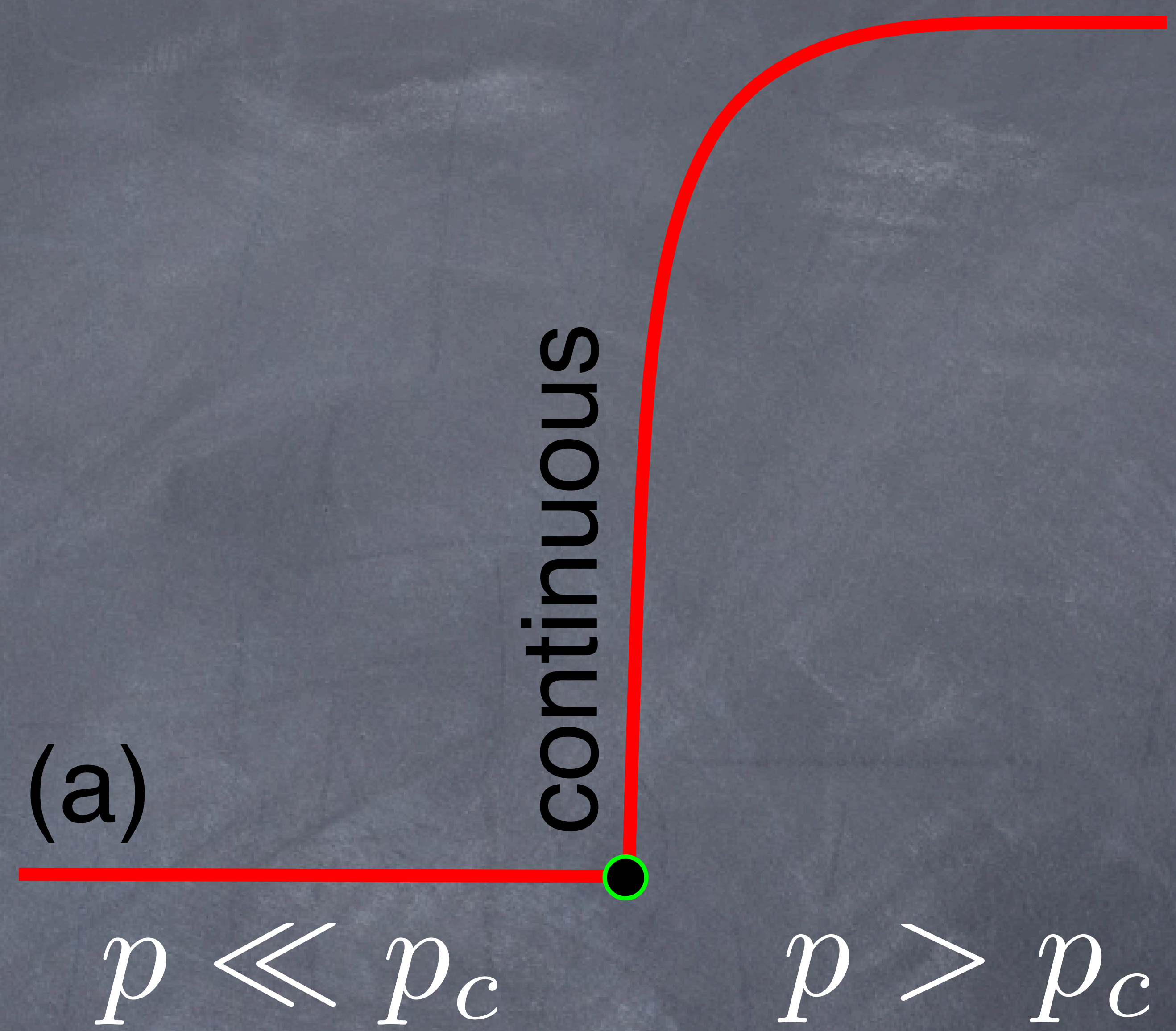
Dirk Brockmann et al. Nature 2006
Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany



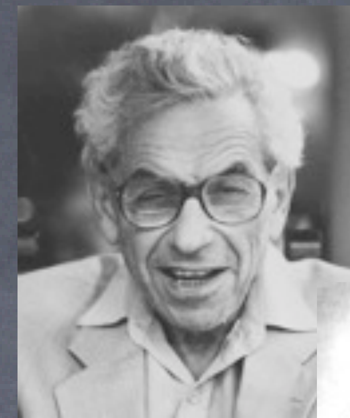
$p \ll p_c$ $p \lesssim p_c$ $p > p_c$

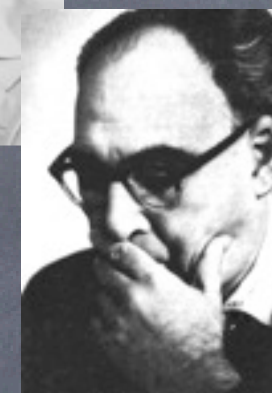
Before: microscopic components only

After: macroscopic component(s)

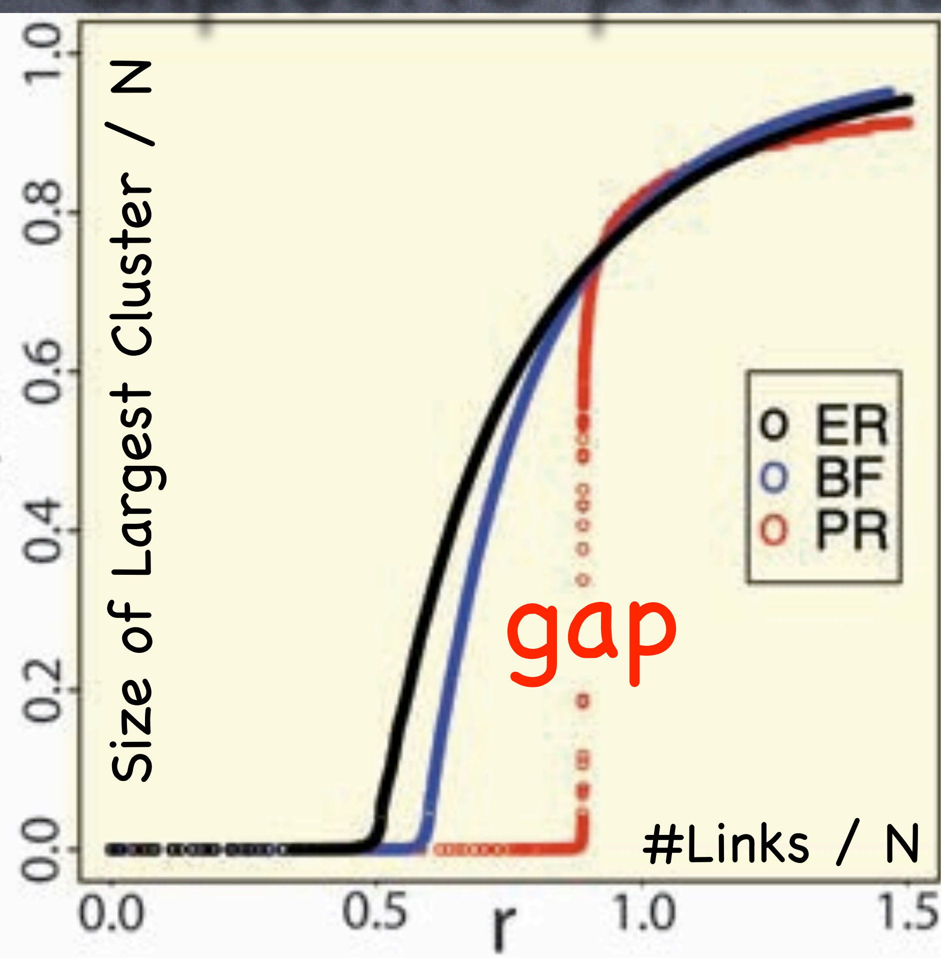

(a)
continuous
$p \ll p_c$
$p > p_c$

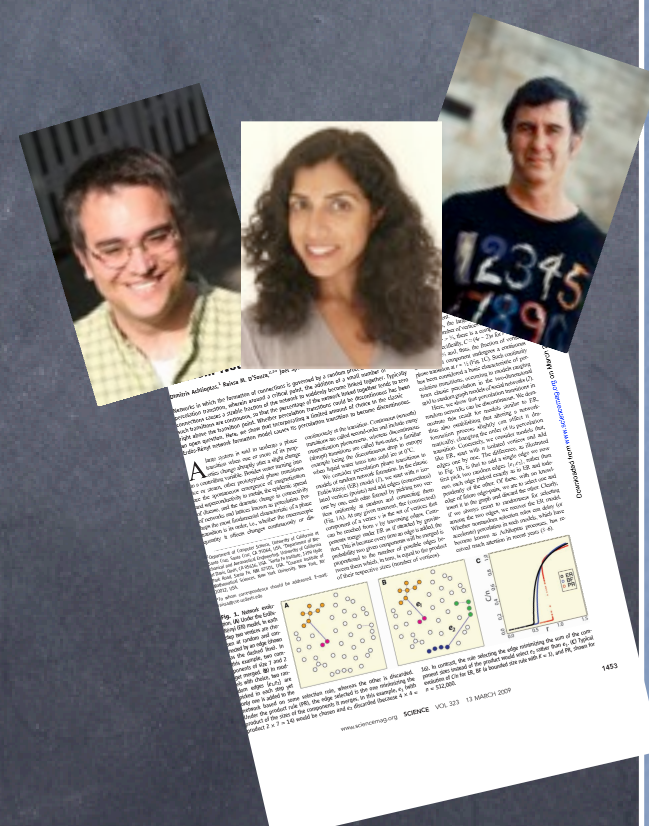
Erdös & Renyi
„Random Network Theory"

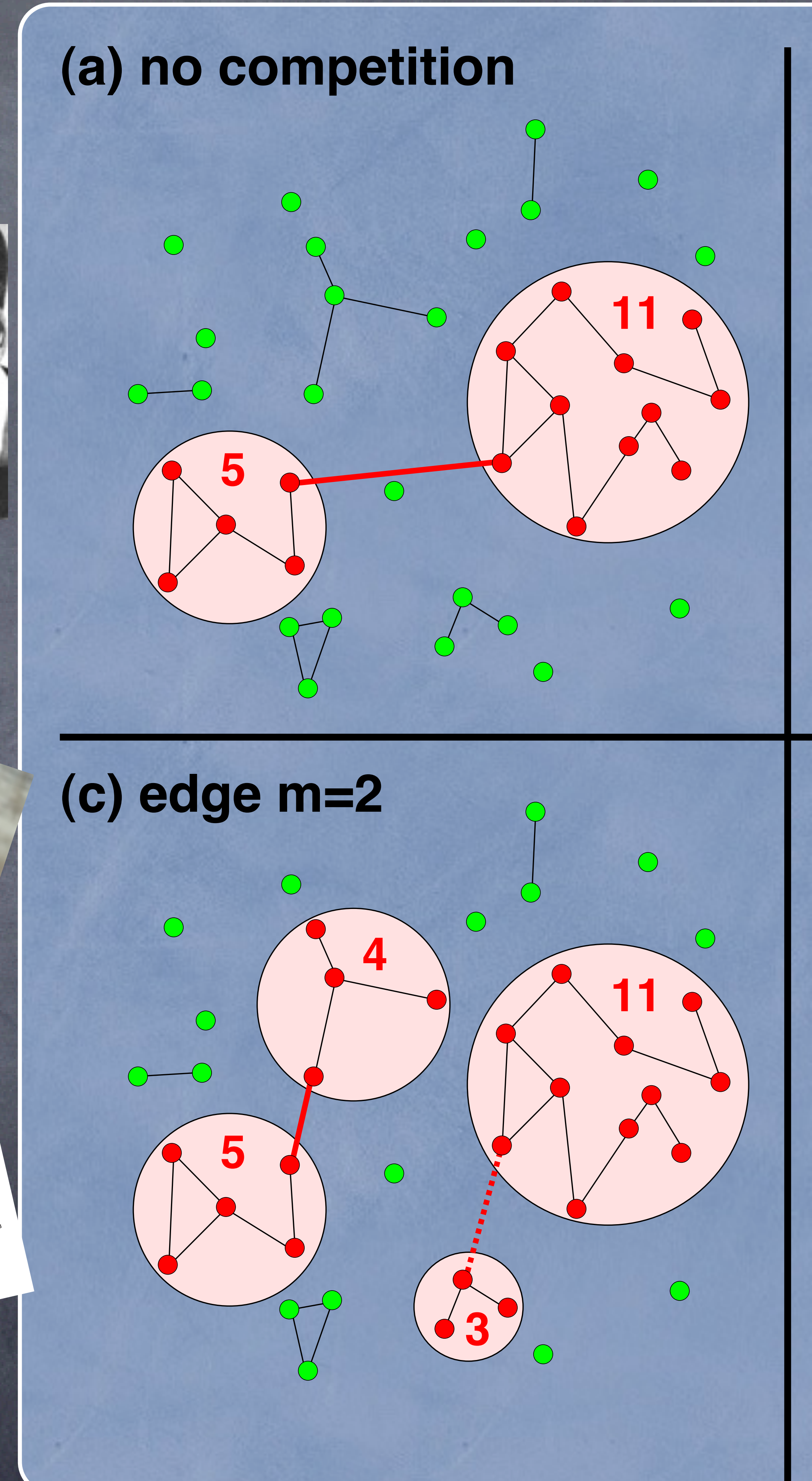




explosive percolation







D Achlioptas, RM D'Souza, J Spencer, Science 323, 1453 (2009)
„Explosive Percolation in Random Networks"

Is explosive percolation discontinuous?

Grid: Tricritical Point in (p,q)=(link density, dilution) diagram, Araujo et. al, PRL 2011

Grid: Suppression of largest cluster, Araujo & Hermann, PRL 2010

Bounded-size rules & cluster aggregation, D'Souza & Mitzenmacher, PRL 2010

Powder Keg & multiple link models, Friedman & Landsberg, PRL 2009

True Grid: Ziff, PRL 2009

Scale-free networks: existence of tricritical point around 2.3 (degree exponent), Cho et al., PRL 2009 & Radicchi & Fortunato, PRL 2009. >3 discontinuous

+ many other publications...

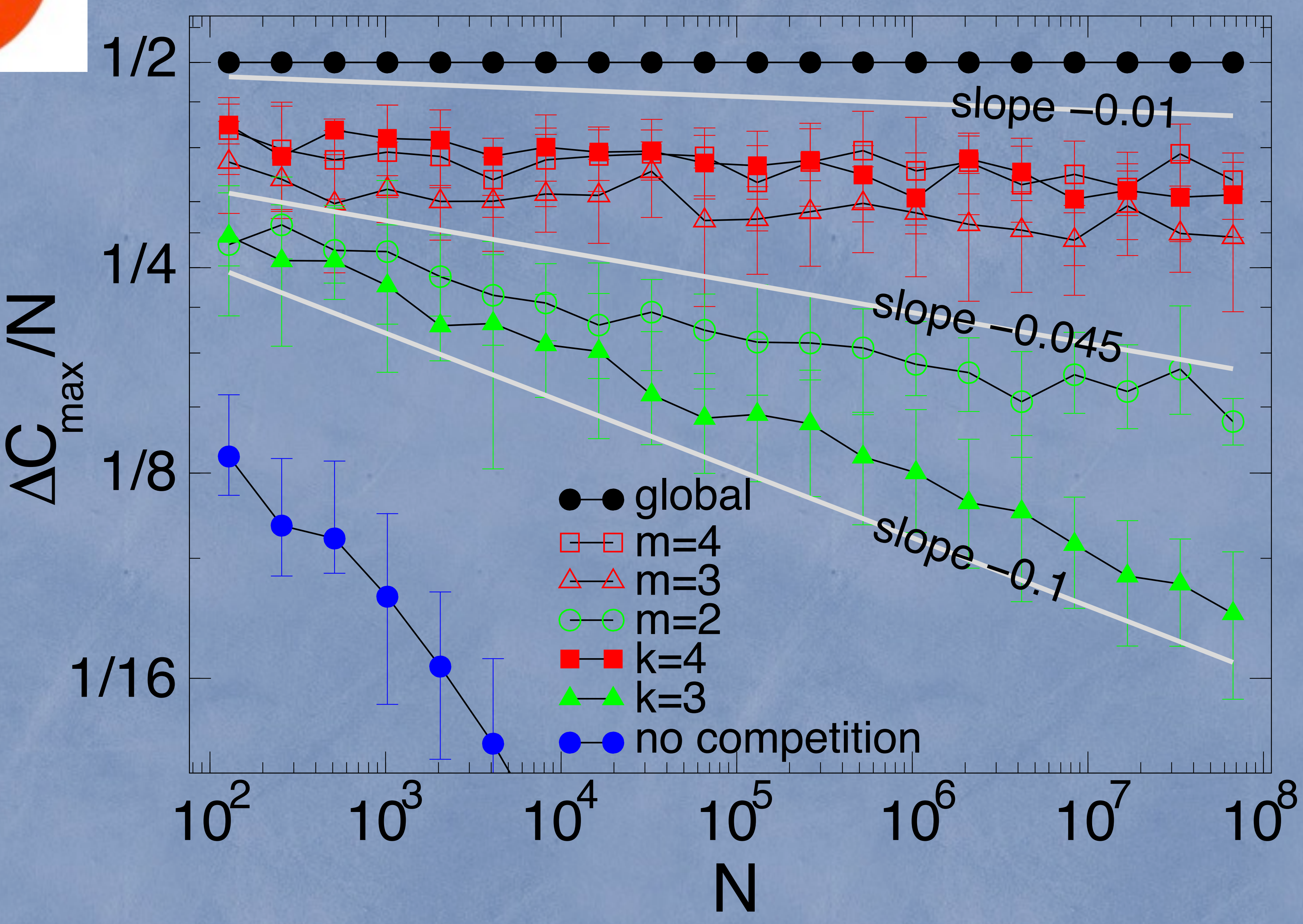
Is explosive percolation continuous?

PRL 2010 (hybrid numerical-analytical):
da Costa et al.: Explosive percolation is continuous

PRL 2011 (strong numerical evidence):
Grassberger et al. : Explosive percolation is continuous but with unusual finite-size behavior

Nature Phys. 2011 (numerics & analytical microdynamical analysis):
Nagler et al.: Impact of single links in competitive percolation

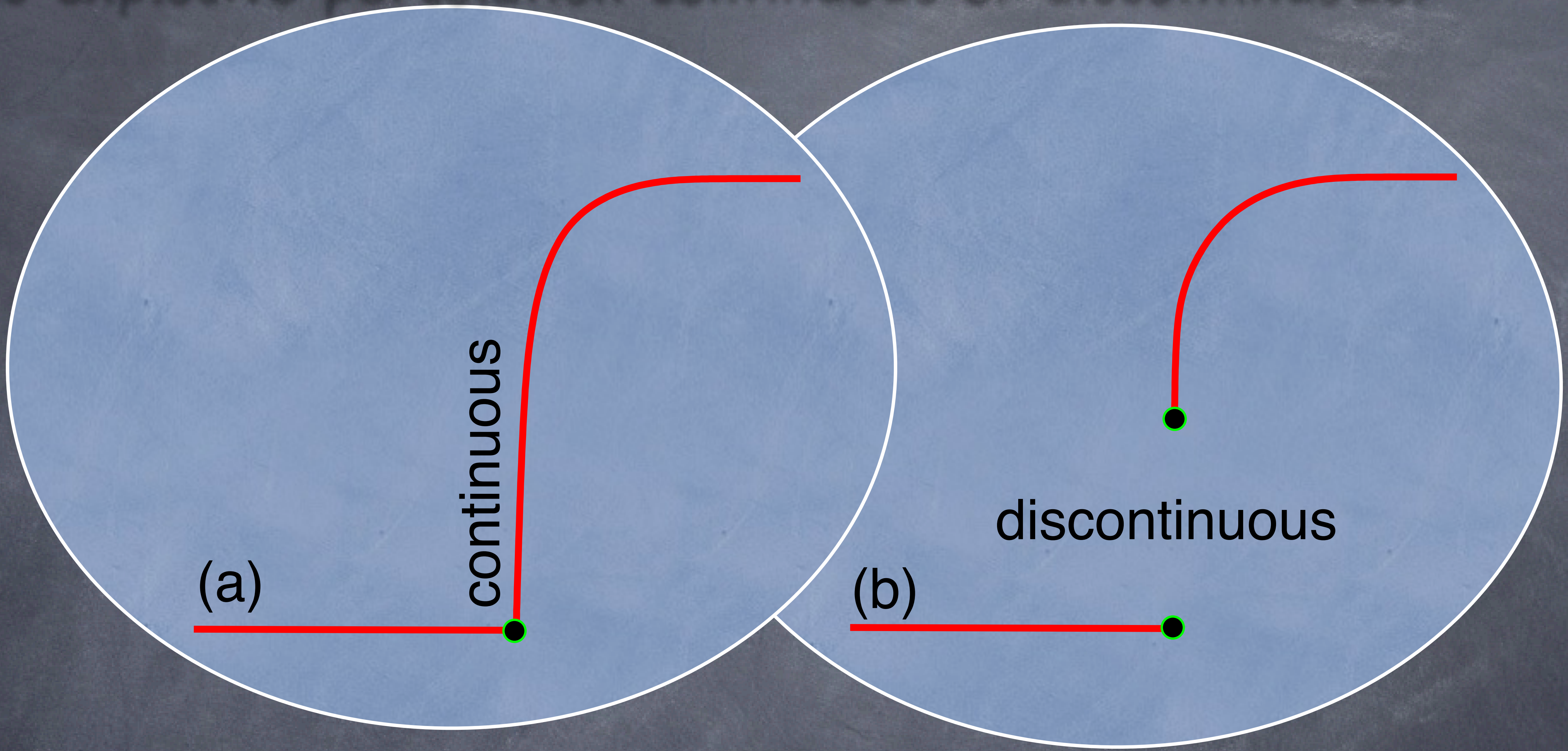
Scaling of Largest Gap



$$\Delta C_{\max}/N \sim N^{-\beta}$$

Nagler, Levina & Timme, Nat. Phys. (2011)

Is explosive percolation continuous or discontinuous?



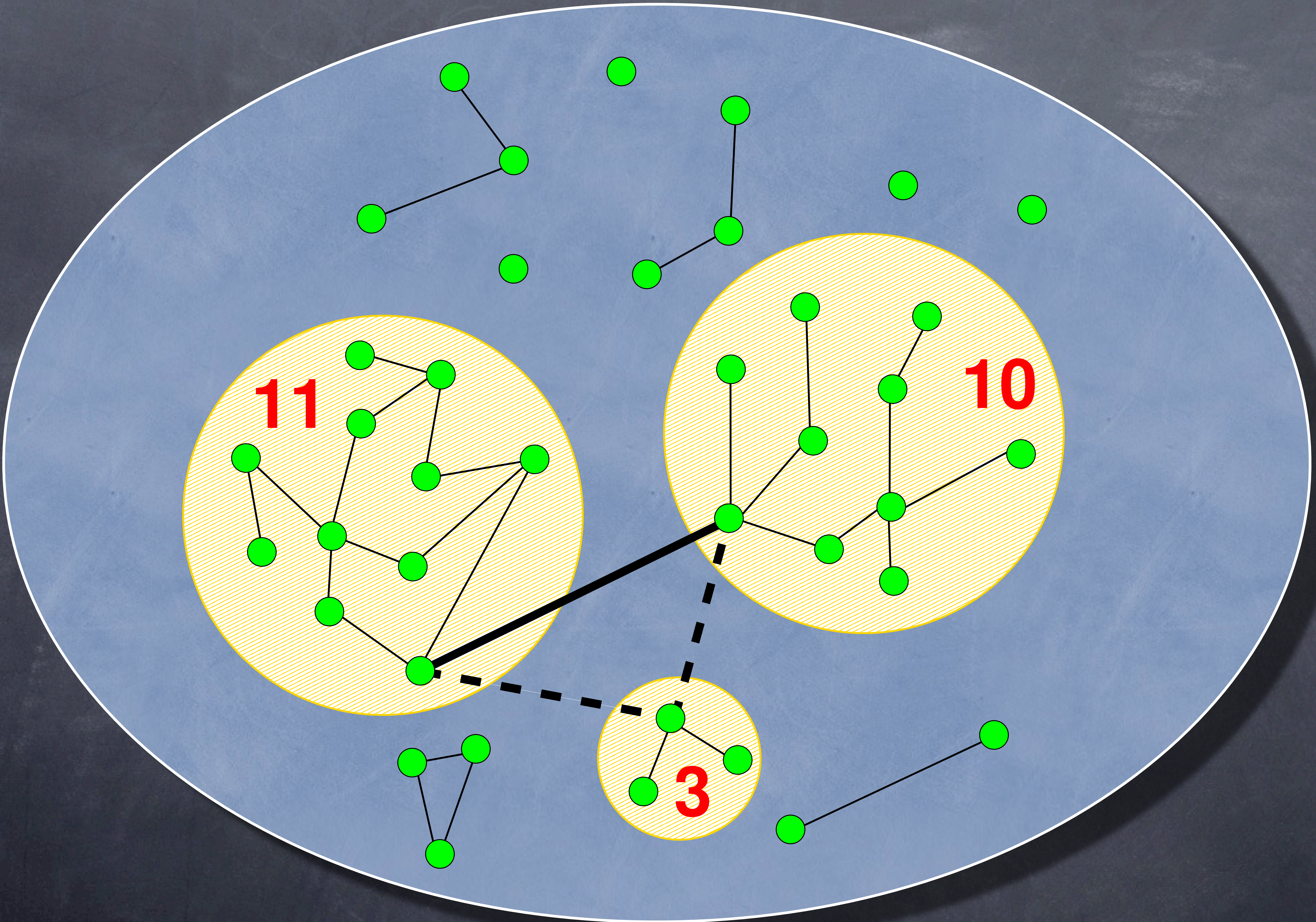
no rigorous proof
but important question

Riordan & Warnke, Science 333, 322 (2011):

„Explosive percolation is continuous“

main conclusion of rigorous proof:

„any rule based on picking a fixed number of random vertices gives a continuous transition“



Nagler, Tiessen & Gutch, to appear in Phys. Rev. X (2012) with L<X<Z

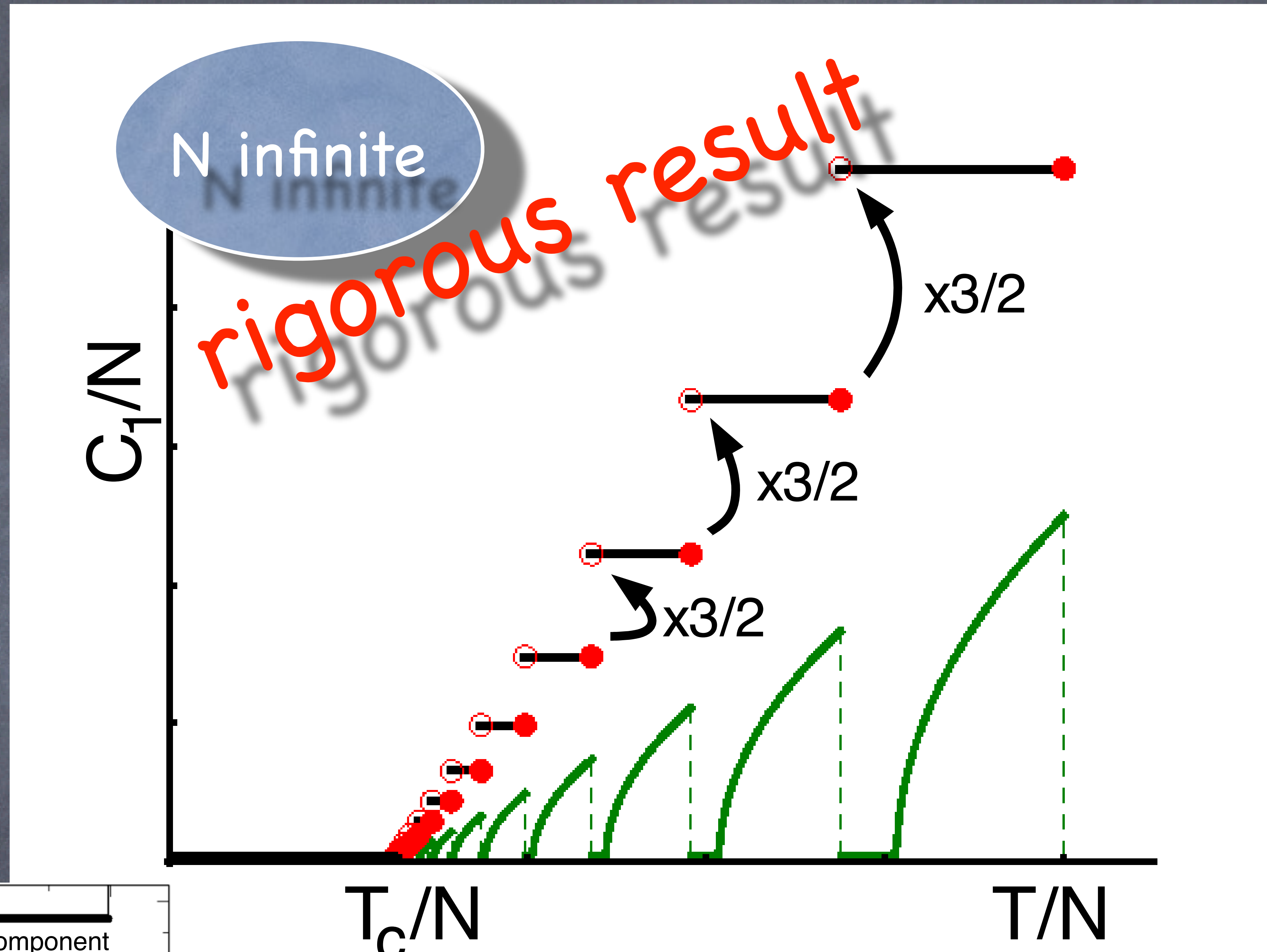
Proof idea

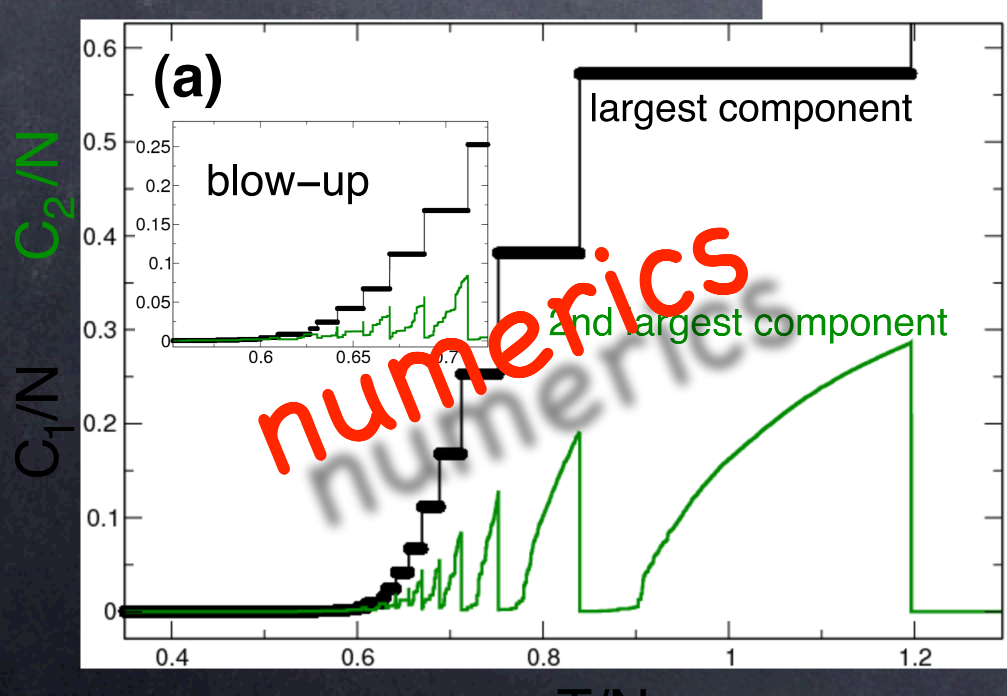
1. Instability of giants:
For any rule based on picking n vertices at random, there cannot be more than n-1 giant components on any finite (time) interval

2. Homophily of mergers:
The largest component cannot join with components smaller than 50% of its own size

3. Impossibility of overtaking of O(N) components:
Giants cannot be overtaken (whp)

Nagler, Tiessen & Gutch, to appear in Phys. Rev. X (2012) with L<X<Z


N infinite
rigorous result
x3/2
x3/2
x3/2
C1/N
Tc/N
T/N


(a)
largest component
blow-up
2nd largest component
numerics
C1/N
C2/N
T/N

Conclusions

Continuous percolation by discontinuities

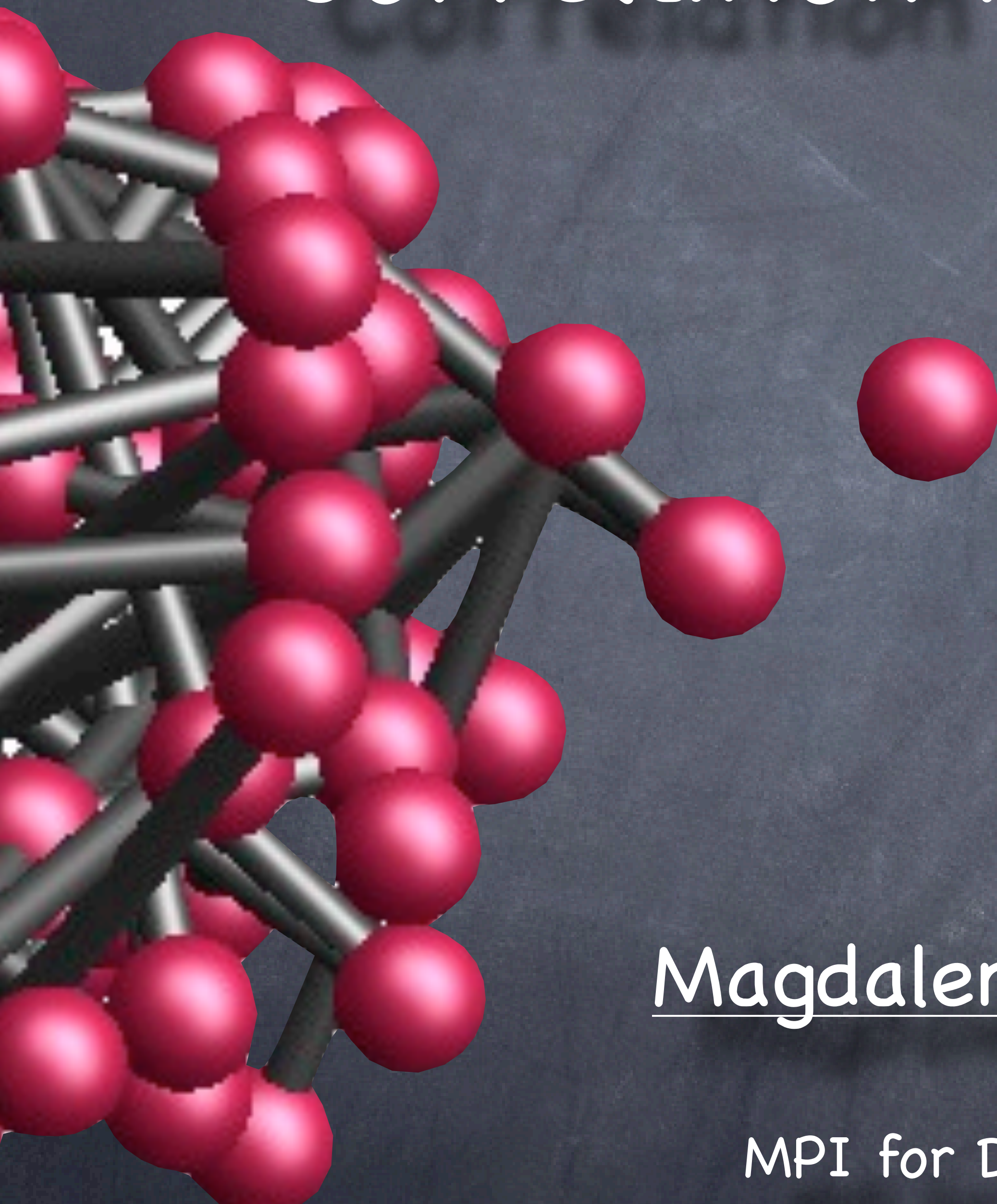
No power law divergence <-> critical phenomena

Coexistence of local continuity and global discontinuities

Please take home our analytical result:
Explosive percolation is not always continuous

Nagler, Tiessen & Gutch, to appear in Phys. Rev. X (2012) with L<X<Z

Part II: Complete Reconstruction of Correlation Networks



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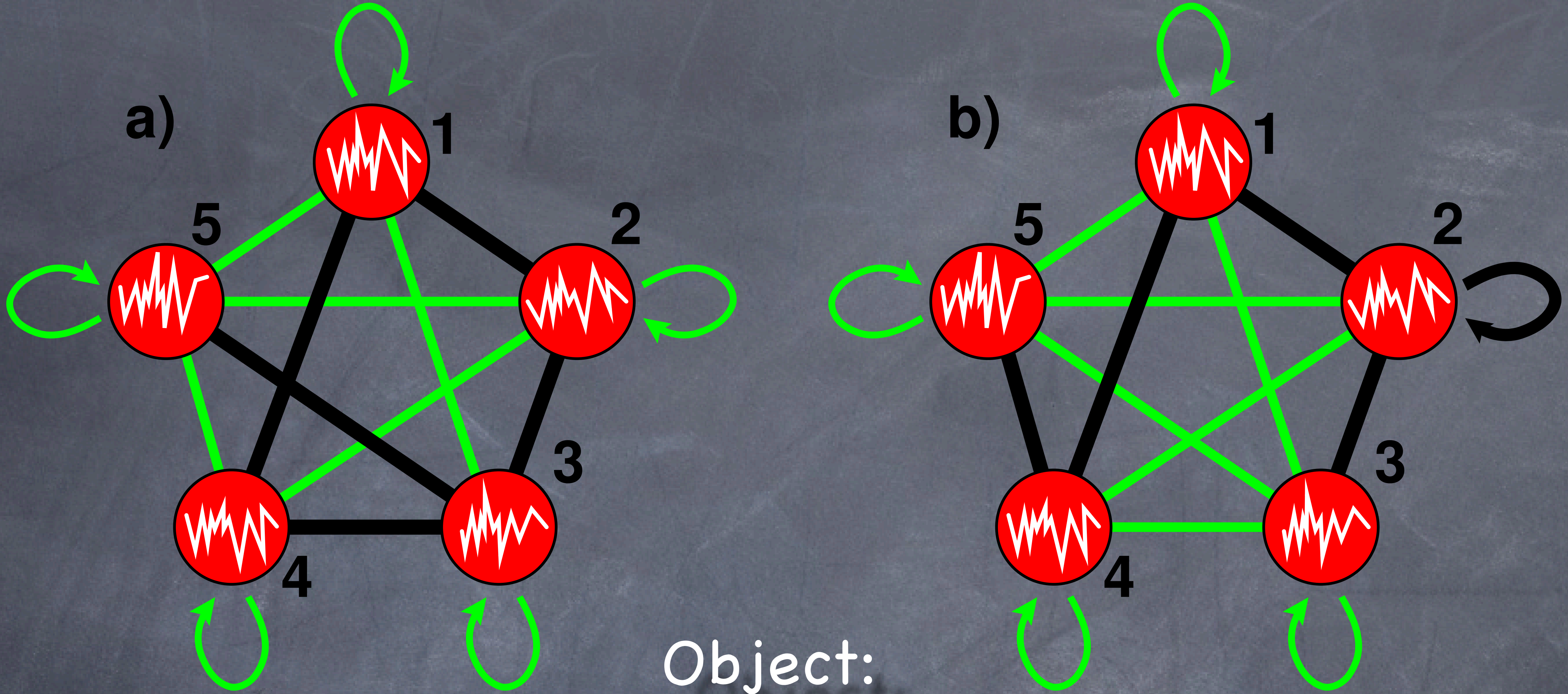
What do we do?

Generalization of the
Crosscorrelation Theorem
to arbitrarily shaped networks
of stochastic processes

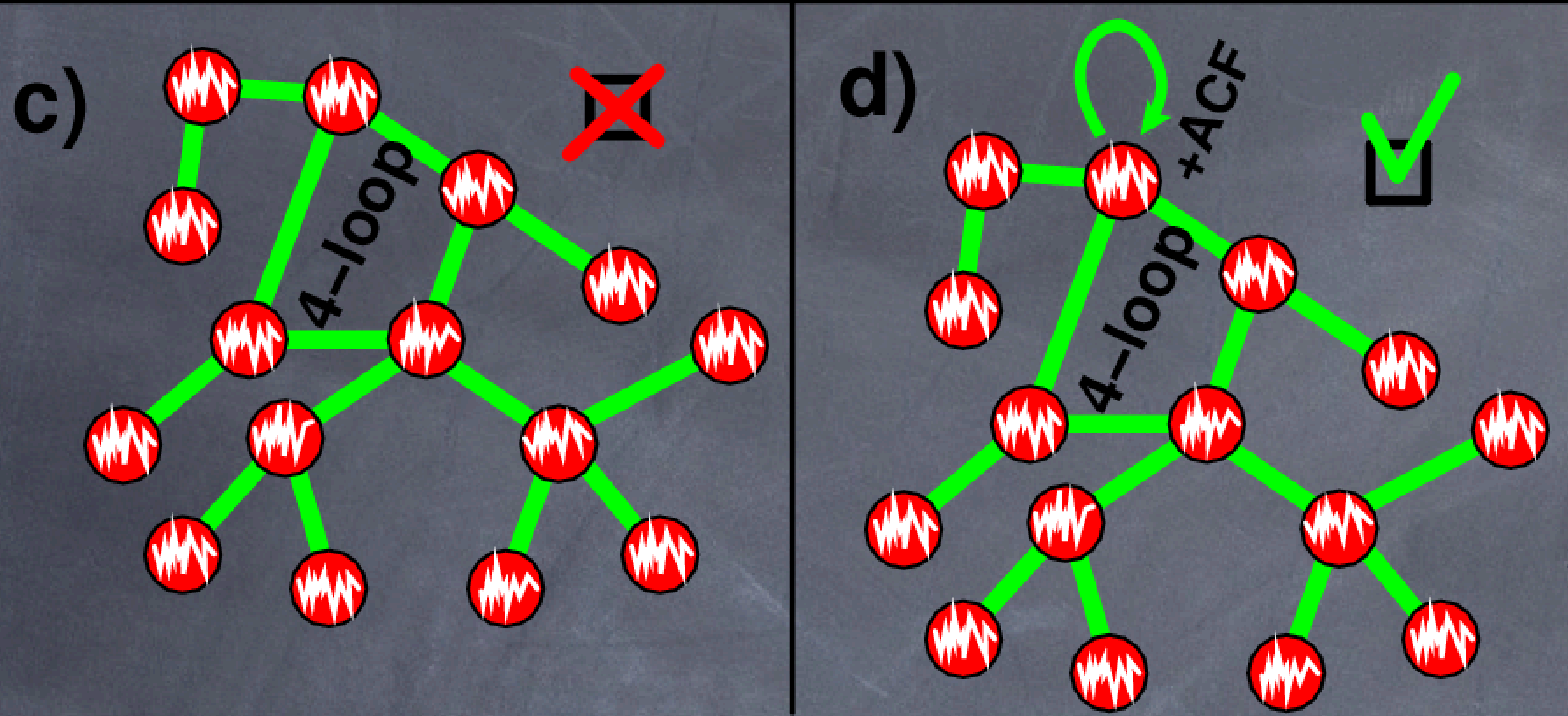


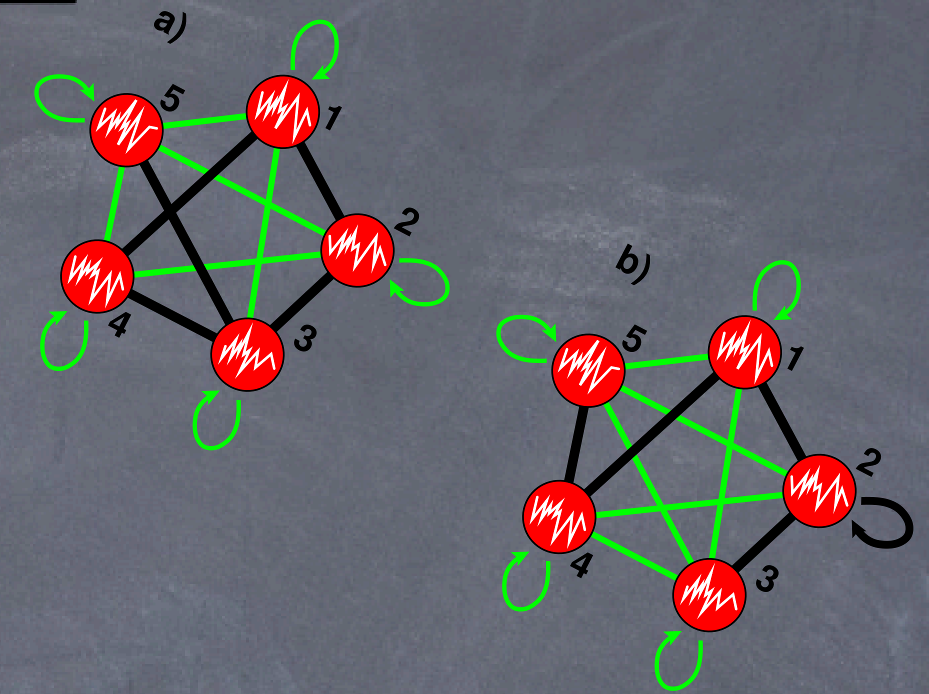
Object:

Network of N wide-sense stationary ergodic processes

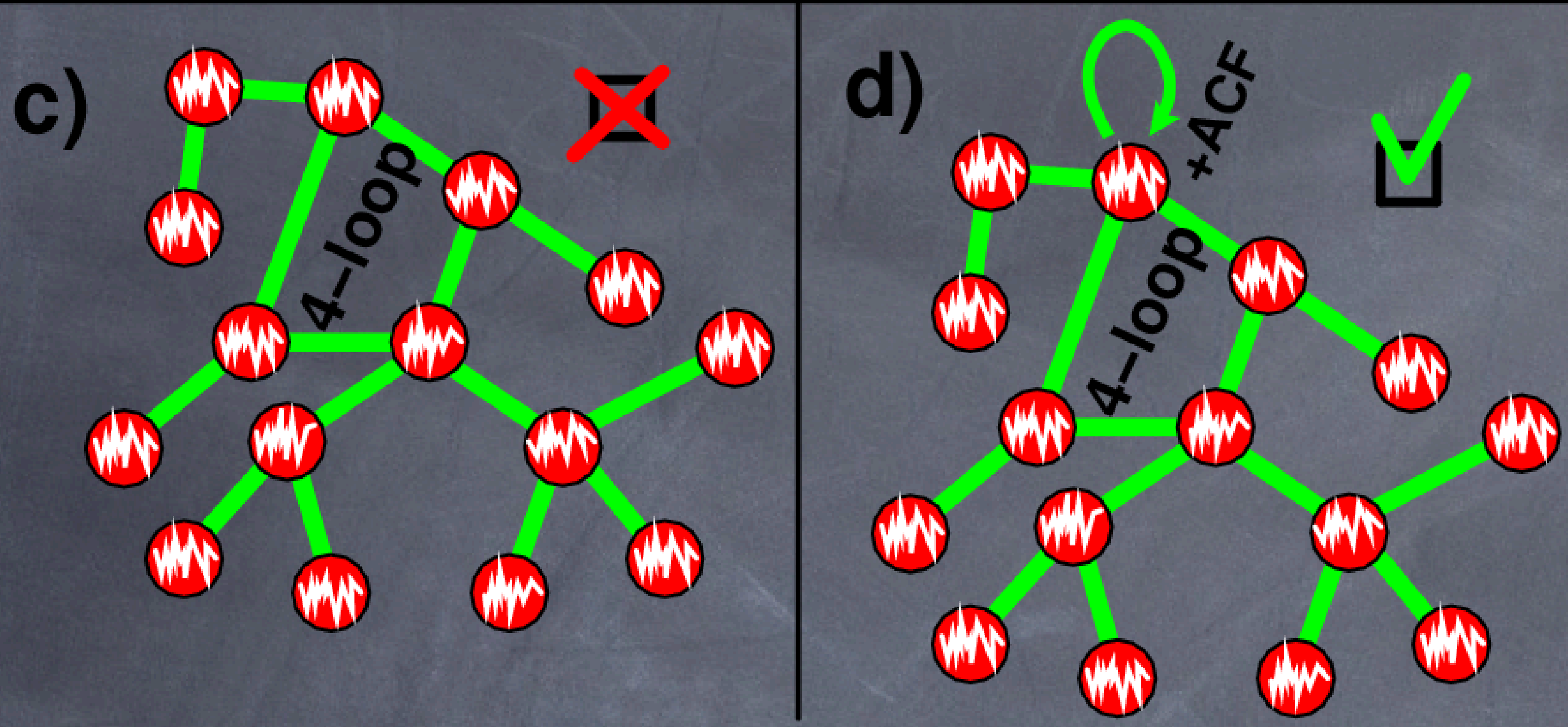
Aim:

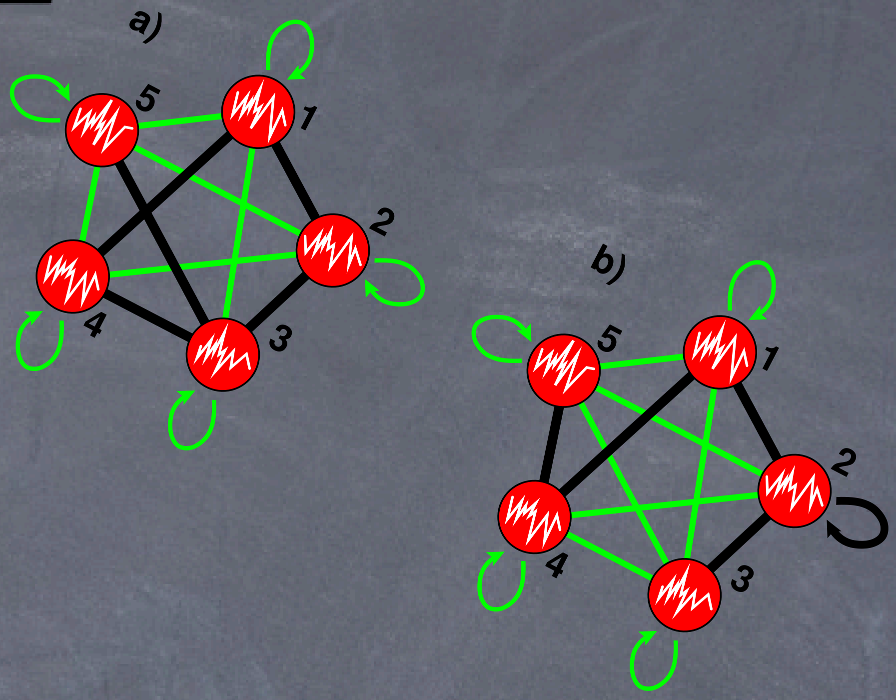
Reconstruction of the entire correlation structure - given a subnetwork formed by correlation functions





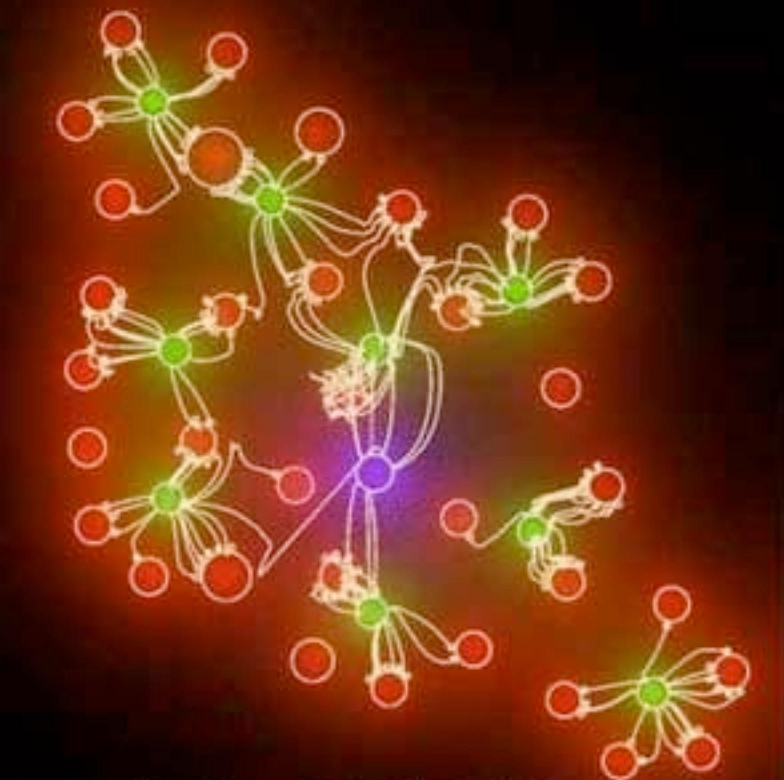
When is the system under- or overdetermined?

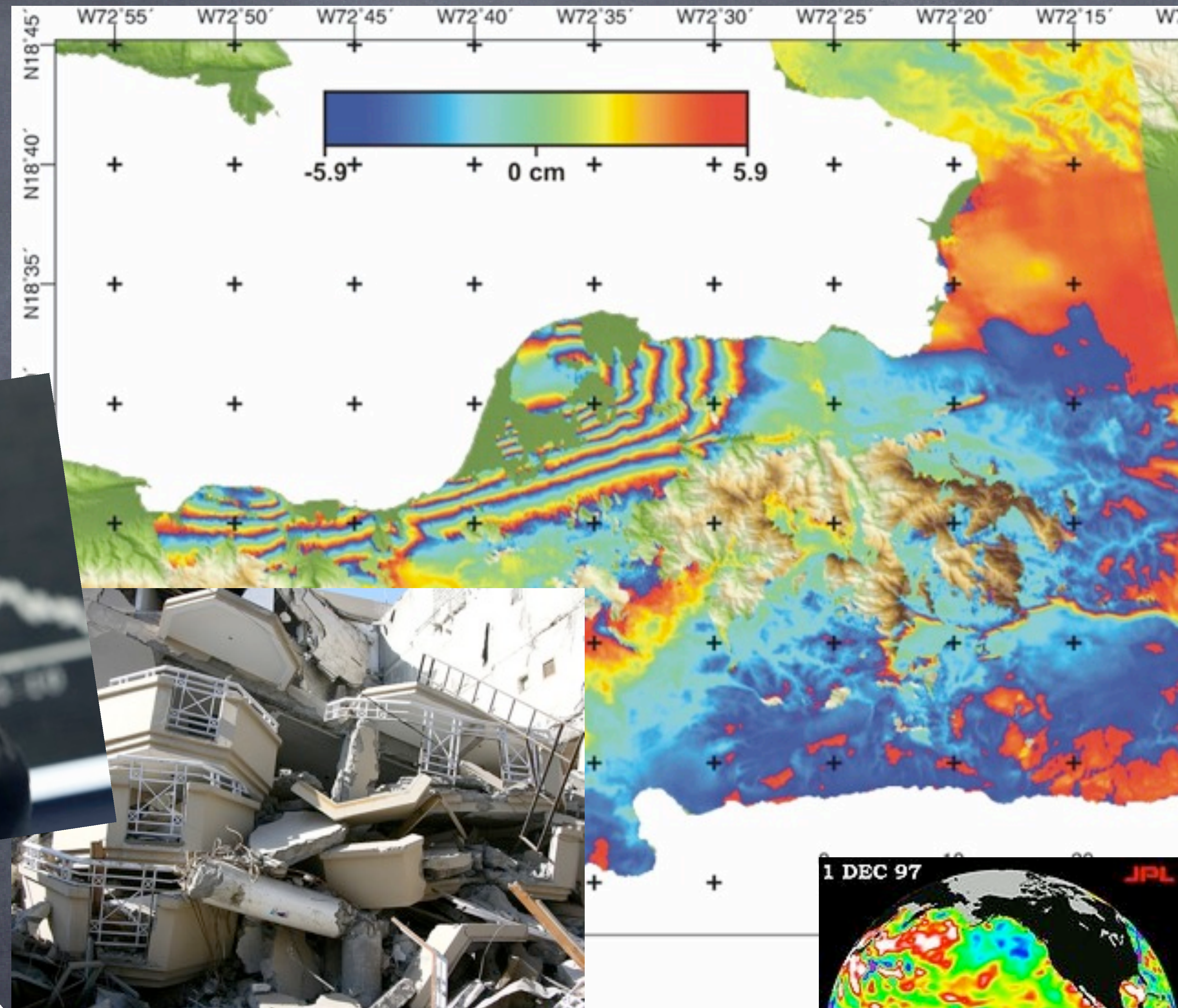


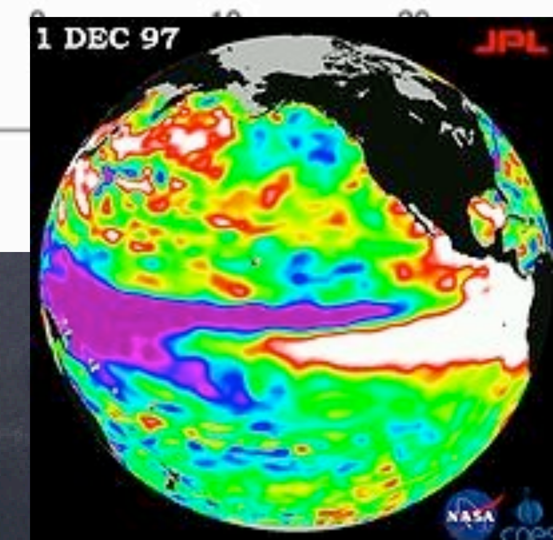


Which topologies are reconstructable?

Why do we care?







Wiener Khinchine:
Fourier(ACF)=Power Spectral Density

$$\mathcal{F}(ACF(\tau)) = X(\omega)X(\omega)^* = |X(\omega)|^2 = S(\omega)$$

Crosscorrelation Theorem:
Fourier(CCF)=Product of Fourier-transformed signals

$$\mathcal{F}(CCF(\tau)) = X(\omega)Y(\omega)^*$$

Assumptions

ACF+CCF:
smooth, non-zero

stochastic processes:
wide-sense stationary & ergodic, unit variance

network:
connected

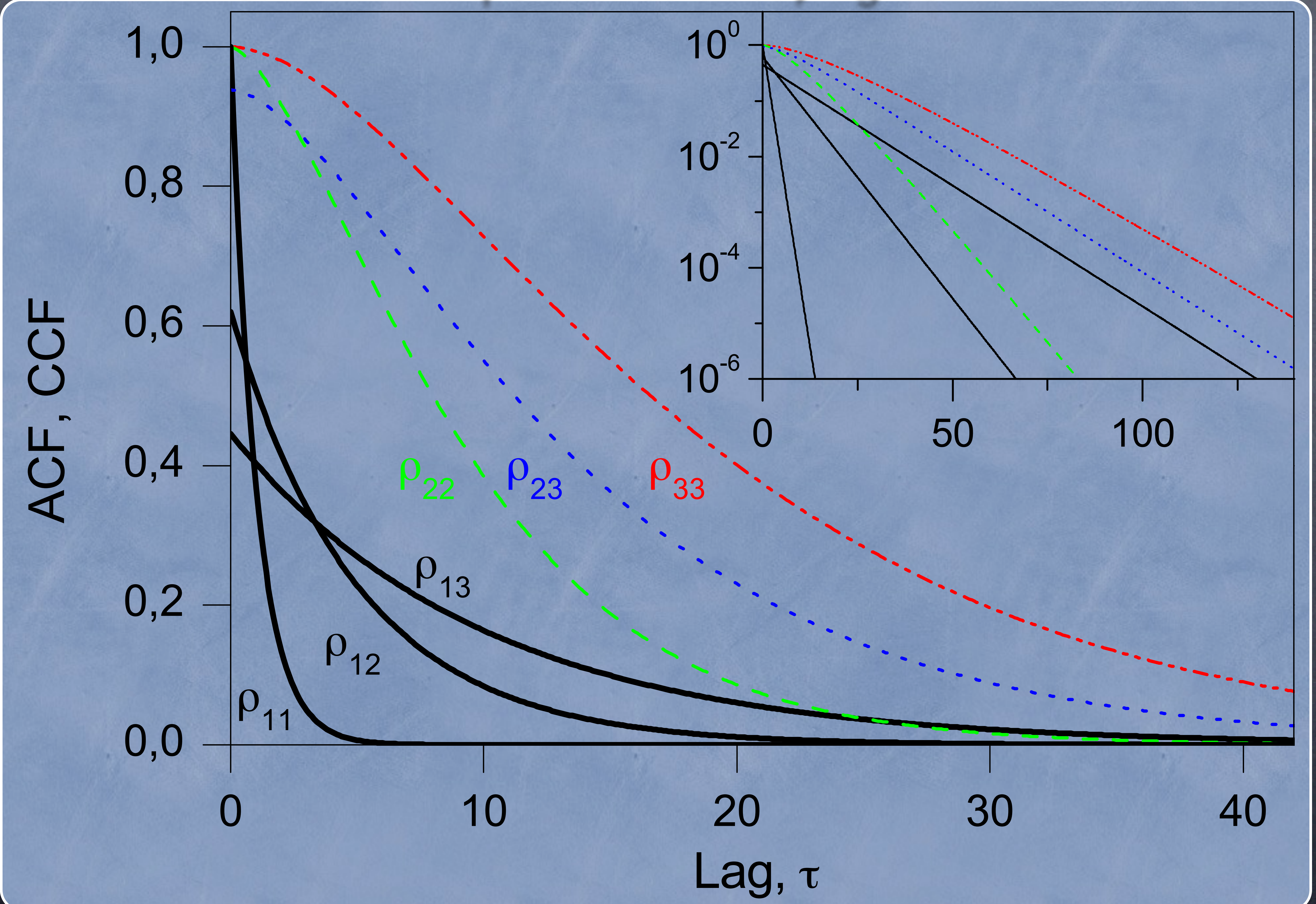
3-network: relation for CCF in Fourier space

$$P_{jk}(\omega) = \frac{P_{jl}(\omega) P_{lk}(\omega)}{P_{ll}(\omega)}, \quad j, k, l \in \{1, 2, 3\}$$

Crosscorrelation Theorem:
Fourier(CCF)=Product of Fourier-transformed signals

$$\mathcal{F}(CCF(\tau)) = X(\omega) Y(\omega)^*$$

3-network with exponential decaying corr. functions



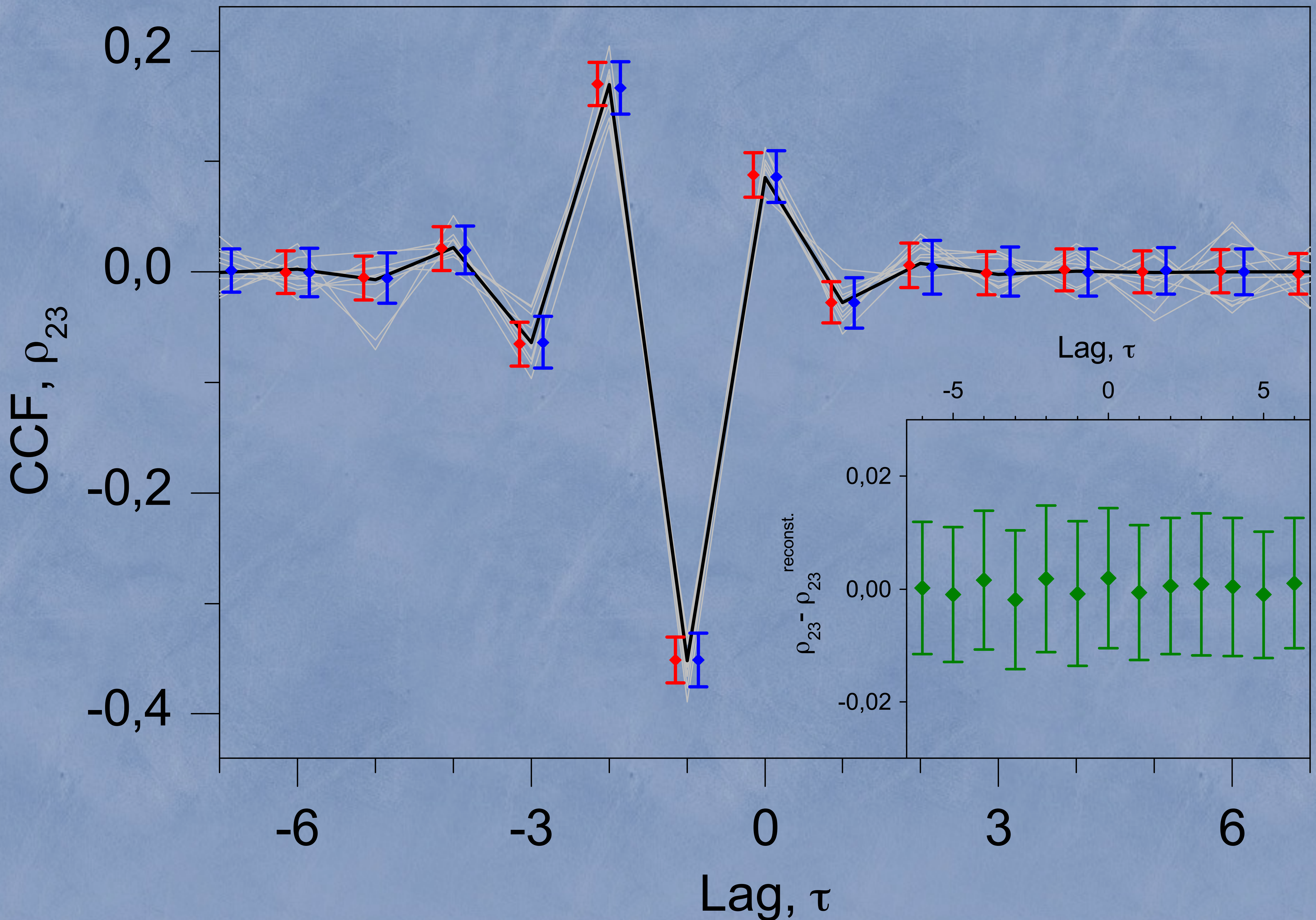
Main Observation (given assumptions)

Except for certain loop structures,
either N crosscorrelation functions (CCF),
or N-1 CCF + 1 single autocorrelation function (ACF)
determine all missing ACF + CCF

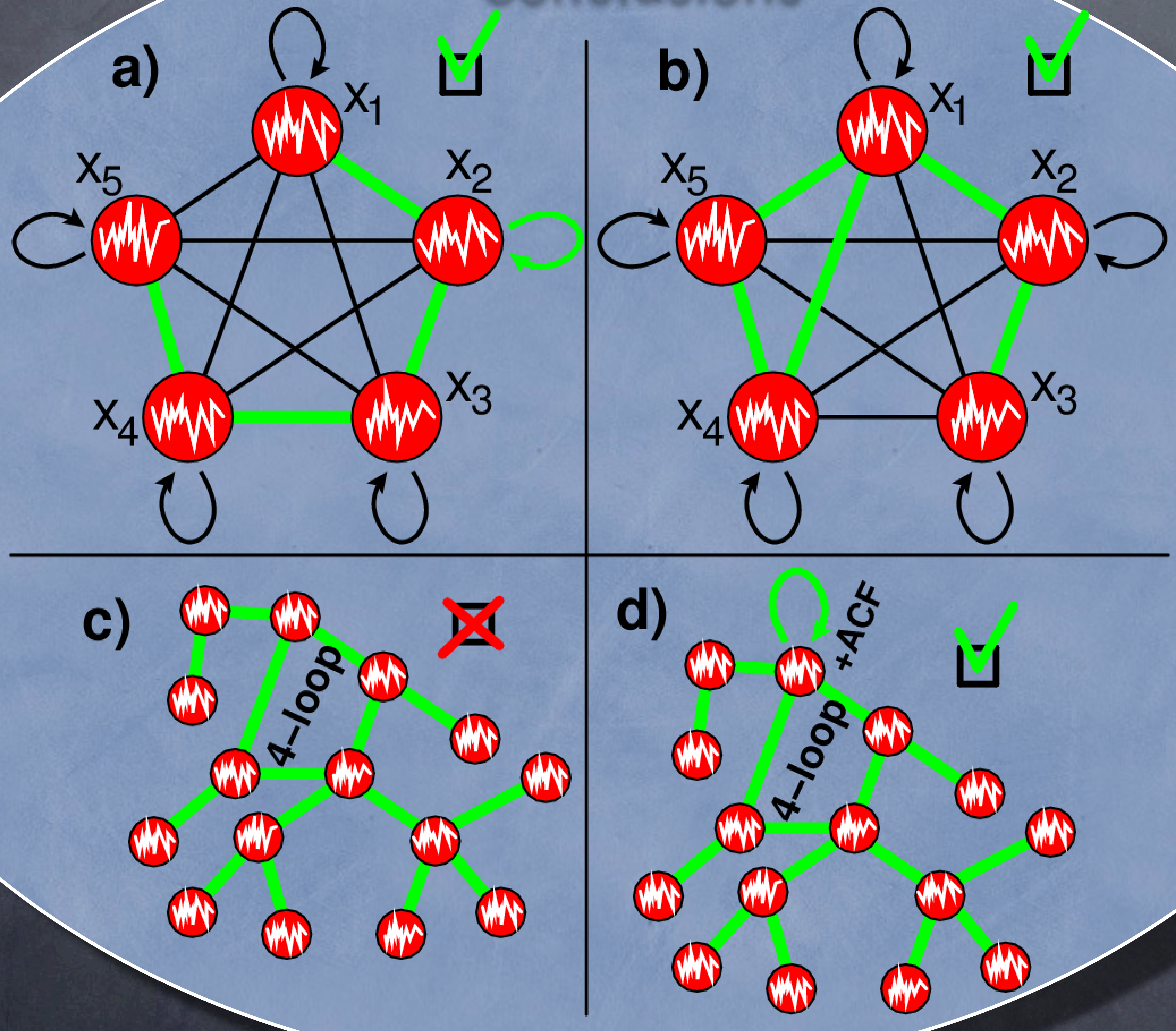
=> for observational data:
1 additional single signal x(t)
determines all other N-1 signals

Kersting, Witt, Geisel, Nagler (2012)

Numerics vs analytics: OK



Conclusions



Kersting, Witt, Geisel, Nagler (2012)

Thanks!

Feedback welcome!

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