Early warning signals for critical transitions: a generalised modelling approach

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Introduction

Critical transitions

Critical transitions, in this context, are sudden, long-term and catastrophic changes in complex systems that occur when a threshold is crossed. Fisheries, coral reefs, productive farmland, planetary climate, neural activity in the brain, and financial markets are all complex systems known to be susceptible to such catastrophic collapse or reorganisation (Scheffer *et al.*, *Nature* 413:591, 2001). Critical transitions can be well modelled by a bifurcation (such as saddle-node) of a dynamical system.

Early warning signals

Some way to anticipate these critical transitions would be highly desirable. A number of data-driven early warning signals for critical transitions have recently been developed (Scheffer *et al.*, *Nature* 461:53, 2009). These include an increasing variance and an increasing autocorrelation as signals of an impending transition.

We propose a new method for calculating early warning signals that makes use of available information about the system.

Generalised modelling

Even if only basic information about the structure of a system is known, its dynamics can be captured through a generalised model (Gross & Feudel, *Phys. Rev. E* 73:016205, 2006). A generalised model combines knowledge about the important variables and processes in a system (see examples to the right). As a first step, this knowledge can be written down in diagrammatic form. In mathematical form, the generalised model can be written down as a set of differential equations, but *where we do not need to specify particular functional forms.* Such a generalised model is the basis for the early warning signal method.

Our approach is based on combining readily available, system-specific knowledge about important variables and processes with time series data through the framework of a generalised model.

Fast-slow decomposition

We assume that the system is in the vicinity of an equilibrium over a *fast* timescale. Our generalised model describes these fast dynamics. However we (like all other current early warning approaches) assume the existence of a *slow* timescale, over which some parameter(s) of the fast subsystem is changing, and which is pushing the system towards the critical transition.

Method

There is concern that a system may undergo a critical transition.

- Write down a generalised model of the system.
- Formally calculate the Jacobian (matrix of derivatives) of the system.
- Use time series data of state variables and processes, together with other available knowledge about these processes, to constrain the entries of the Jacobian
- Calculate the eigenvalues of the Jacobian
- Monitor the changes in eigenvalue over time.
- **An eigenvalue moving consistently towards a stability boundary (real part zero for flows, absolute value one for maps) is a warning signal that a critical transition may be about to occur.**

System

One-population model with Allee effect

This is a one-dimensional population model with a strongly density-dependent (nonlinear) reproduction rate, leading to the possibility of multiple equilibria and bifurcations involving loss of those equilibria.

Fishery simulation of Biggs et al. *(2009)*

Simulation data from the model of Biggs *et al.* (PNAS 106:826, 2009)

Tri-trophic food chain

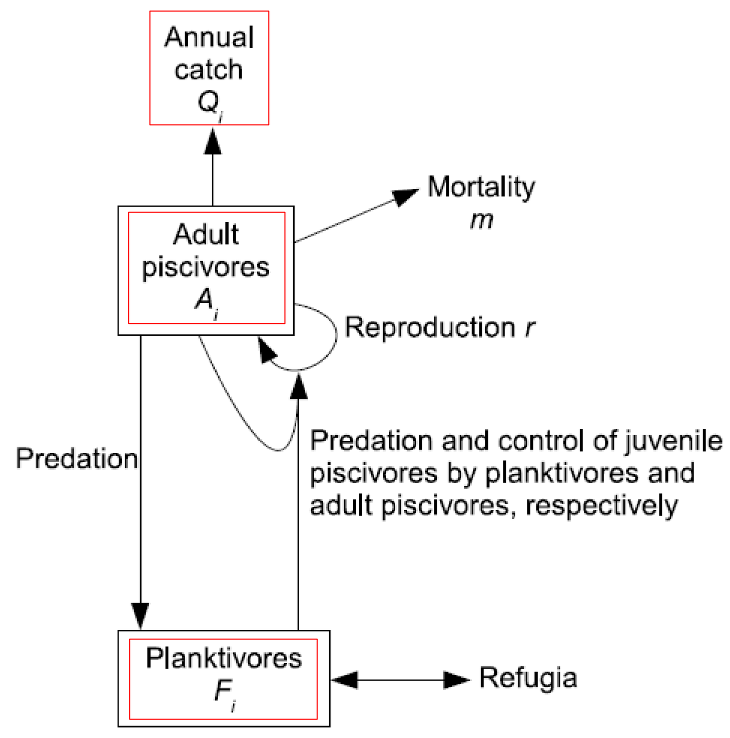
Generalised model

One-population model with Allee effect

$$\frac{\mathrm{d}}{\mathrm{d}t}X = B(X) - M(X,\mu)$$

X (*t*)= Population size (observed)
B = birth rate (observed)
M = death rate, assumed linear in *X*
μ = slowly changing external parameter (e.g. environmental conditions)

Fishery simulation



We constructed a discrete-time generalised model based on this causal loop diagram. Observed quantities are marked in red.

Tri-trophic food chain

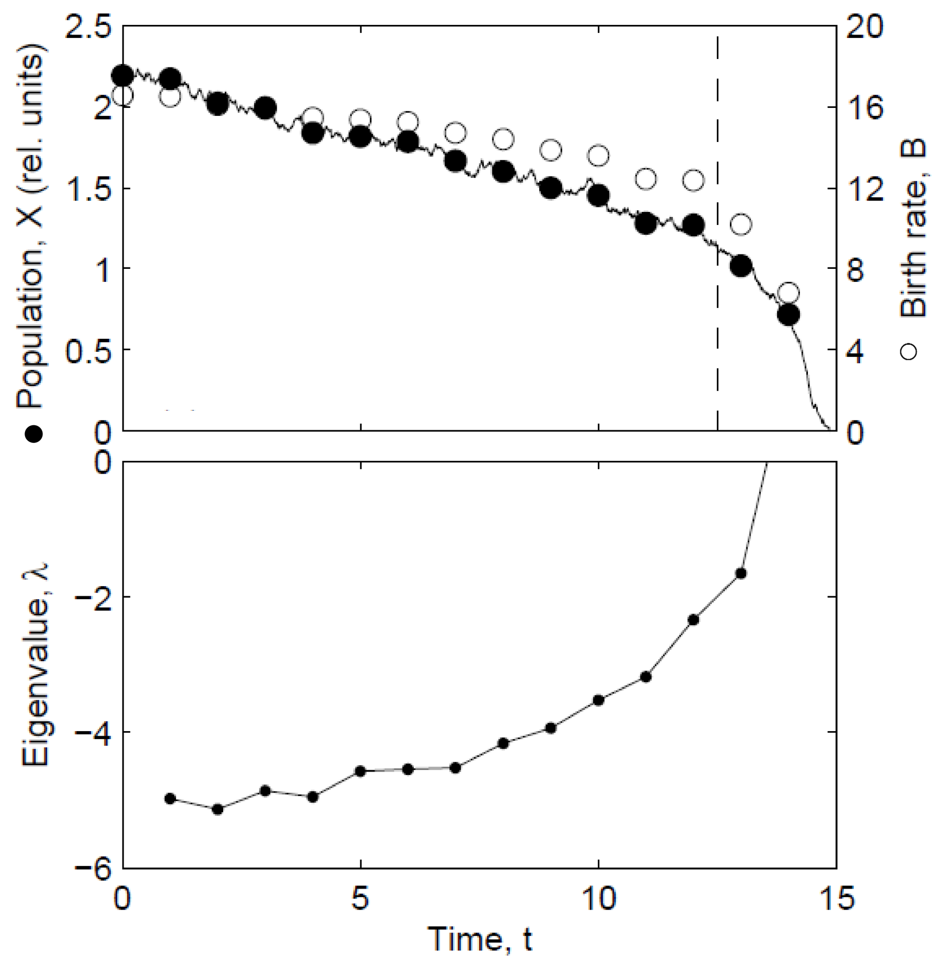
$$\frac{d}{dt}X_1 = A(X_1) - G(X_1, X_2)$$
$$\frac{d}{dt}X_2 = G(X_1, X_2) - H(X_2, X_3)$$
$$\frac{d}{dt}X_3 = H(X_2, X_3) - M(X_3, \mu),$$

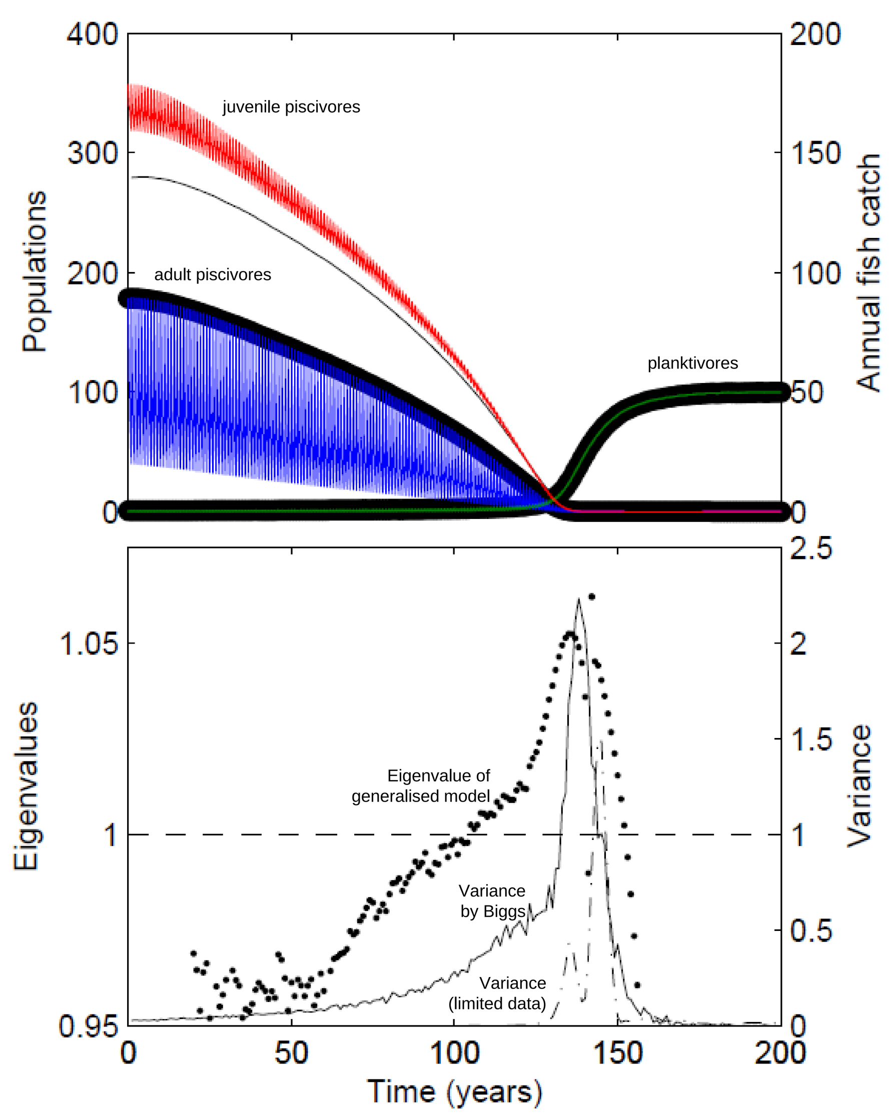
$X_1(t)$, $X_2(t)$, $X_3(t)$ = Biomasses of primary producer, predator, and top predator (observed)
A = birth rate of primary producer
G = rate of consumption of X_1 by X_2, assumed linear in X_2
H = rate of consumption of X_2 by X_3, assumed linear in X_3
M = death rate of top predator, assumed linear in *X*
μ = slowly changing external parameter

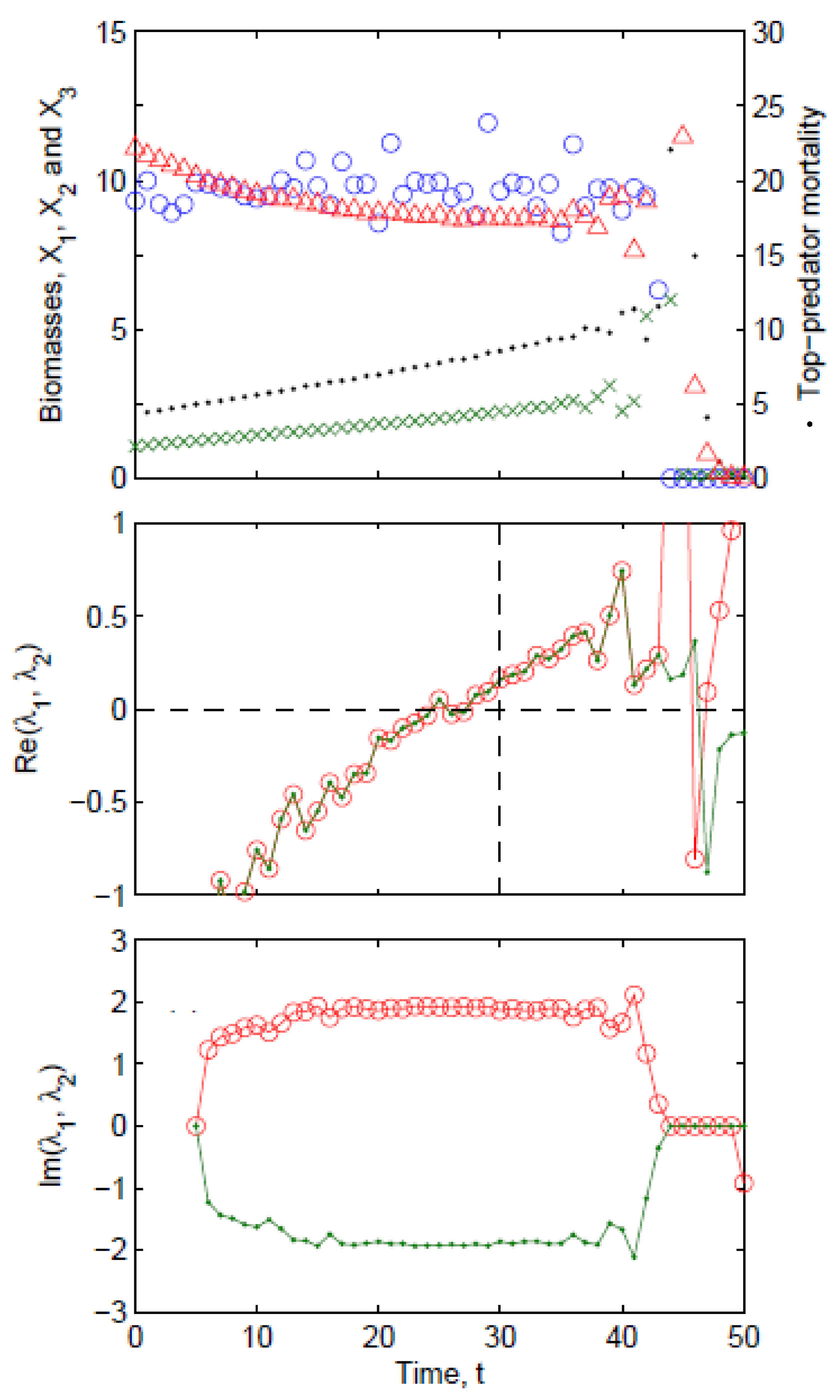
Results

Simulation

Early warning signal







Conclusions

The generalised modelling approach can produce an effective early warning signal from very few (as few as 10) time points.

The generalised modelling approach is effective in a realistic scenario where the generalised model has not accurately reflected all the properties of the actual system. The generalised model does not explicitly model the dynamics of the juvenile piscivore population that are included in the simulation model.

In this test, the generalised modelling warning signal was of a similar quality to the variance signal computed by Biggs *et al.*, and better than a variance signal computed with the same amount of data (one datum per year only) as was available to the generalised modelling calculation.

The generalised modelling approach can distinguish different types of critical transitions. Here the approach of the real part of a complex conjugate pair of eigenvalues towards zero indicates a critical transition associated with a Hopf bifurcation.

Future work

Apply the generalised modelling approach to observational data from the 1980s fishery transition in the Baltic Sea.