Emergence of fast agreement in an overhearing population : The case of Naming Game

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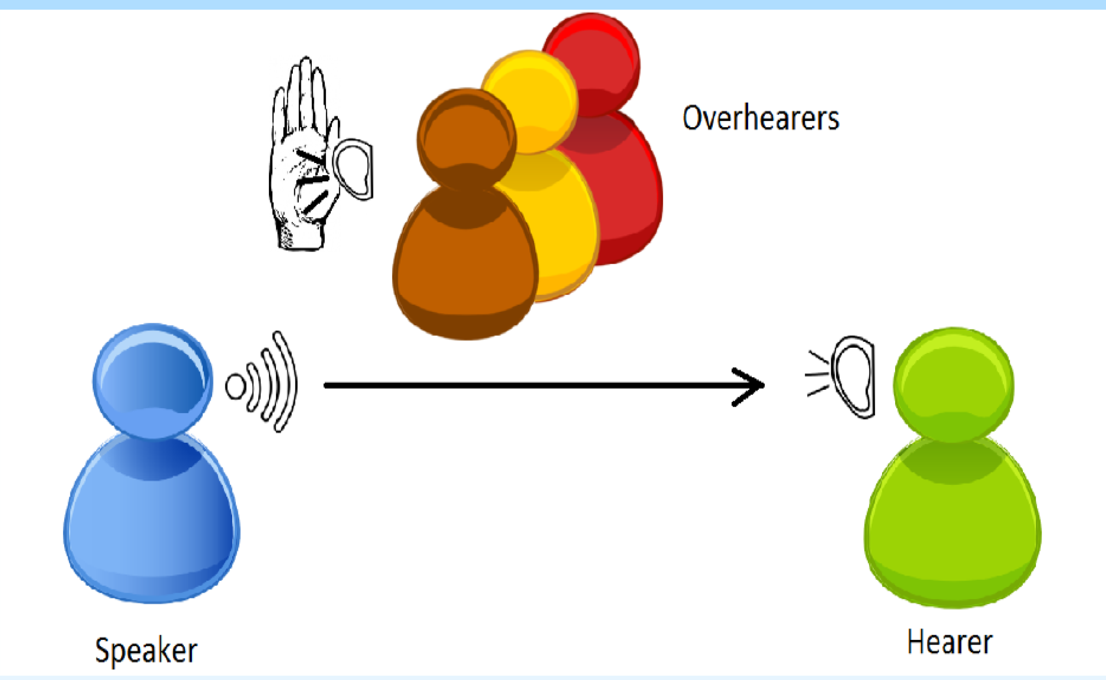
The Naming Game

- The Naming Game (NG) is a model of non equilibrium dynamics for the self-organized emergence of a linguistic convention or a communication system in a population of agents with pair-wise local interactions.
- This model has its relevance in the novel fields of semiotic dynamics and specifically to opinion formation. The application of this model ranges from wireless sensor networks as spreading algorithm, leader election algorithm to user based social tagging systems and language evolution.
- The game is played by a population of agents in pairs to agree upon a single name for an object . One of them assumes the role of a "speaker" and other as "hearer".
  - The speaker conveys a name to the hearer;
  - the hearer tries to search for it in his memory, if he gets it then the interaction is a success and both the agents delete other competing names from their memories ;
  - otherwise the interaction is a failure and the hearer learns the name.

Multi-party communication on NG

- We recast the naming game on the "multi-party" communication framework.



The model

The model consists of an interacting population of N artificial agents observing a single object to be named. Each agents are endowed with an inventory which is empty at the beginning (t = 0).

At each time step (t = 1, 2, . . .):
two agents are randomly selected and interact: one of them plays the role of **speaker**, the other one that of **hearer**. In addition, a set of N^{δ} individuals are randomly selected in each step who behave as **overhearers**. δ is a parameter of the model.

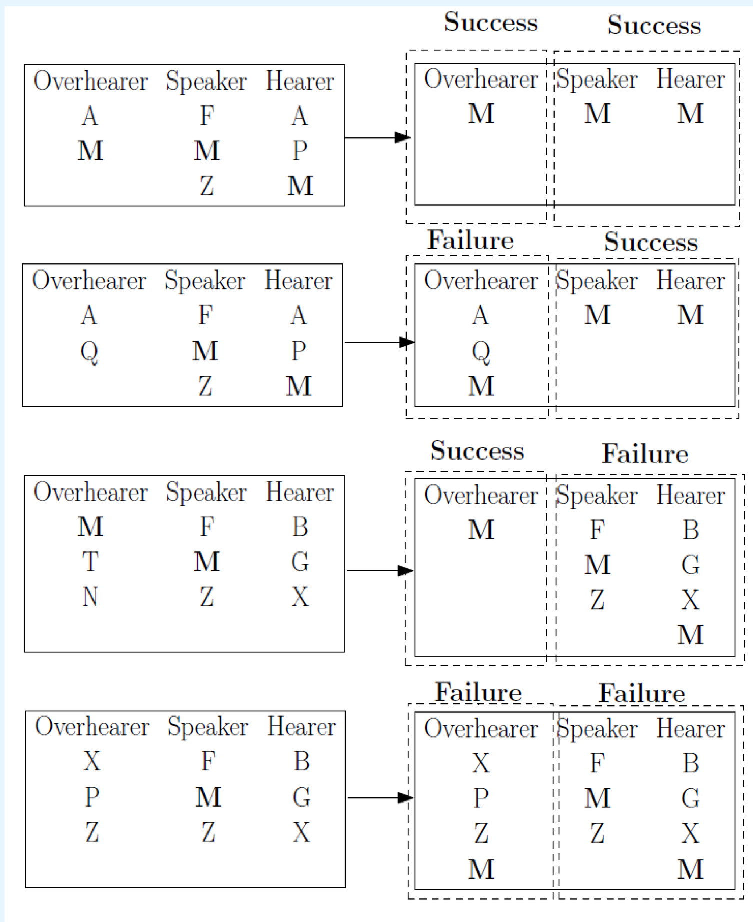
In each game the following steps are executed:

- The speaker transmits a name to the hearer. If its inventory is empty, the speaker invents a new name, otherwise it selects randomly one of the names it knows.

- If the hearer has the uttered name in its inventory, the game is a success, and both agents delete all their names, but the winning one.

- If the hearer does not know the uttered name, the game is a failure, and the hearer inserts the name in its inventory.

- Each overhearer overhears the word uttered by the speaker; if the word is in its inventory, it removes all the words from its inventory except this word (i.e., treats the event as a success) else it adds this word in its inventory (i.e., treats the event as a failure).



Results

The basic quantities to be measured in NG are :

- $N_w(t)$ – total number of names/words in the system at time t
- $N_d(t)$ – number of different names in the system at time t
- S(t) – average success rate at time t
- N_w^{max} – maximum memory required by the system
- t_{max} – the time required to reach the memory peak
- t_{conv} – the time required to reach the global consensus

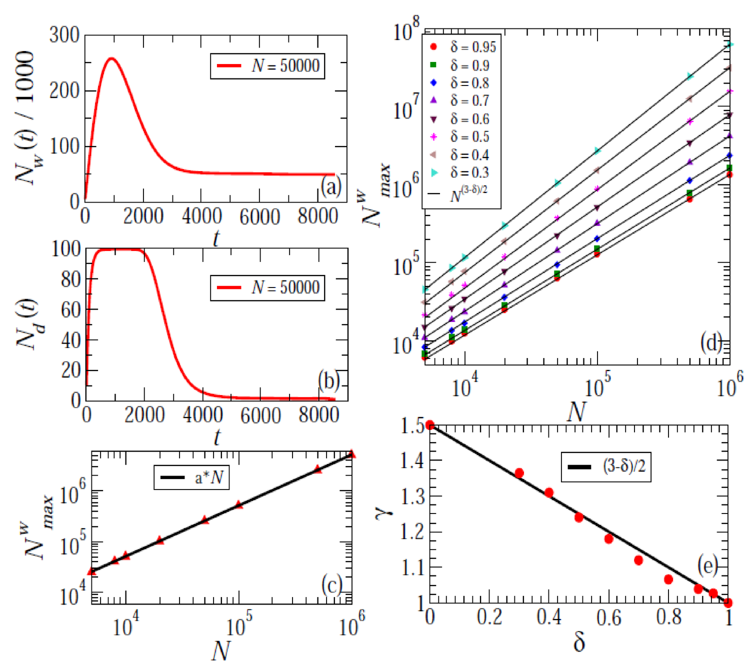


Figure 2. Evolution of (a) $N_w(t)$, (b) $N_d(t)$ when no. of overhearers = ηN where η = 0.05. (c) Scaling of N_w^{max} with N when no. of overhearers = ηN where η = 0.05. (d) Scaling of N_w^{max} with N for different values of δ. (e) The figure expresses the relation of γ vs δ

At the beginning both $N_w(t)$ and $N_d(t)$ grow linearly as the agents invent new words. As invention ceases, $N_d(t)$ reaches a plateau whose height is α N/N^{δ} (when δ approaches 1, the height of the plateau = O(1)). On the other hand, $N_w(t)$ keeps growing till it reaches a maximum at time t_{max}. The total number of words then decreases and the system reaches the convergence state at time t_{conv}. At convergence, all the agents share the same unique word, so that $N_w(t_{conv})$ = N and $N_d(t_{conv})$ = 1.

Analysis of scaling of N_w^{max}

- The number of unique words in the system when the total number of words is close to maximum is $N / N^{\delta} = N^{1-\delta}$
- Assume that each agent has on an average cN^a words when the total number of words is close to the maximum

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^a}\left(1-\frac{cN^a}{N^{1-\delta}}\right)N^{\delta} - \frac{1}{cN^a}\frac{cN^a}{N^{1-\delta}}cN^aN^{\delta}$$

- The first term is related to unsuccessful games (increase in N_w is proportional to N^{δ} times the probability of a single failure)
- The second term is for successful games (the decrease in N_w is proportional to cN^a N^{δ} times the probability of a single success)
- At steady state,

$\frac{dN_w(t_{max})}{dt} = 0$ implies $a = \frac{1-\delta}{2}$ So, $N_w^{max} \sim N^{(3-\delta)/2}$

References

1. Baronchelli et al., Sharp transition towards shared vocabularies in multi-agent systems 2006.

2. Baronchelli et al., In-depth analysis of the Naming Game dynamics : the homogeneous mixing case, 2008.

Analysis of scaling of t_{max}

At early stages, most successful interactions involve agents which have already met in previous games. Thus the probability of success is proportional to the ratio between the number of couples that have interacted before time t i.e., $tN^{\delta}(N^{\delta} - 1) / 2$ and total number of possible pairs which is $N(N - 1) / 2$. At early stage, $S(t) = tN^{(2\delta - 1)}$

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^{(1-\delta)/2}}\left(1 - tN^{2(\delta-1)}\right)N^{\delta} - \frac{1}{cN^{(1-\delta)/2}}tN^{2(\delta-1)}cN^{(1-\delta)/2}N^{\delta}$$

At steady state, $\frac{dN_w(t_{max})}{dt} = 0$ implies $t_{max} \propto N^{\frac{3(1-\delta)}{2}}$

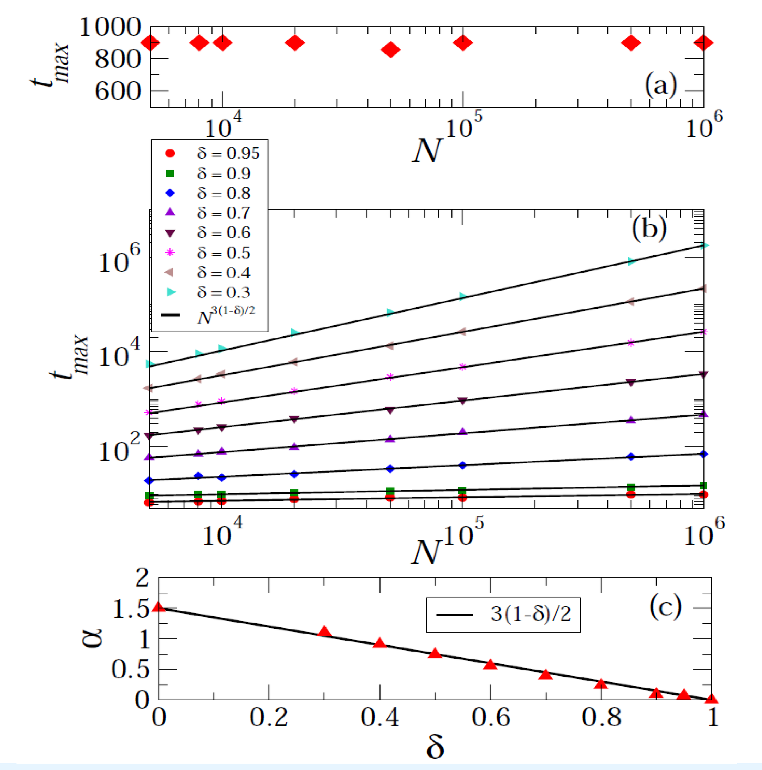
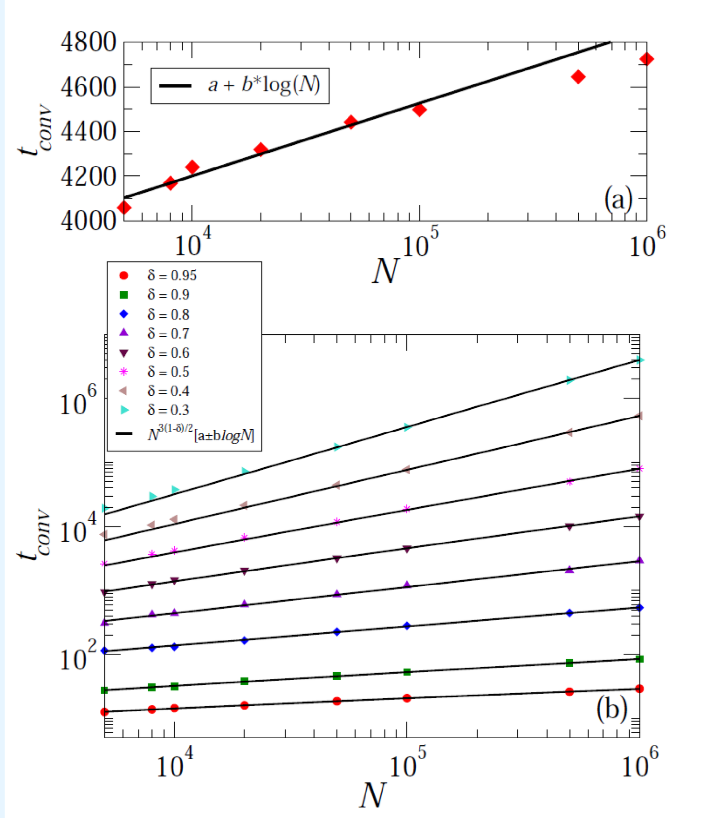


Figure 3. (a) The peak time t_{max} scales as a constant when no. of overhearers = ηN where η = 0.05. (b) Scaling of t_{max} with N for different values of δ. (c) The figure expresses the relation of α vs δ

Analysis of scaling of t_{conv}



The exponent for t_{conv} is much more involved and we could give only empirical evidence which shows that as one varies δ, t_{conv} varies as

$$N^{\frac{3(1-\delta)}{2}}(a \pm b\log N)$$

Figure 2. (a) the consensus time t_{conv} scales as log N when no. of overhearers = η N where η = 0.05. (b) Scaling of t_{conv} with N for different values of δ

Conclusions and future work

- We have introduced the agreement dynamics of naming game to describe the convergence of population in the domain of multi-party communication.
- With the increase in the number of overhearers, the system reaches fast agreement with a significantly low memory requirement.
- It will be interesting to explore this model in probabilistic β framework.
- The model could also be studied in different complex topologies to understand the effect of agent topology.

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