Collaboration in social networks: Incentives and topology

Luca Dall'Asta,

Politecnico Torino

Matteo Marsili

Abdus Salam ICTP, Trieste

and Paolo Pin

Dept. Economics, Universita' di Siena

Luca Dall'Asta, Matteo Marsili, and Paolo Pin

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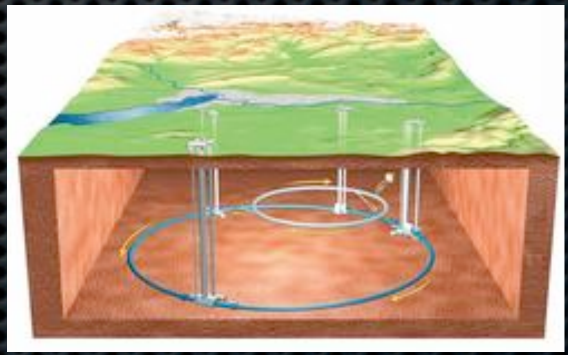
The puzzle of cooperation

- Why do we see so much cooperation around?

- Failed states, why do societies collapse?



- Will Euro collapse if Greece drops out?

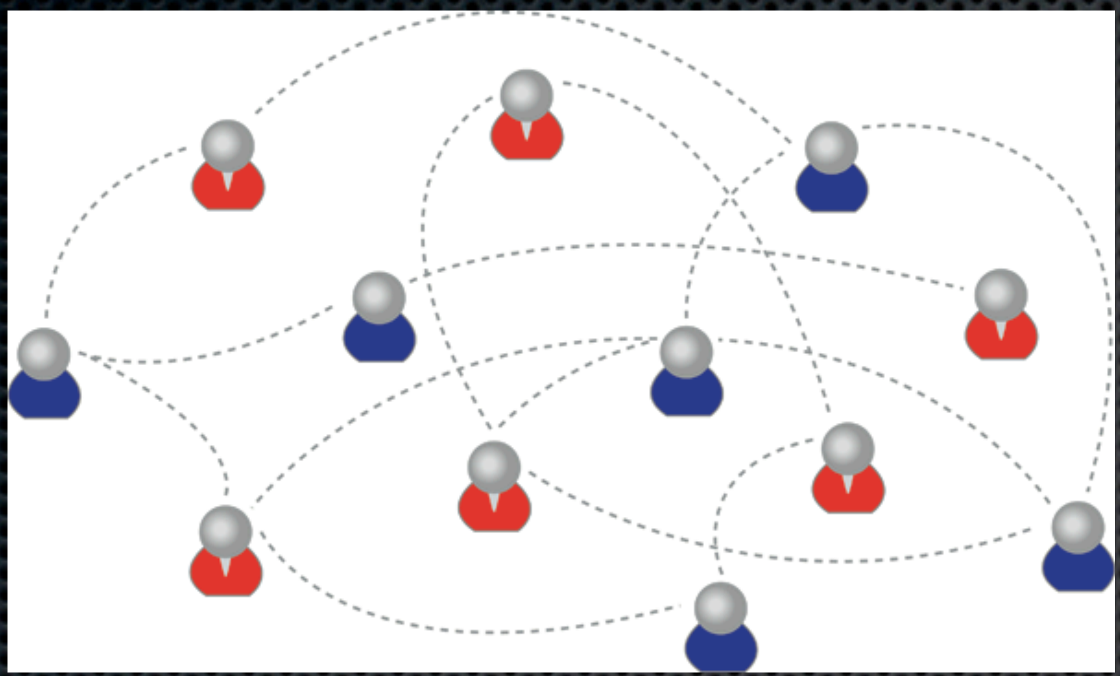
Much has been written on the emergence of cooperation on networks

- Repeated games, reputation and trust (Myerson 1991)
- Endogenous network games (Vega-Redondo 2007, Jackson 2008, Goyal 2009)
- Repeated games on evolving networks (Ellison 1994, Haag Lagunoff 2006, Vega-Redondo 2006).
- Cooperation in evolutionary games without mutation (Boyd 1999, Hofbauer Sigmund 2003, Poncela et al 2010)
- Repeated games and punishment on specific structures (Eshel et al 1998, Haag Lagunoff 2007, Fainmesser 2009, Karlan et al 2009)
- Focus here: social network = pattern of repeated interactions
  repeated interaction = forward looking behavior
  collaboration = incentives + credibility of threats
  How difficult is this in large games on complex structures?

Outline

- The prisoners dilemma
- Collaboration in repeated interaction: 2 players
  - Collaboration is supported by credible threats of punishment
- Collaboration in N players games on a network: Local contribution game
  - Conditional collaboration has to be reciprocal and limited to a subset of neighbors
- How does collaboration depend on incentives and topology?
  - Collaborative equilibria are subgraphs of the social network
- The complexity of collaboration:
  - Counting collaborative equilibria with message passing
- Conclusions

Defection is the only possible outcome in one shot prisoner's dilemma



	C (s=1)	D (s=0)
C (s=1)	1-x, 1-x	-x, 1
D (s=0)	1, -x	0, 0

N players on graph G=(N,L)
Each player either cooperates (C) or defects (D) with all neighbors

Payoff: 1 for each neighbor that collaborates minus X_i (=cost of collaboration)

$$u_i(s_i, s_{-i}) = -X_i s_i + \sum_{j \in \partial_i} s_j$$

All D (s_i=0) is the only Nash equilibrium

$$s_i = 0, 1$$

N=2: When the game is played many times cooperation is possible, among other things

- Strategies become plans of actions, decided at time 0, to optimize future payoffs

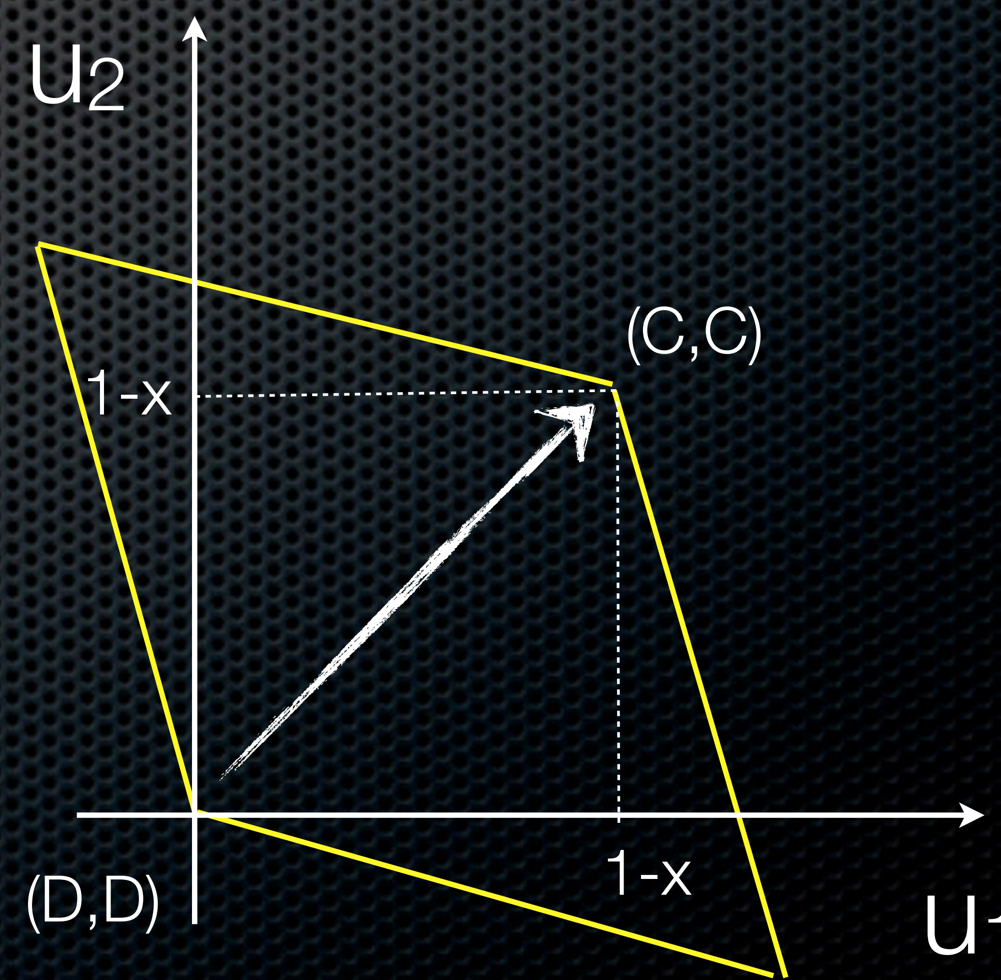
$$U_i = (1-d)\sum_{t=0}^{\infty} d^t u_i\left(s_i^{(t)}, s_{-i}^{(t)}\right), \qquad d \in [0,1]$$

- Cooperation under trigger strategies T:
  T= {start with C;
  C as long as opponent plays C,
  D forever, if opponent plays D}

  If d is large enough, (T, T) is a Nash equilibrium

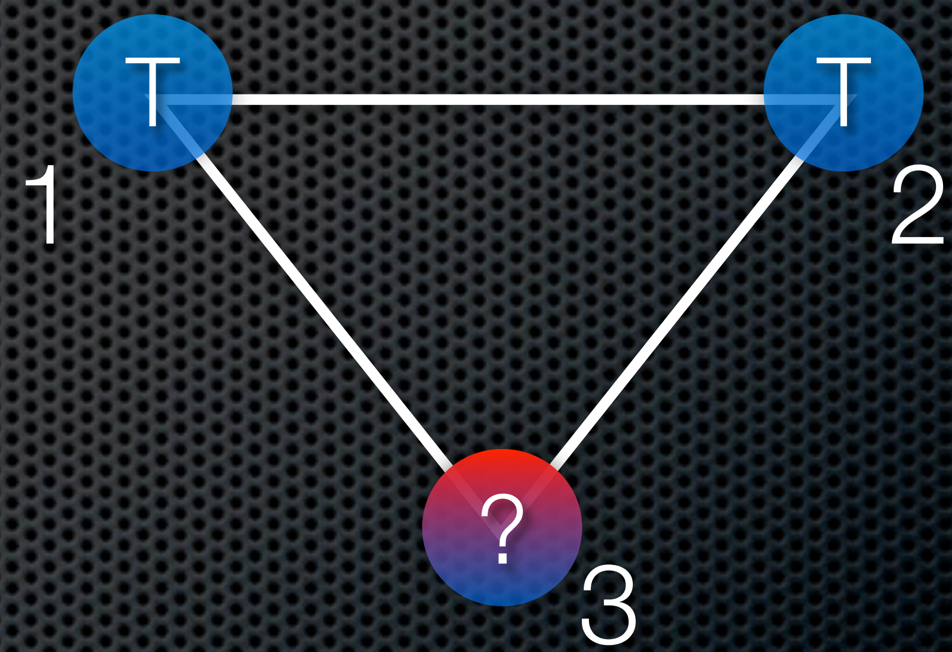
- Folk's theorem: many other outcomes can be supported as a Nash equilibrium

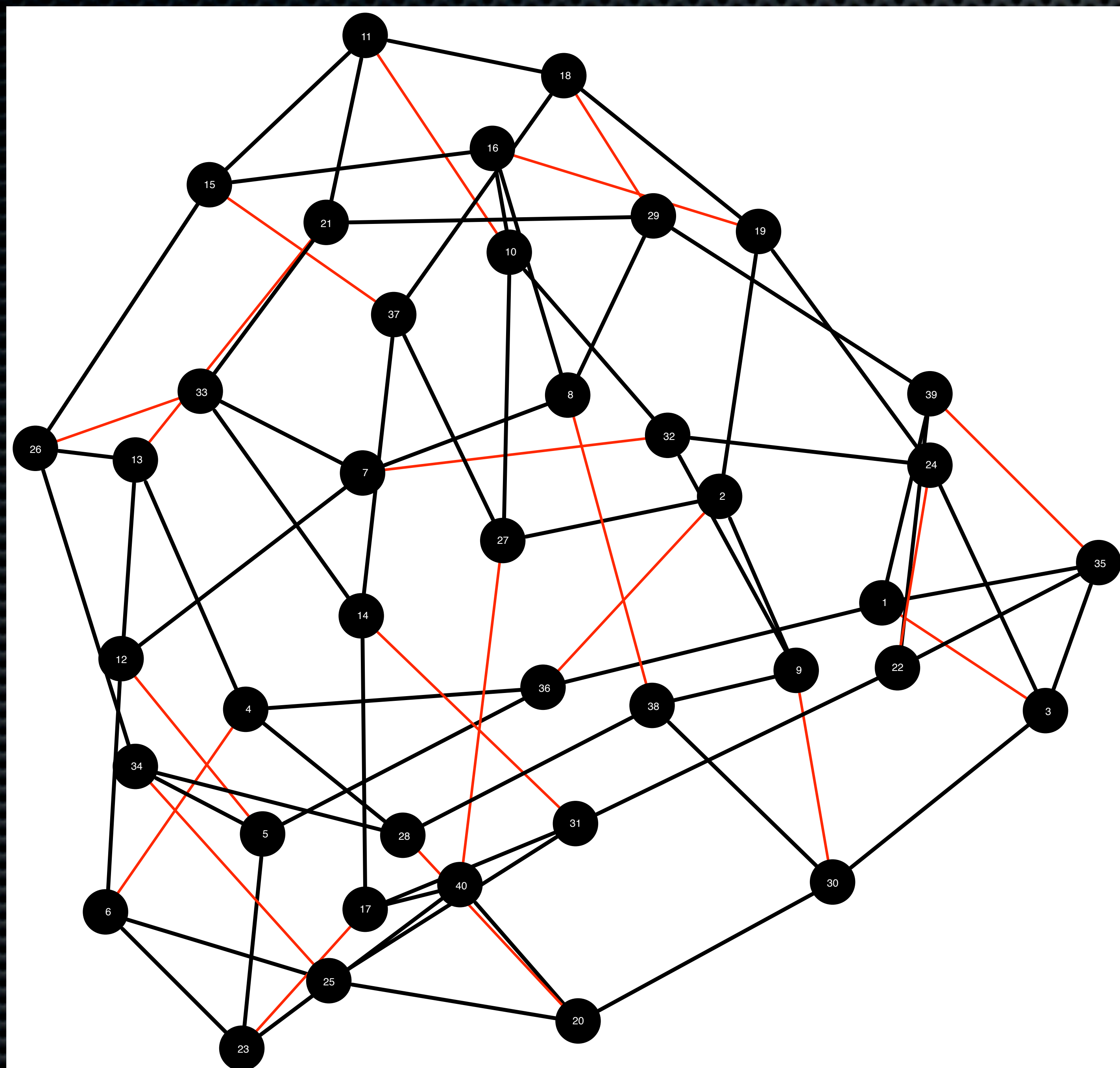
- ! d=1 in what follows



But threats should be credible

- N=3
- Is it credible that 1 and 2 punish 3?
- Not if $u_1(C,C,D) > u_1(D,D,D)$!
- Players need to condition C only to a subset of their neighbors
- If i conditions on j, j should condition on i
- Emergent heterogeneity





?

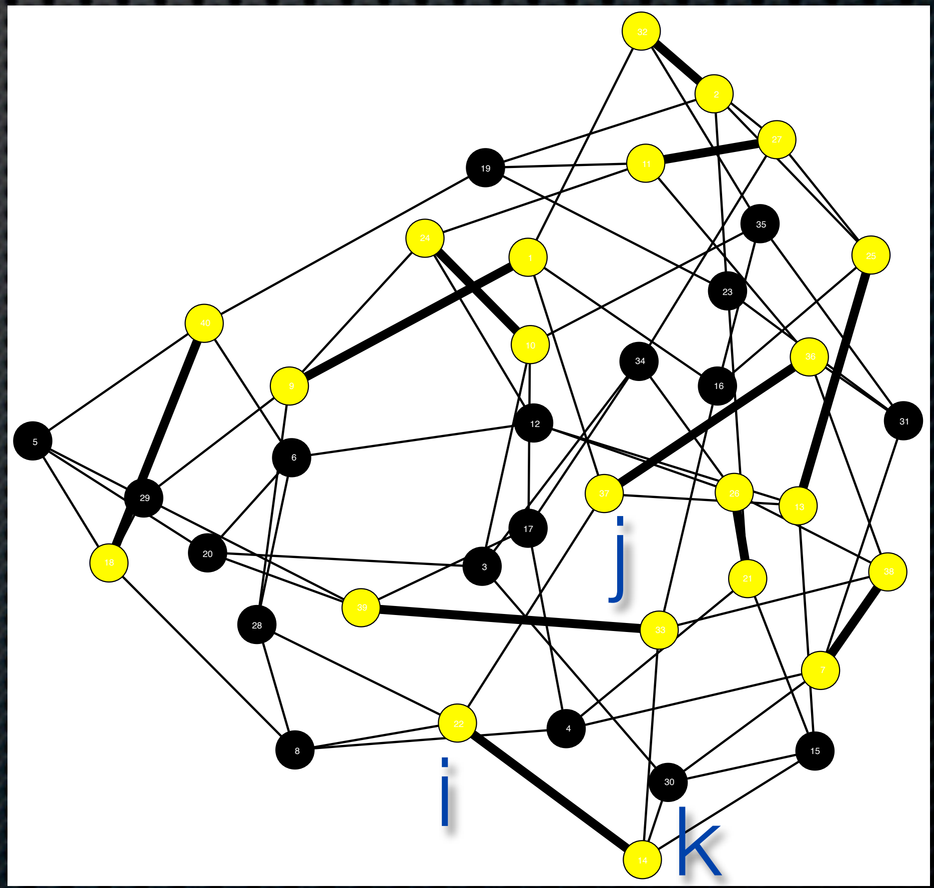
- Trees
- Graphs

On trees, Nash equilibria are subtrees

- Given an undirected tree G=(N,L)
  $k_i = |\partial_i|$ = degree of node i
  m_i = smallest integer larger than X_i
  c_i = number of collaborators in ∂_i

$$u_i(s_i, s_{-i}) = c_i - X_i s_i$$

- Any collection of disjoint undirected subgraphs $\Gamma=(V,\Lambda)$ of G is a collaborative equilibrium where all $i \in V$ cooperate conditionally to neighbors in Γ and $|\partial_i \cap \Lambda| = m_i$



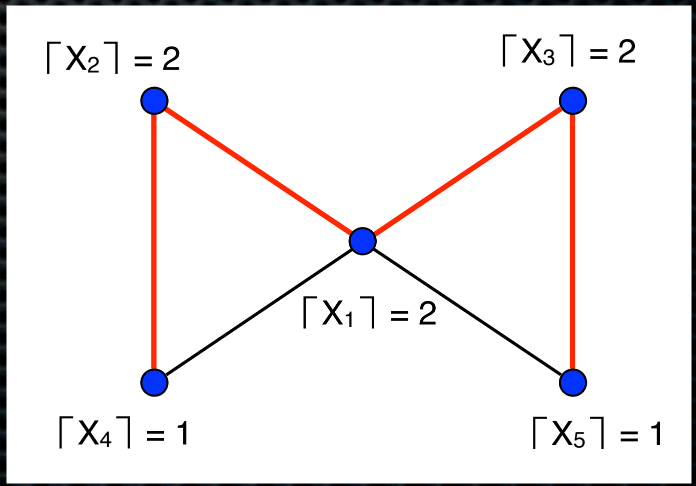
- Incentives: $i \in V$ $c_i - X_i \geq c_i - m_i \Rightarrow m_i \geq X_i$

- Reciprocity: $i,j \in V$, if j does not punish i
  $\Rightarrow$ i should not punish j when j defects

- Credibility:
  $i,k \in V$, $(i,k) \in \Lambda$ if k defects $c_i - 1 - X_i < c_i - m_i \Rightarrow m_i < X_i + 1$

On generic graphs cascades of defection make things more complex

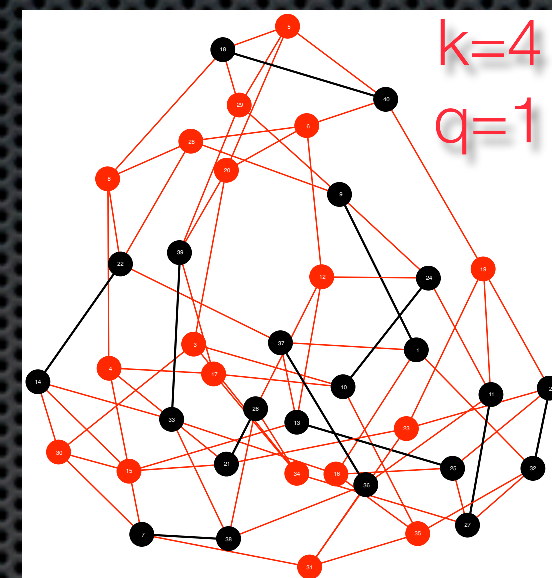
- Indirect defections: As a result of the defection of $j \in \partial_i$ other neighbors $k \in \partial_i$ may also defect because of loops
- A collection of disjoint undirected subgraphs $\Gamma=(V,\Lambda)$ of G is a collaborative equilibrium where all $i \in V$ cooperate conditionally to neighbors in Γ and $|\partial_i \cap \Lambda|=m_i$ provided
  i) the indirect effects caused by the defection of all $j \in \partial_i \cap \Lambda$ have the same consequence of the defection of i itself.
- i) holds provided removing i from V does not disconnect Γ
  - Works on trees, for dimers and loops, for the complete graph
  - Likely works on random graphs and on dense graphs

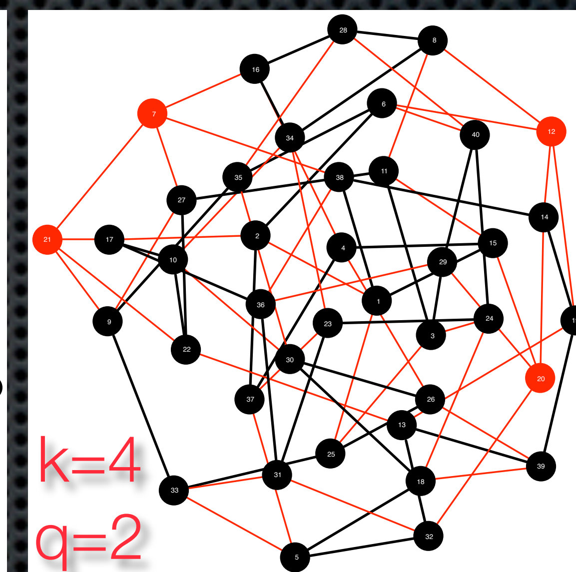


counter example

Nash equilibria on random graphs

Regular random graphs: k_i=k, X_i=X for all i

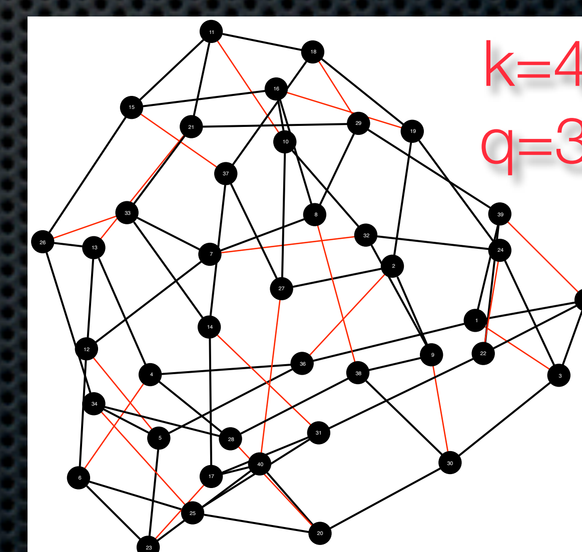




- X ≤ 1 dimers
  1 < X ≤ 2 circuits
  ...

  q-1< X ≤ q q-regular subgraphs
  ...

  k-1< X ≤ k back to dimers



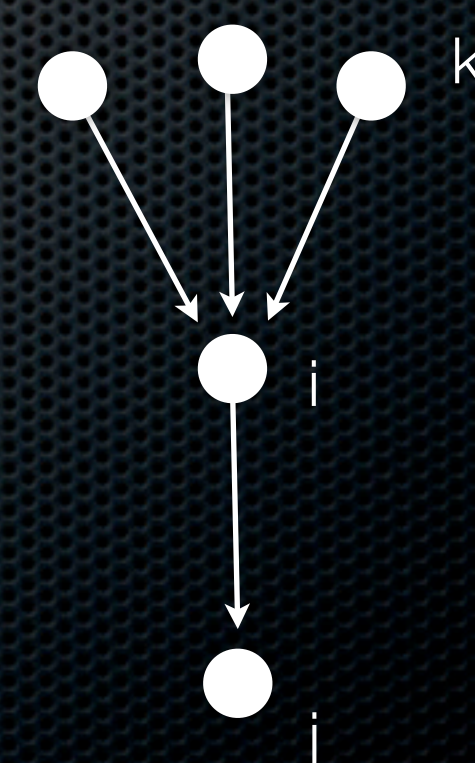
- Do NE exist? How many? How hard is it to find them?

Circuits: Marinari, Monasson, Semerjian 2006
q-regular subgraphs: Pretti, Weigt 2006

Counting NE by message passing

- $x_{i\to j}$ = 1 if i conditions C on j, $x_{i\to j}$ = 0 otherwise
  - there are m_i-1 k ∈ ∂_i/j with $x_{k\to i}$=1 ⇒ $x_{i\to j}$ =1
  - m_i k ∈ ∂_i/j with $x_{k\to i}$=1 ⇒ $x_{i\to j}$ =0
  - no k ∈ ∂_i/j with $x_{k\to i}$=1 ⇒ $x_{i\to j}$ =0
- Marginals:

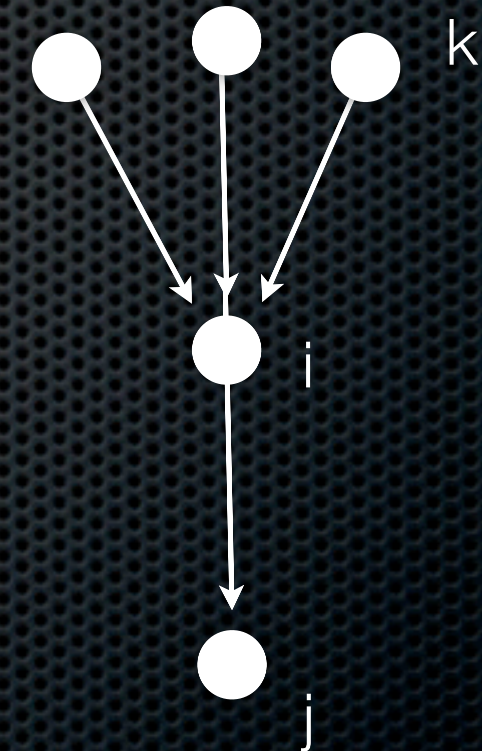
$$\mu_{i\to j} = P\{i \in V, i \text{ punishes } j\}$$



Circuits: Marinari, Monasson, Semerjian 2006
q-regular subgraphs: Pretti, Weigt 2006

Message passing equations:

$$\mu_{i\to j} = \frac{e^{-\epsilon} Z^{m_i-1}_{N_i\backslash j\to i}}{Z^0_{N_i\backslash j\to i} + e^{-\epsilon} Z^{m_i-1}_{N_i\backslash j\to i} + e^{-\epsilon} Z^{m_i}_{N_i\backslash j\to i}}$$



$$Z^q_{V\to i} = \sum_{U\subseteq V} \mathbb{I}_{|U|=q} \prod_{j\in U} \mu_{j\to i} \prod_{k\in V/U} (1-\mu_{k\to i})$$

$$P\{i\in C\} = \frac{e^{-\epsilon} Z^{\gamma_i}_{N_i\to i}}{Z^0_{N_i\to i} + e^{-\epsilon} Z^{\gamma_i}_{N_i\to i}}$$

$$P\{i\in \Gamma_j\} = \frac{\mu_{i\to j}\mu_{j\to i}}{\mu_{i\to j}\mu_{j\to i} + (1-\mu_{i\to j})(1-\mu_{j\to i})}.$$

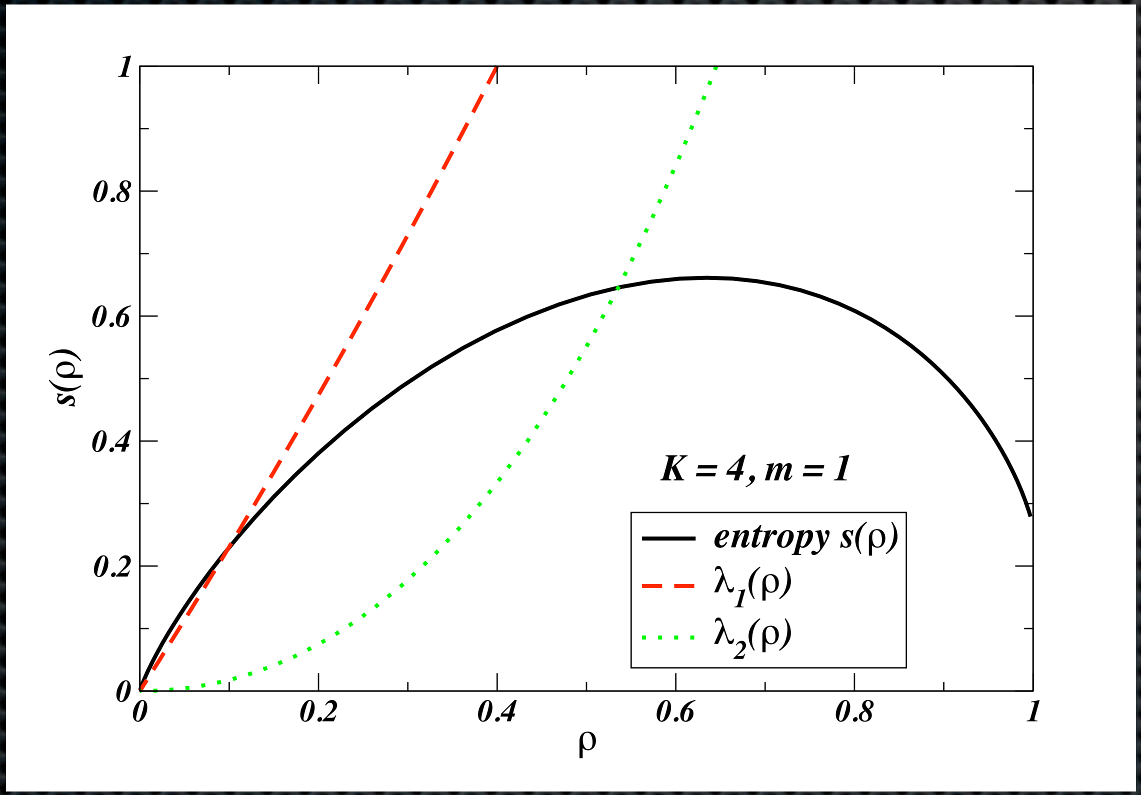
- Fixed point ⇒ number of subgraphs (entropy)

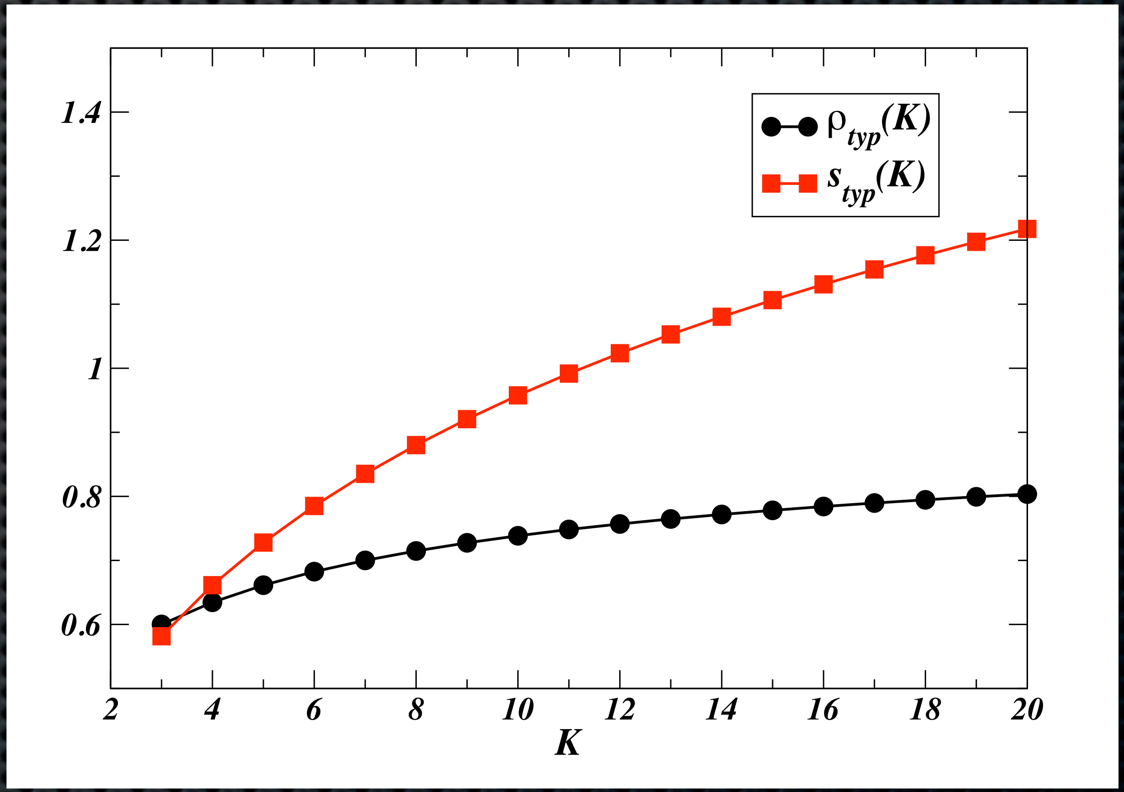
Circuits: Marinari, Monasson, Semerjian 2006
q-regular subgraphs: Pretti, Weigt 2006

Regular random graphs: dimers

(m_i=1)

- Exponentially many NE's





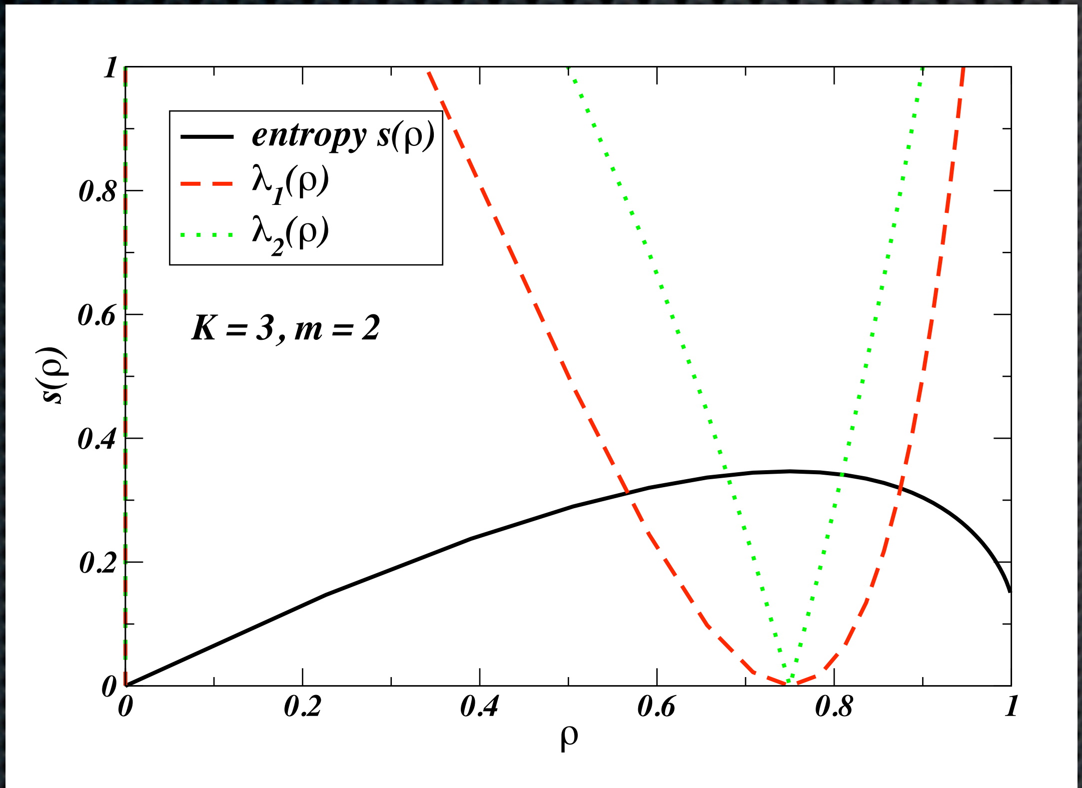
- ρ = fraction of cooperators
  $s(\rho)$ = log(number of NE|ρ)/N

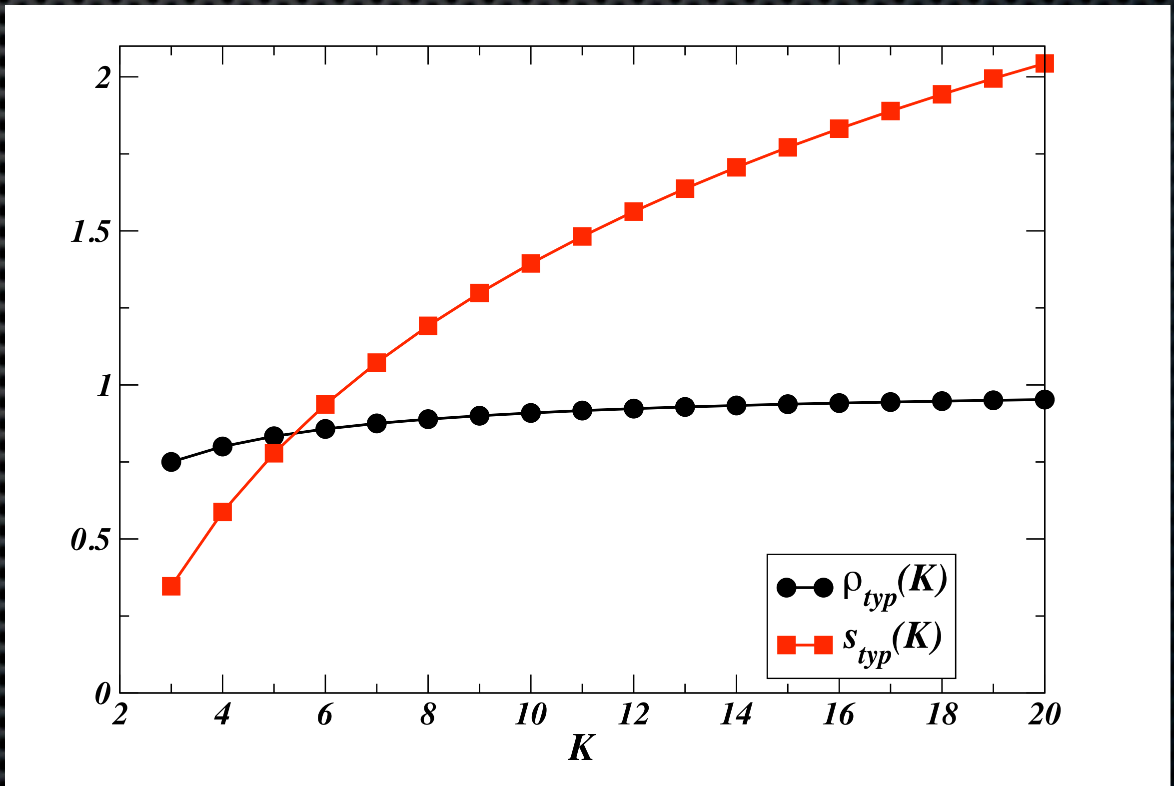
NE $\exists$ $\forall\rho\in[0,1]$

Regular random graphs: circuits

(m_i=2)

- Exponentially many NE's



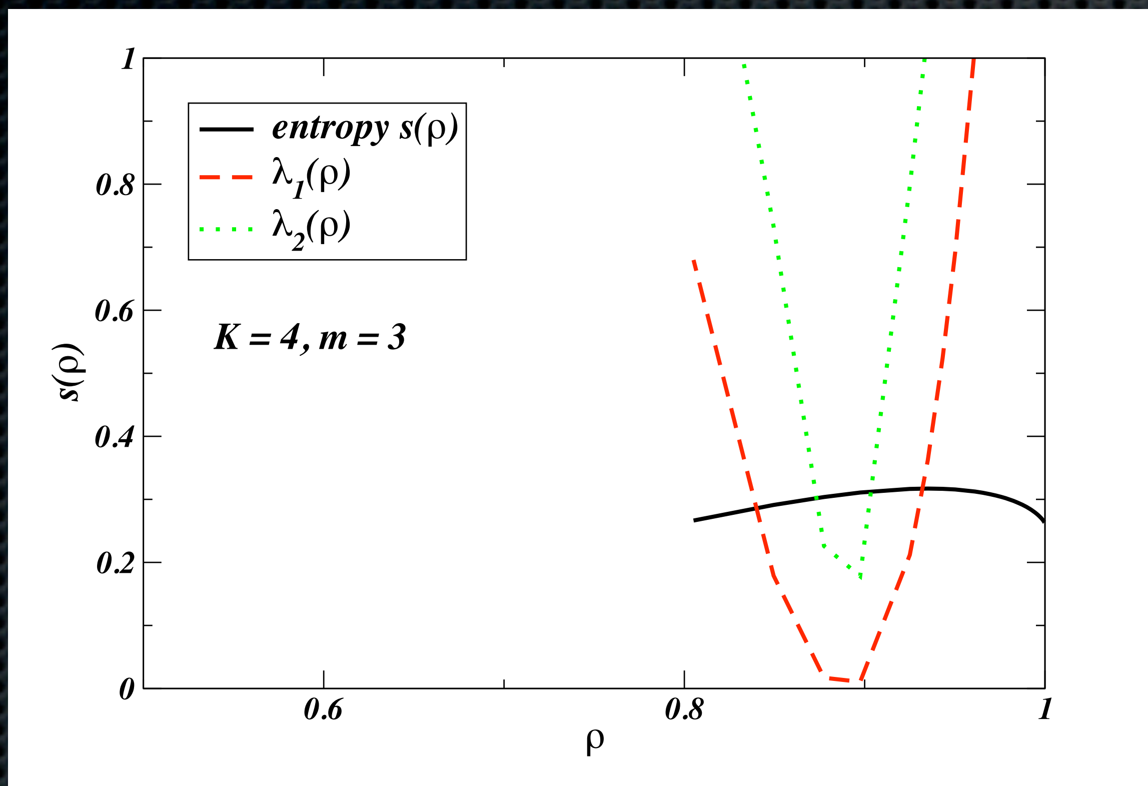


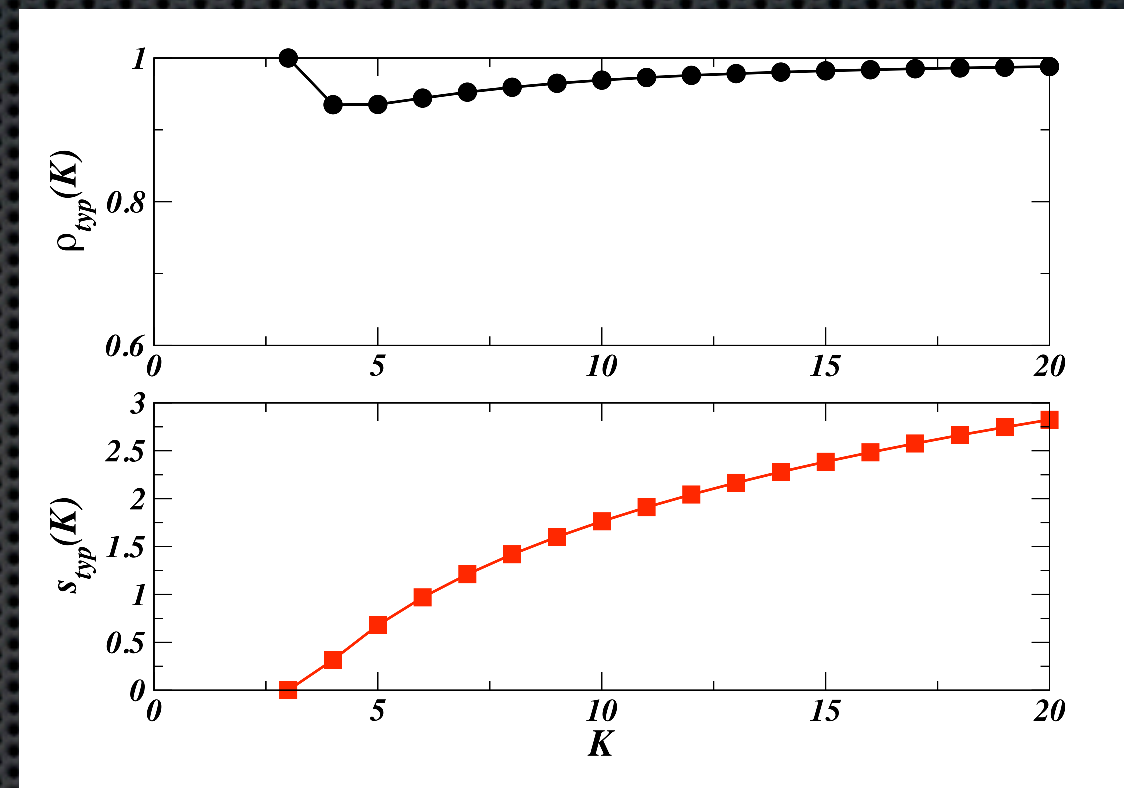
- ρ = fraction of cooperators
  s(ρ) = log(number of NE|ρ)/N

NE ∃ ∀ρ∈[0,1]

(Marinari, Monasson, Semerjian 2006)

Regular random graphs: m_i=3

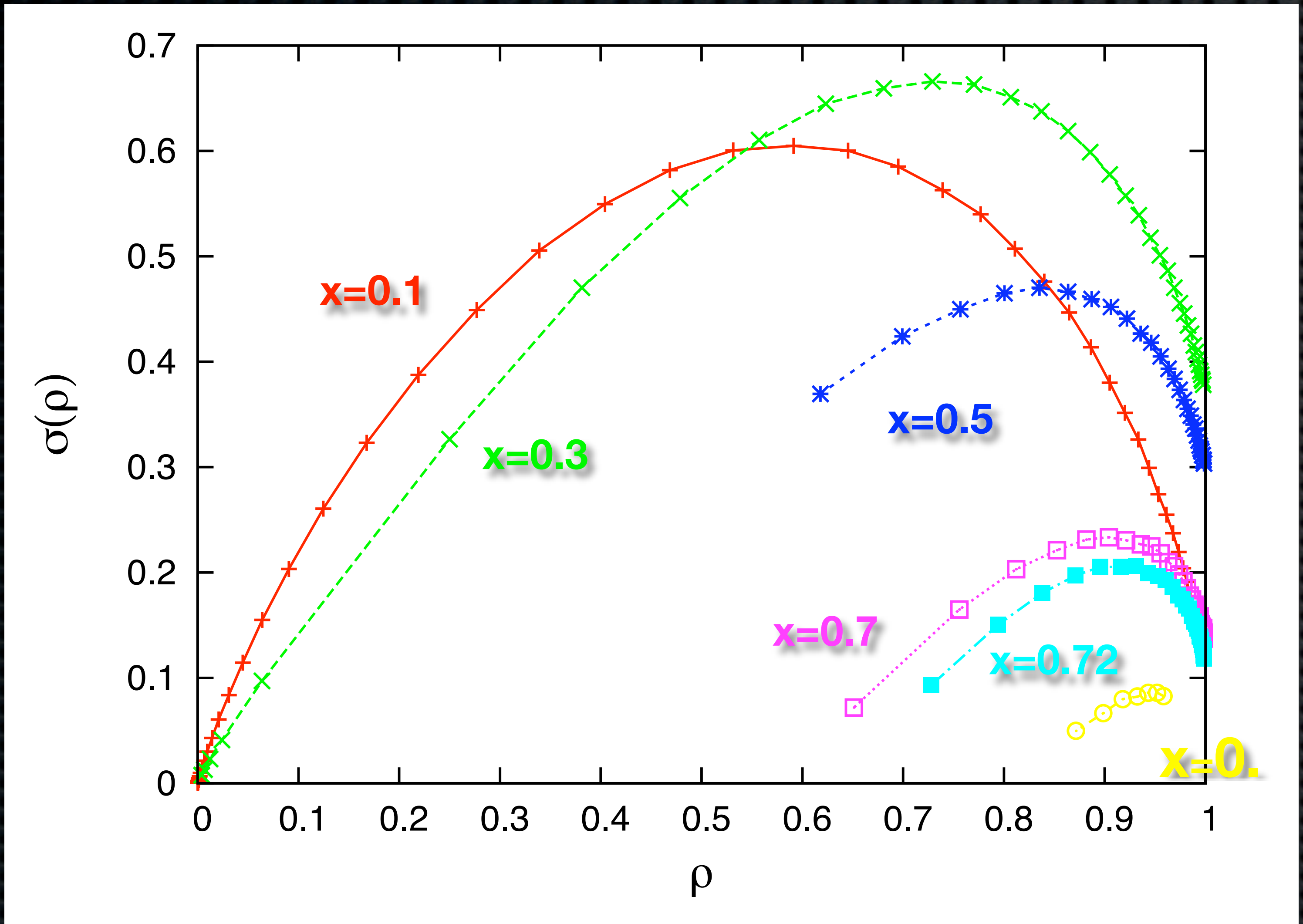




- Exponentially many NE's
- NE ∄ $\forall \rho < \rho_c$
- NE are non-local and fragile

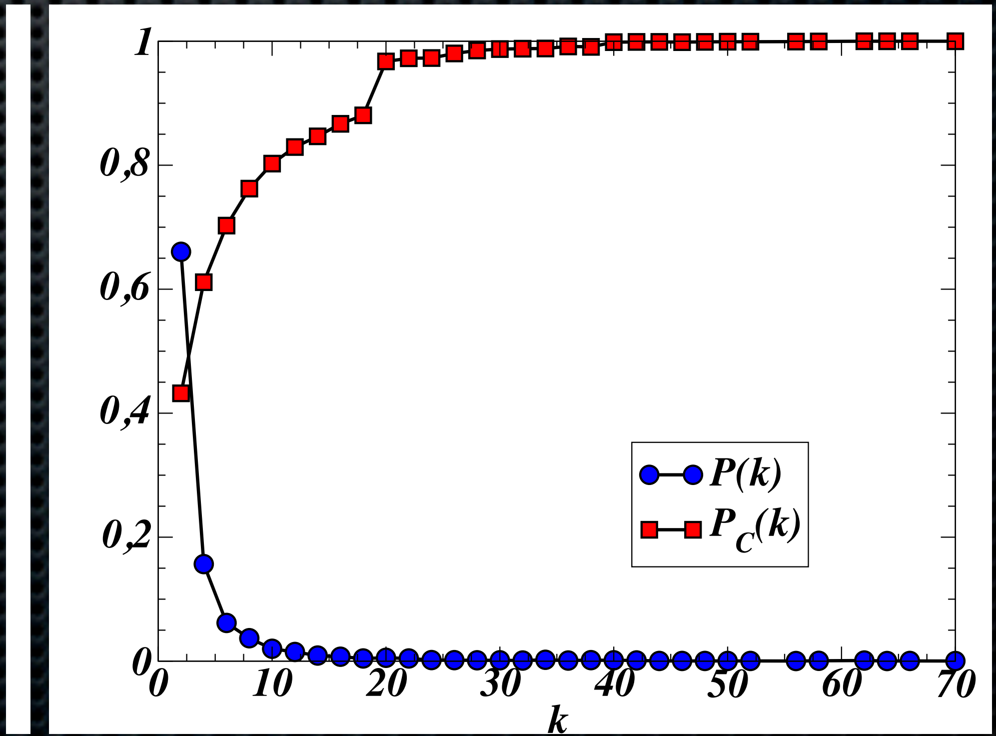
(Pretti, Weigt 2006)

Heterogeneous random graphs



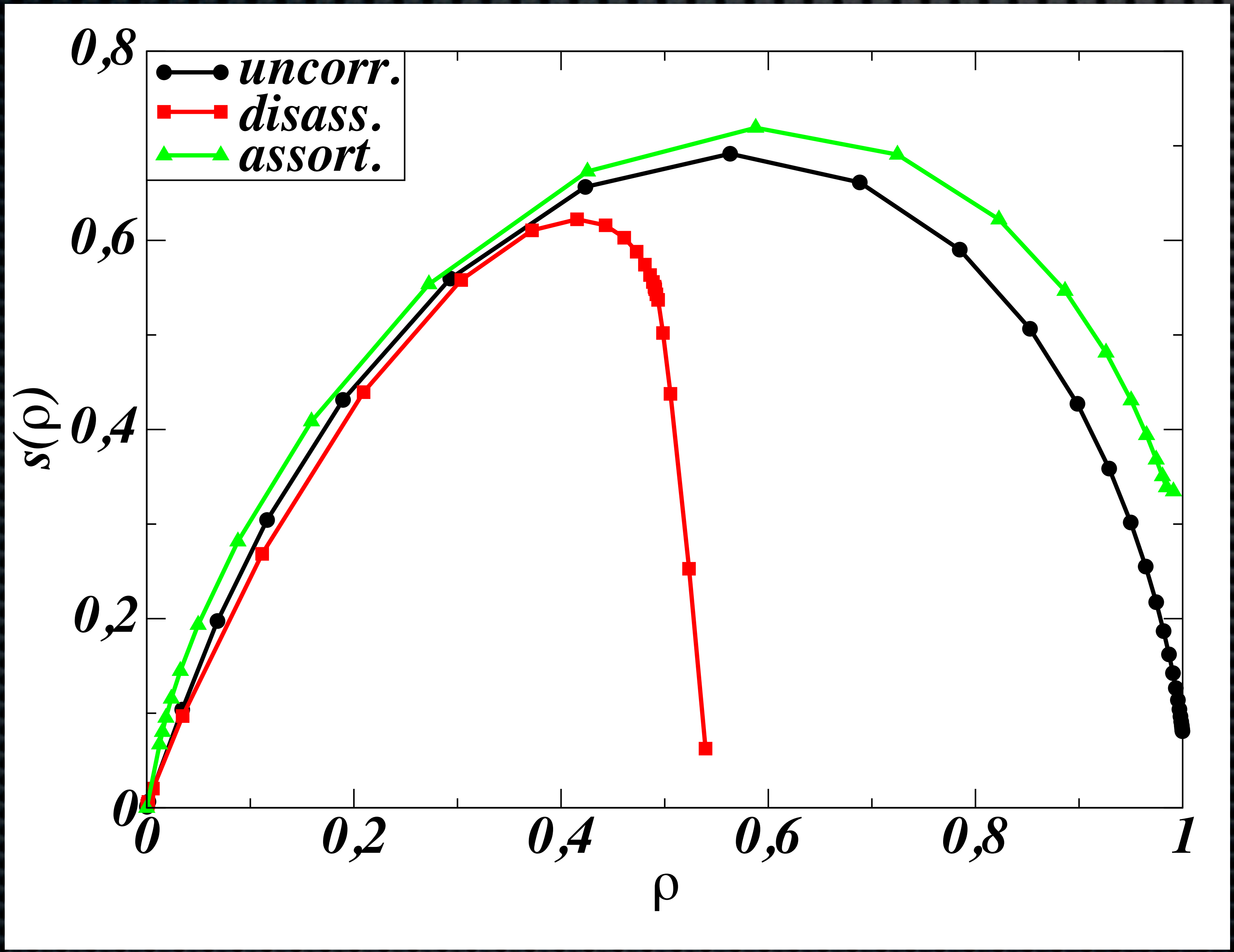
Erdös-Rényi: E[k]=4

$X_i = xk_i$



Scale free: hubs collaborate more likely than spokes

Assortative networks are more conducive to collaboration



Scale free network
$P(k) \sim k^{-2.5}$
$X_i = xk_i$, $x=0.1$

Conclusions: Theory

- Collaboration in repeated prisoners dilemma as a graph theoretical problem:
  1- make sure enough neighbors collaborate
  2- not credible to monitor more neighbors
  3- checks should be reciprocal
- If incentives to defect (x)
  - is small then cooperation is easy
  - is large
    i) collaboration requires critical mass
    ii) Nash equilibria are fragile
    iii) effect of defection are non-local
- Topology: Collaboration is easier on
  i) trees
  ii) densely connected graphs
  Collaboration is harder on networks which can be disconnected
  (e.g. quasi 1d graphs)

Conclusion: Empirical evidence

- Individuals condition collaboration on that of others (Fishbacher, Gachter, Fehr 2001)
- Weak and strong ties (Granovetter, 1973)
  Individuals do not condition collaboration to all contacts, not even to all those who collaborate, only to a subset of them
- Critical mass theory of collective action (Oliver, Marwell 1993)
  If the cost of collaboration is large enough, collaborative equilibria only arise if a finite fraction of agents participate
- Collaboration easier in dense networks (Kirchkamp, Nagel 2007; Cassar 2007)
- More connected agents are more likely to collaborate (Cassar 2007)
- Collaboration is not contagious (Suri, Watts 2011)
  The more of my contacts are engaged in conditional collaboration with others, the less likely I am to find neighbors with whom to collaborate conditionally