Extreme Event-Size Fluctuations in Biased Random Walks on Networks

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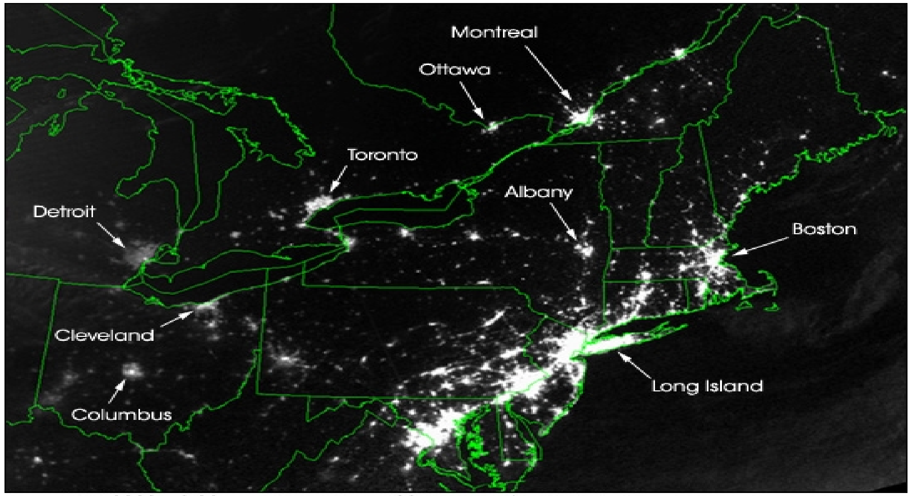
Plan of the talk:

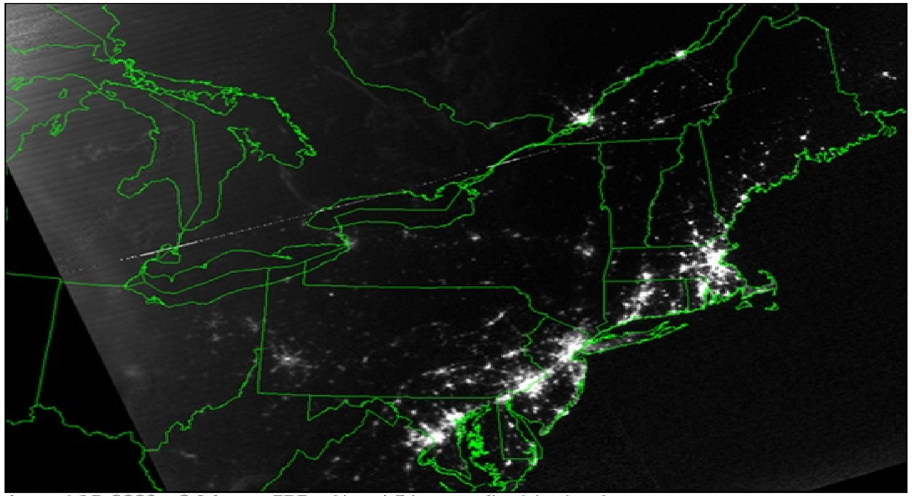
- Introduction
  Extreme events
- Dynamics on networks
  Biased Random walk model
- Extreme events on networks
  how frequent are extreme events on network?
- Extreme fluctuations
  Fluctuations, OK!!! but how large?
- Conclusion

Extreme Events

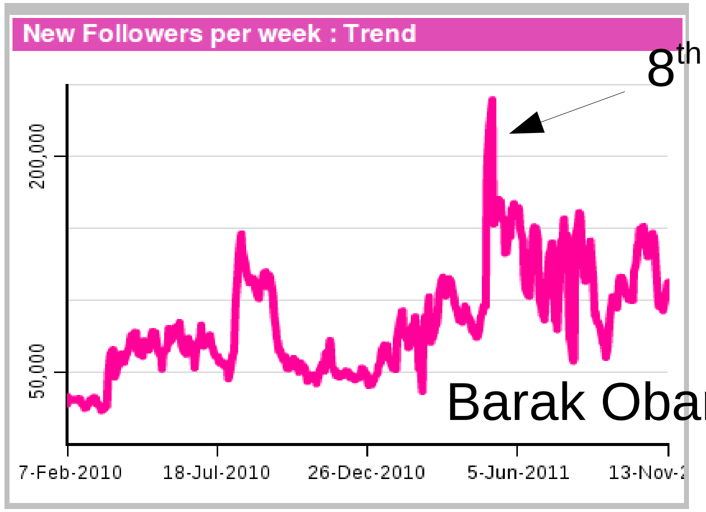
Internet slowdown



August 14, 2003 • 9:29 p.m. EDT • About 20 hours before blackout


August 15, 2003 • 9:14 p.m. EDT • About 7 hours after blackout

Power blackout in US

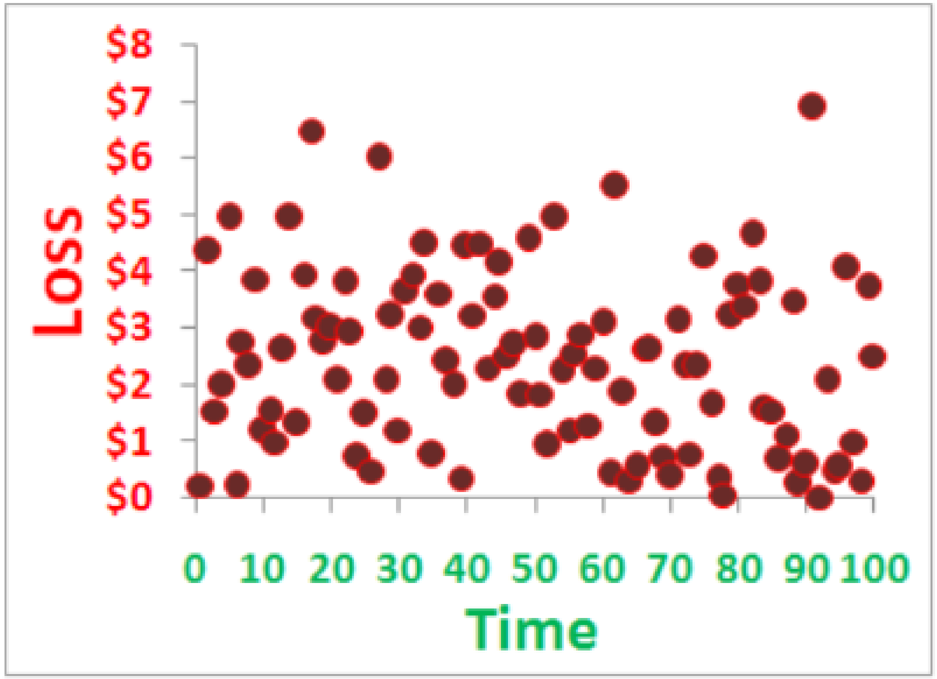


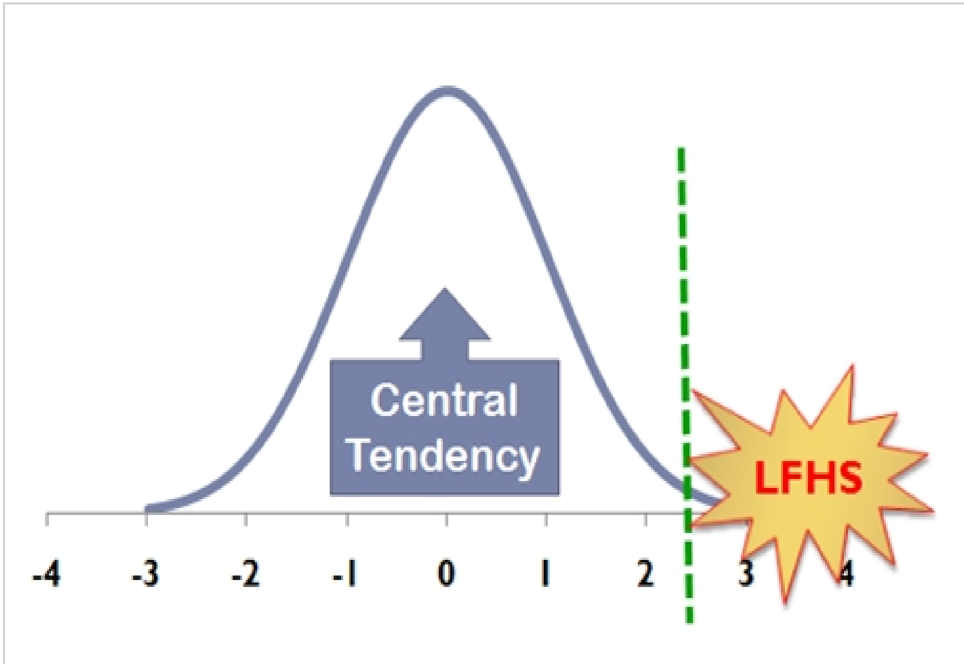
8th May 2011

Barak Obama fan page

These extreme events take place on some underlying lattice structures and hence, are our inspiration behind studying E*xtreme Events on networks*.

Financial loss





Courtsey: David Harper, Bionic Turtle

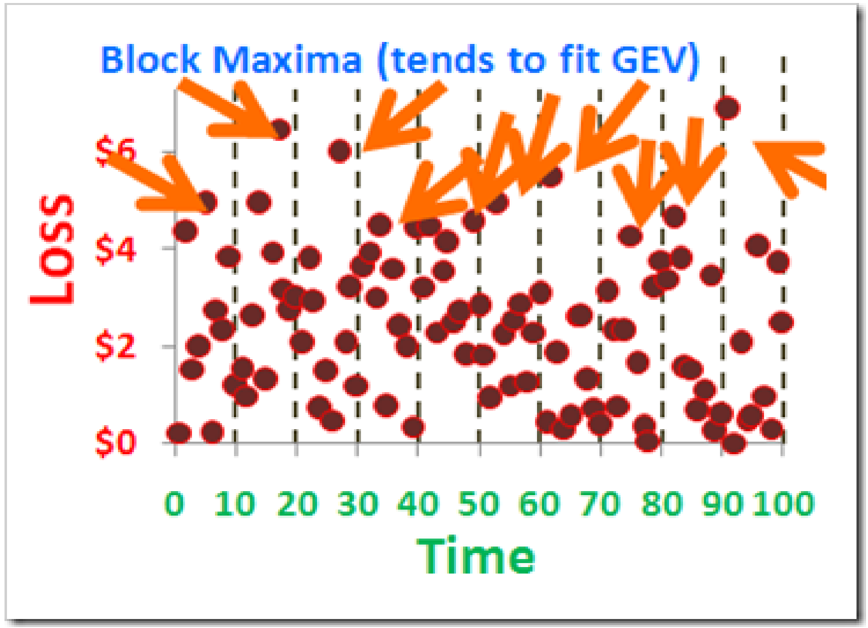
What is an extreme event?

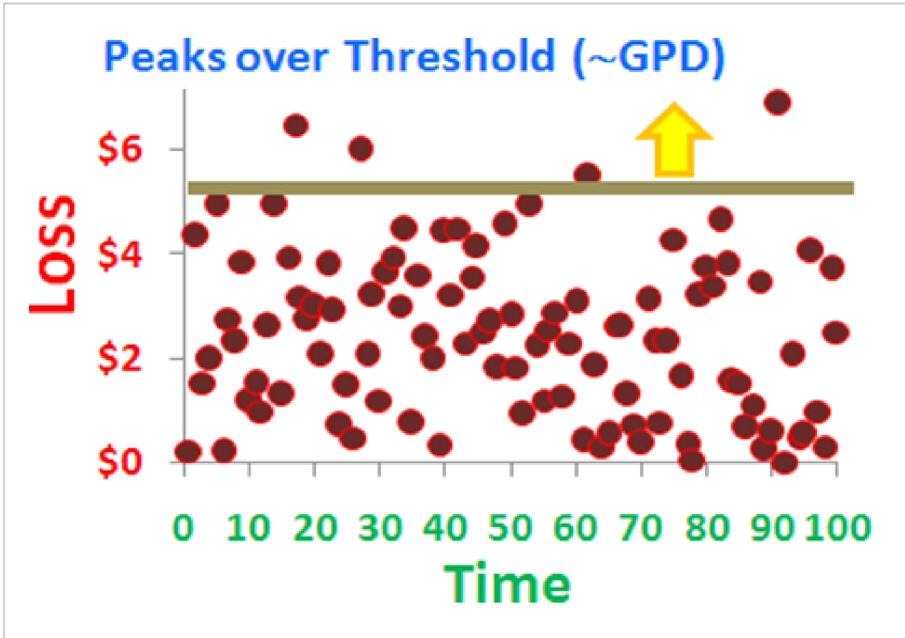
An Extreme event is one which is associated with the tail of the Probability distribution P(m) of events of size m.

Basic features:

- They are rare
- They are recurrent
- Which are inherent to the system under study
- To which we can assign a variable (“magnitude”)

Ways of defining an Extreme Event (EE)





Courtsey: David Harper, Bionic Turtle

Framework for studying EE on Network:

Dynamics on network-
Dynamics supported by the network...

Stationary distribution-
Does it exist? Necessary to define extreme events...

Defining an event and an extreme event-
How to define an event and based on that, what is extreme event?

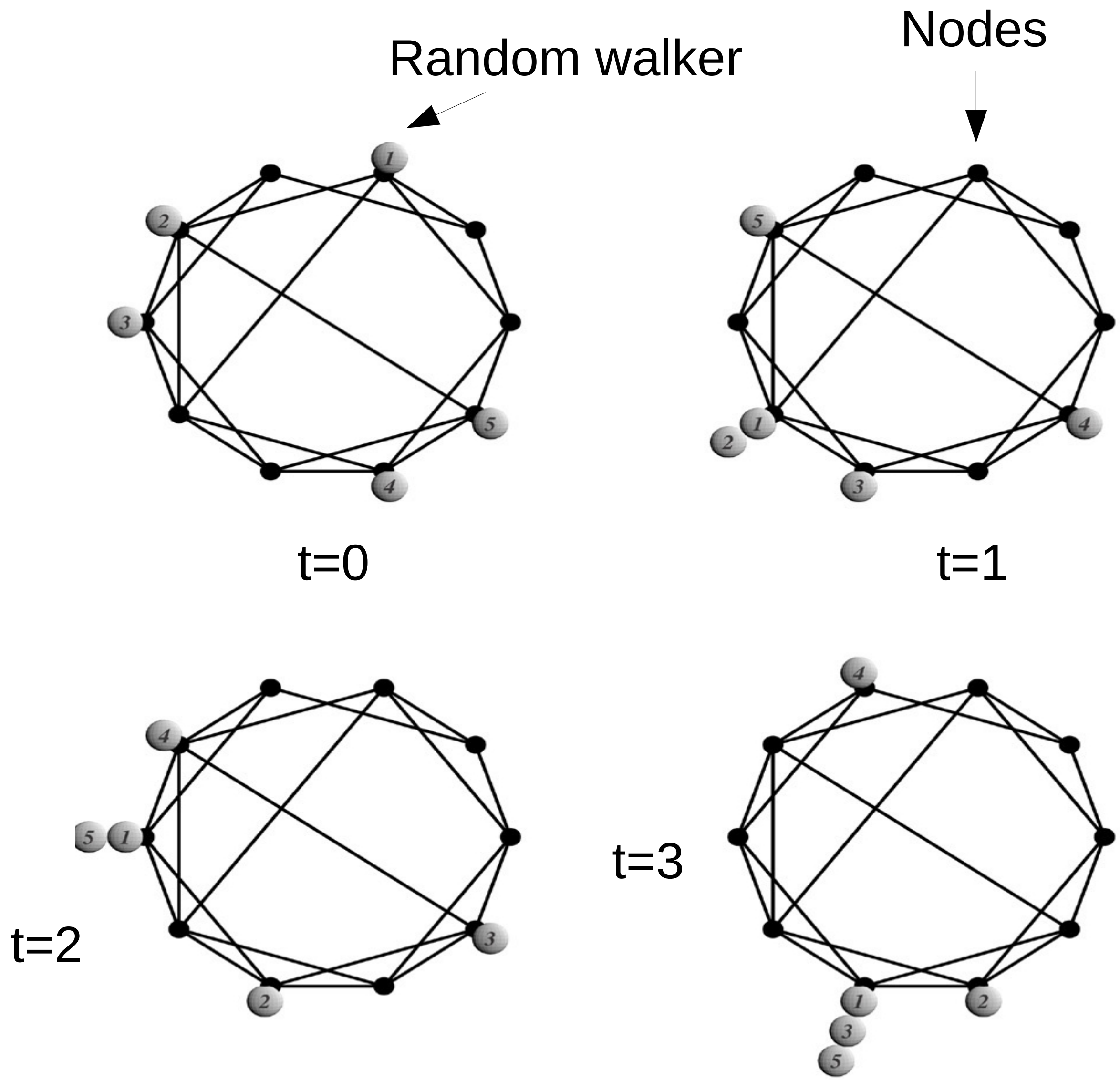
Cutoff-
How to decide the threshold?

Questions-
Probability distribution of extreme events
Role of topolgy

Random walk on network:

Standard Random walk:

A random walker on a node can hop to a neighboring node with equal probability.

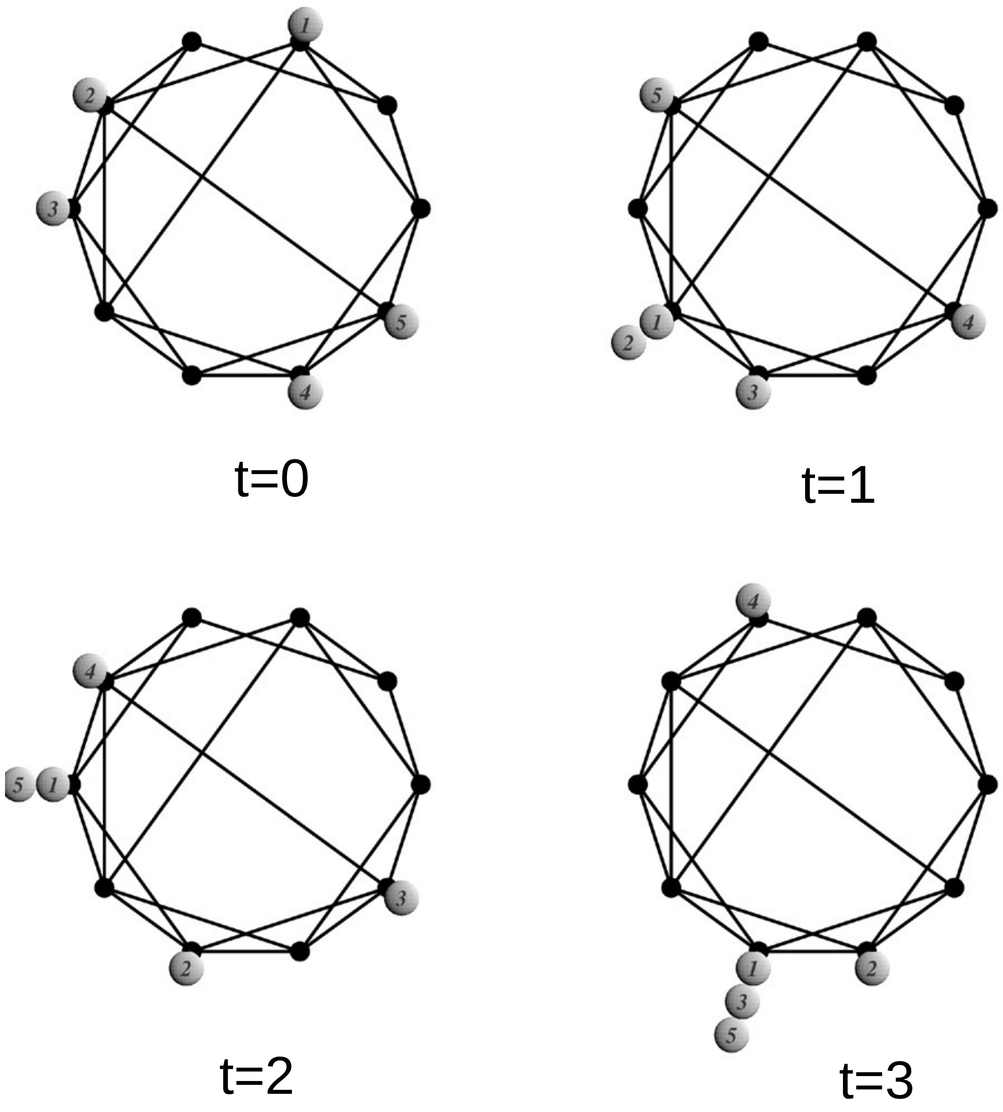


Biased Random walk:
A walker on a node can hop to a neighboring node but with some preferences.

What is an event:

- Event:
  No. Of walkers on a node

Size->	0	1	2	3	4	5
t=0	5	5	0	0	0	0
t=1	6	3	1	0	0	0
t=2	6	3	1	0	0	0
t=3	7	2	0	1	0	0

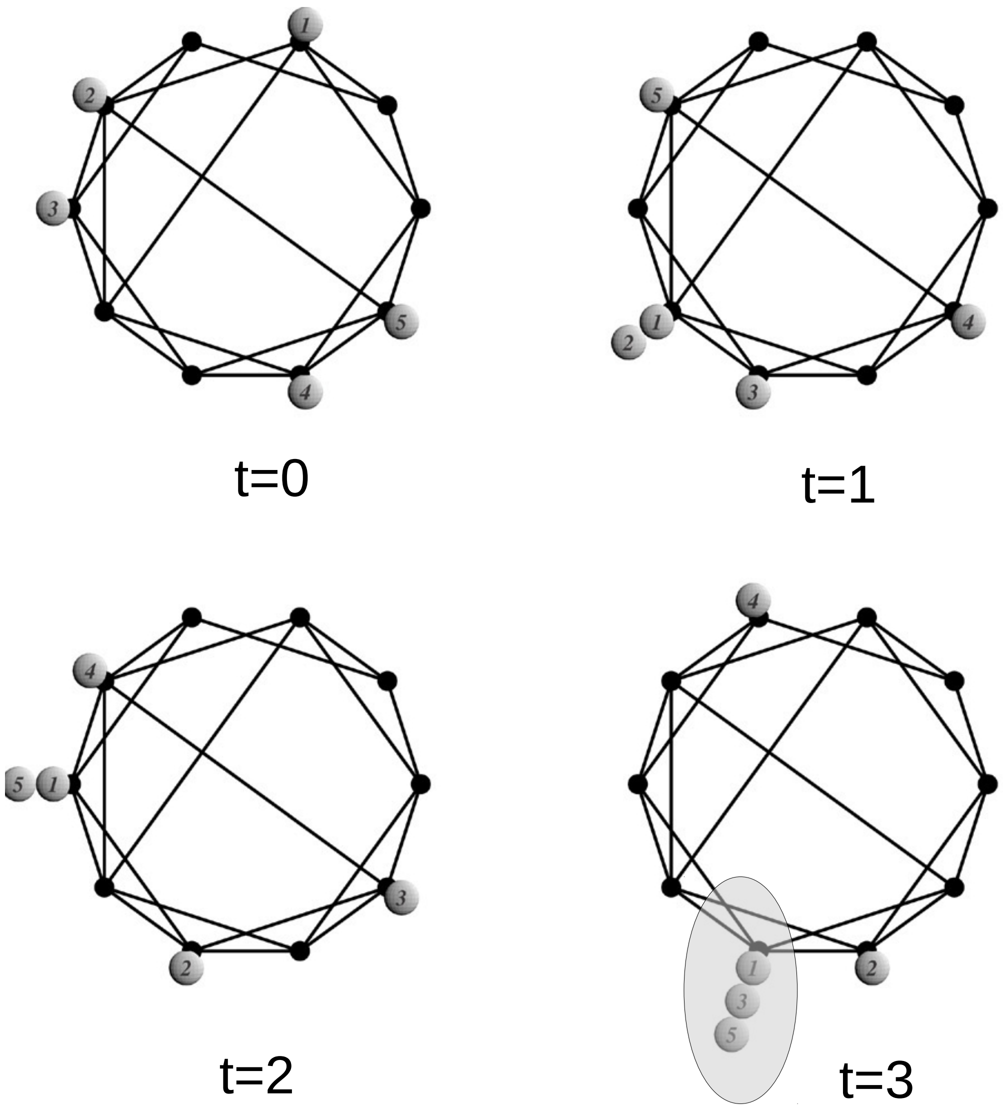


Defining an extreme event:

- Event:
  No. Of walkers on a node

Size->	0	1	2	3	4	5
t=0	5	5	0	0	0	0
t=1	6	3	1	0	0	0
t=2	6	3	1	0	0	0
t=3	7	2	0	1	0	0

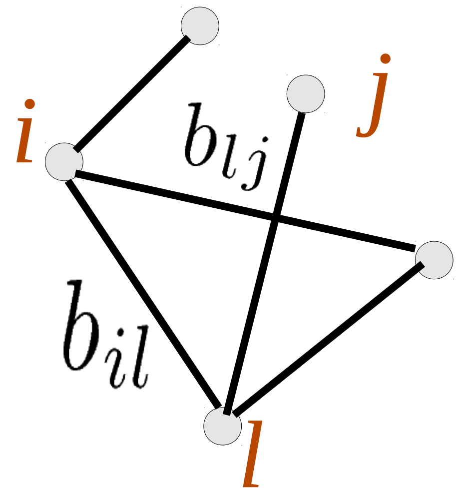
- Extreme event:
  No. Of walkers on a node more than the threshold



Biased Random Walk on Networks

Consider a connected, undirected network with *N* nodes, *E* edges with *W* non-interacting walkers.

Probability for a walker to go from node `i(t=0)` to `j(t=n+1)` *with transition probabilty* b_{ij} :



$$P_{ij}(n+1) = \sum_l A_{lj}\, b_{lj}\, P_{il}(n)$$

For hopping from *l-th* to *j-th* node, walkers discriminate among neighbors on the basis of their degree:

$$b_{lj} \propto k_j^{\alpha},$$

Stationary probability for finding a walker at node j :

$$\lim_{n\to\infty} P_{ij}(n) = p_j = \frac{k_j^\alpha \sum_{l=1}^{k_j} k_l^\alpha}{\sum_{m=1}^{N} \left(k_m^\alpha \sum_{l=1}^{k_m} k_l^\alpha \right)}.$$

Let me define the generalized strength of i-th node to be:

$$\phi_i = k_i^\alpha \sum_{j=1}^{k_i} k_j^\alpha.$$

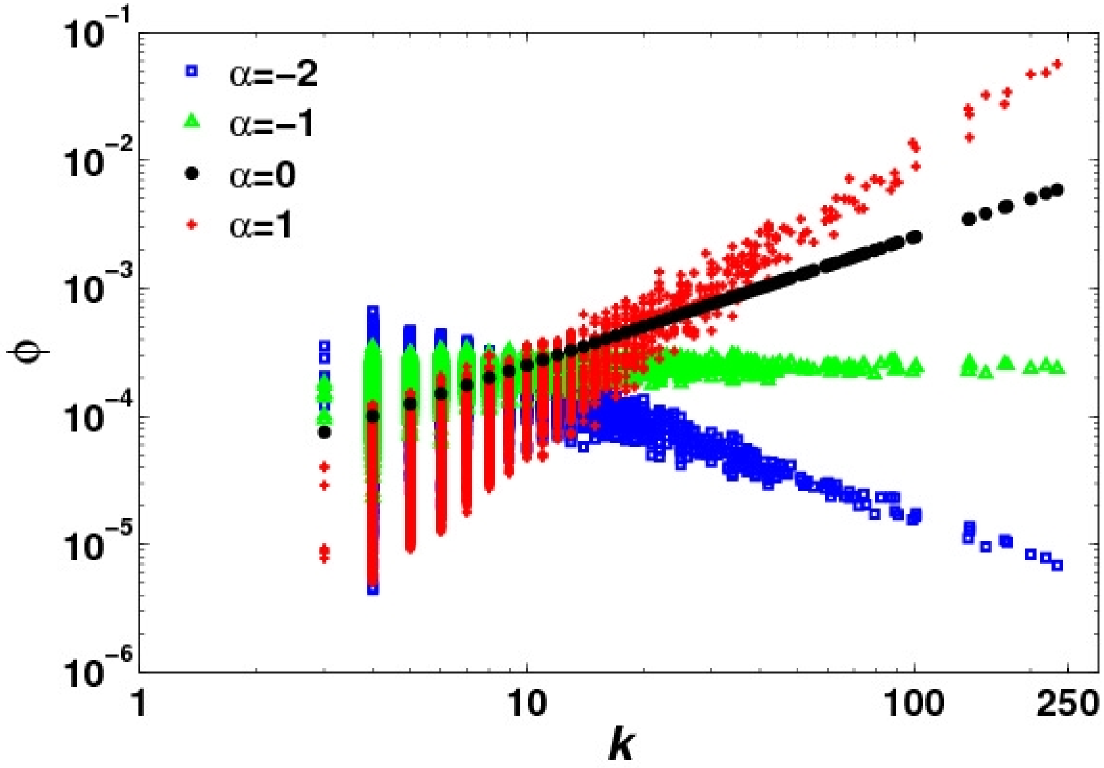
Now, Stationary probability :

$$p_j = \frac{\phi_j}{\sum_{l=1}^{N} \phi_l}.$$

$\alpha > 0$ Walk biased towards low degree nodes

$\alpha = 0$ Standard random walk

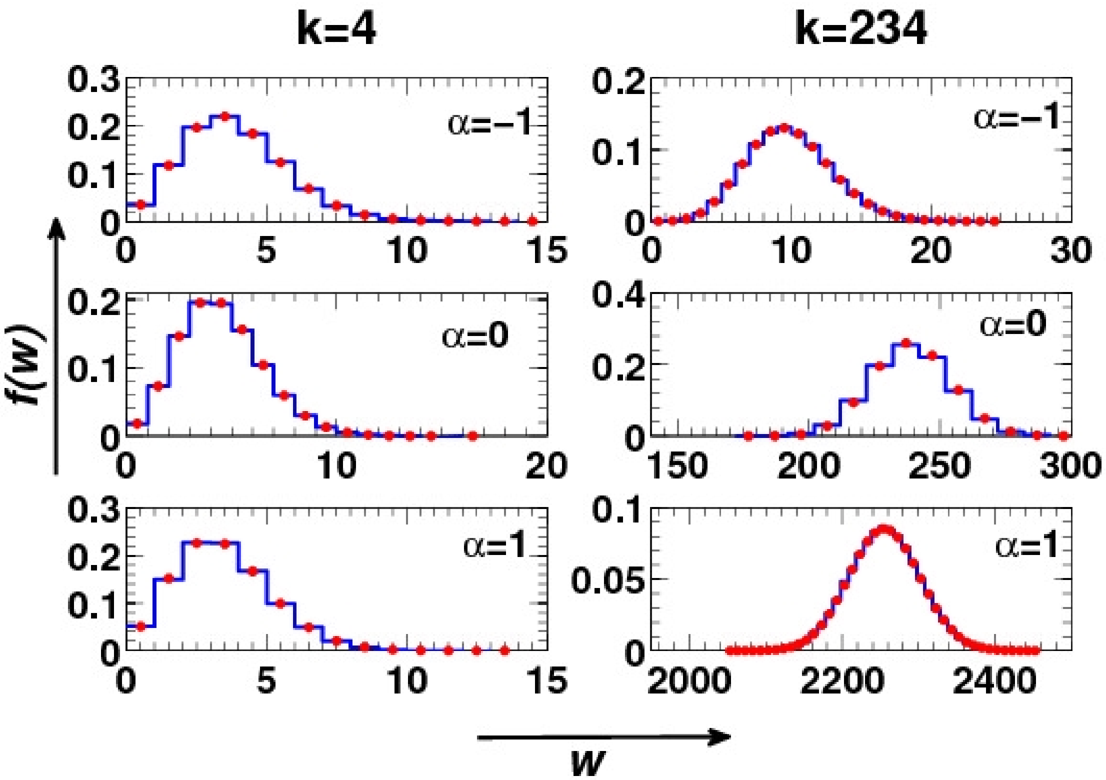
$\alpha < 0$ Walk biased towards hubs



Nodes with same degree can have different strengths because of their local environment.

Probability of *m* walkers on node *i* : Binomial distribution

$$F_i(w) \;=\; \binom{W}{w} p_i^w (1-p_i)^{W-w}$$



Analytical
Simulation

Mean flux and variance

$$\langle f_i \rangle = \frac{W\phi_i}{\sum_{j=1}^{N} \phi_j}, \qquad \sigma_i^2 = W \frac{\phi_i}{\sum_{j=1}^{N} \phi_j} \left(1 - \frac{\phi_i}{\sum_{j=1}^{N} \phi_j} \right)$$

In case of SRW ($\alpha = 0$)

$$\langle f_i \rangle = \frac{W k_i}{2E}\ ; \qquad \sigma_i^2 = W \frac{k_i}{2E} \left(1 - \frac{k_i}{2E} \right)$$

Noh et. al., PRL (2004).

Some Numbers for Simulations

W = 39830

Scale free network (N = 5000, E = 19815)

10^7 time steps, 100 Ensembles

Probability distribution of EE on a node

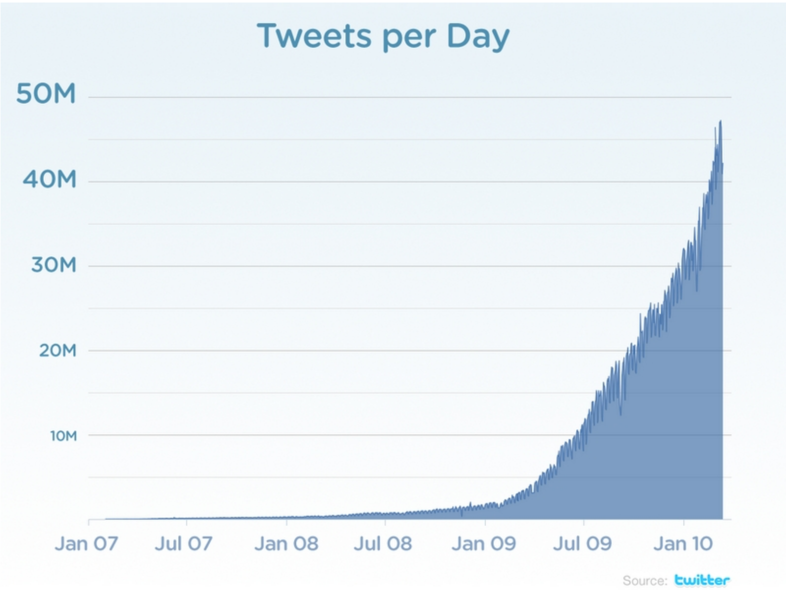
$$\mathcal{F}_i = \sum_{w=\tau_i}^{W} \binom{W}{w} p_i^w \, (1 - p_i)^{W-w}$$

Where τ_i is some threshold. This gives,

$$\mathcal{F}(\phi_i) = I_{\frac{\phi_i}{\sum_{j=1}^{N} \phi_j}} \left(\lfloor \tau_i \rfloor + 1, W - \lfloor \tau_i) \right\rfloor$$

Nodes with same generalised strengths should have the same probability for extreme events. But, how to decide the threshold over the network?

Defining extreme events for nodes in a network



50 million tweets per day.
On an average, 600 tweets/second.

Source : twitter.com

In 2009, Google resolved 87.8 billion search queries per month.
About 34000 queries/second.

Source : comscore.com

For most sites on the www, these represent extreme events.

Constant threshold : What is extreme in one node will not be so in another.

R. K. Laxman

delhi.gov.in

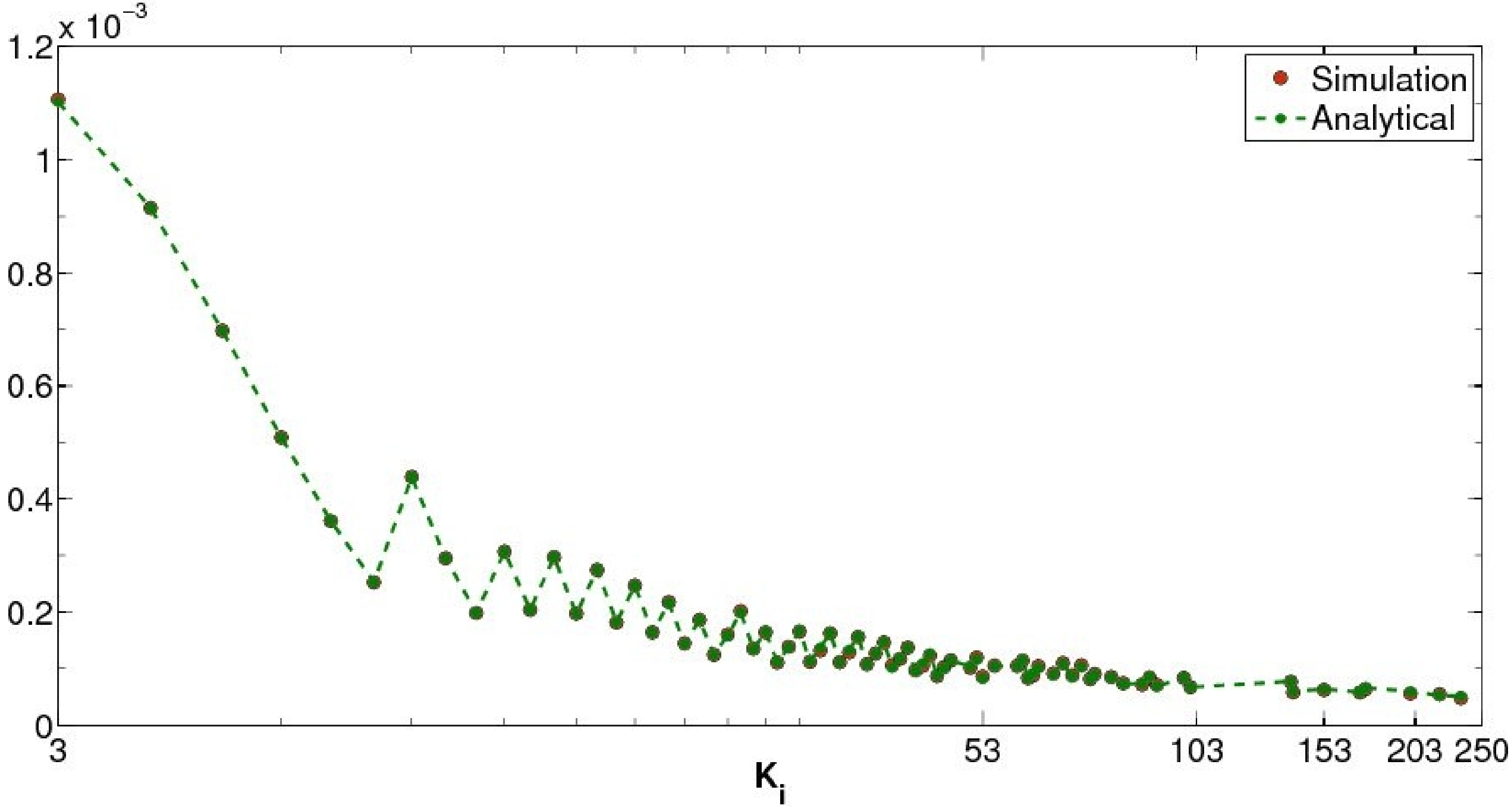
Threshold based on variance in each node :

$$\tau_i = \langle f_i \rangle + q\sigma_i \qquad (q > 0)$$

Depends on the flux passing through the node.

Probability distribution of EE as a function of degree of node (SRW)

F(K)



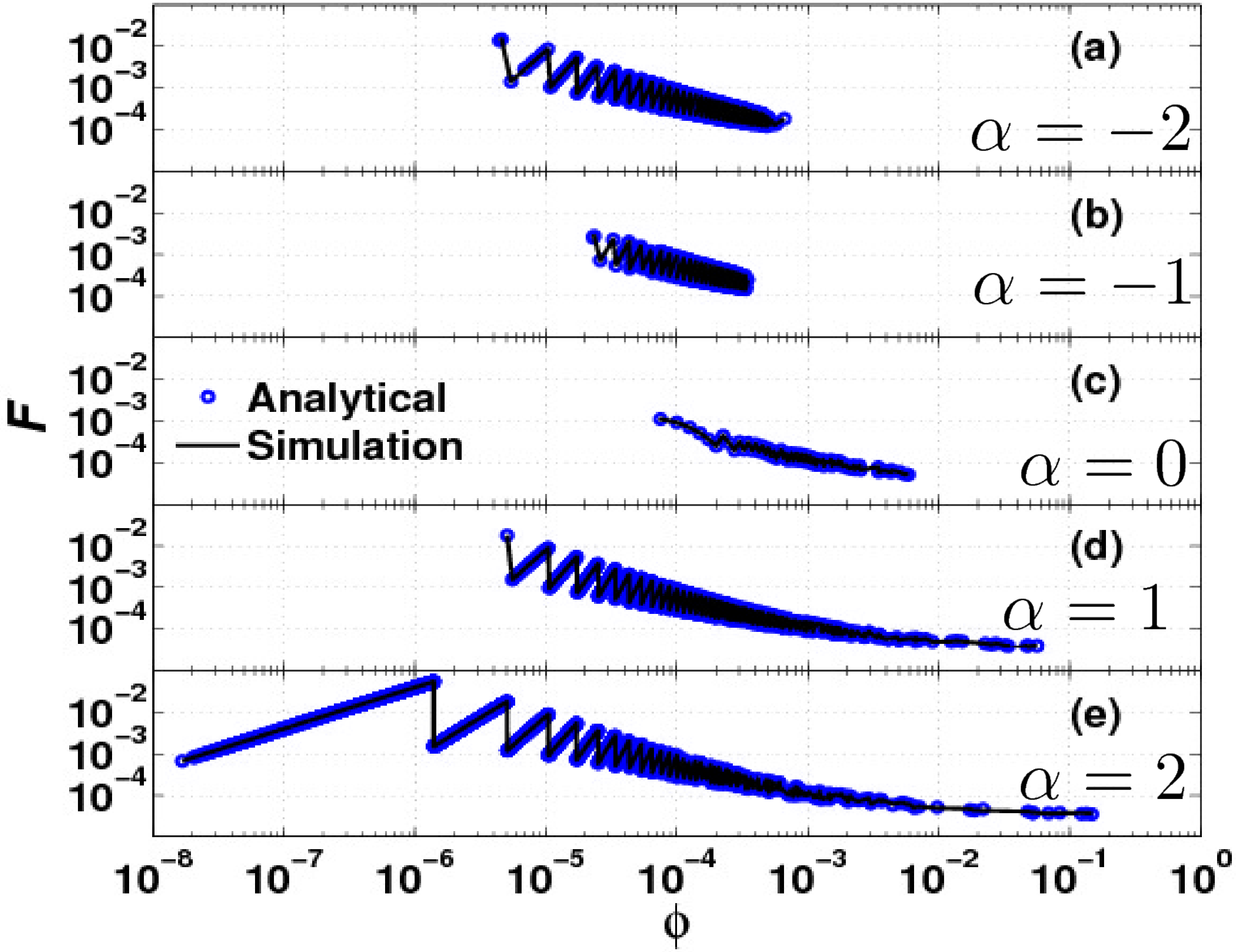
Vimal Kishore et. al. PRL(2011)

EE probability as a function of strength

Biased towards
low degree nodes

Standard Random
walk

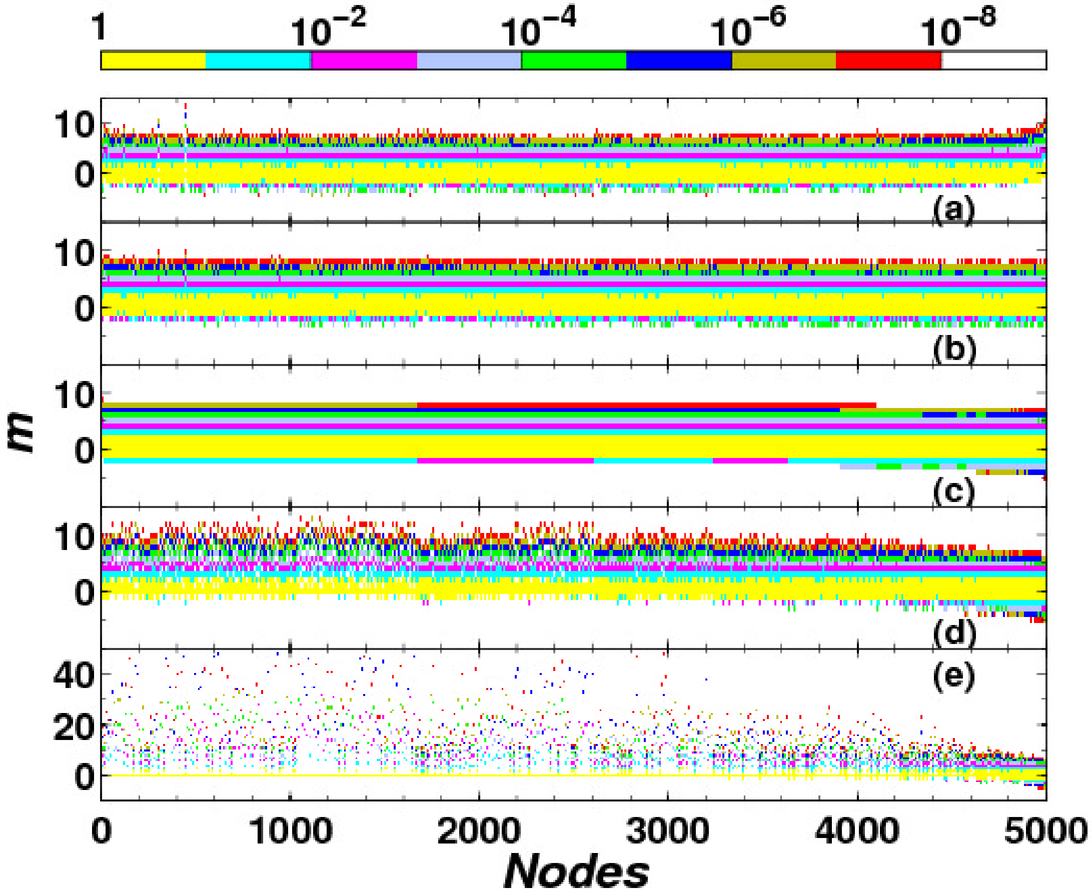
Biased towards
hubs



Extreme fluctuations

Event size: $m = m\sigma \leq (w - \bar{w}) < (m+1)\sigma$

Node numbers (arranged in ascedning order of degree)



$\alpha = -2$

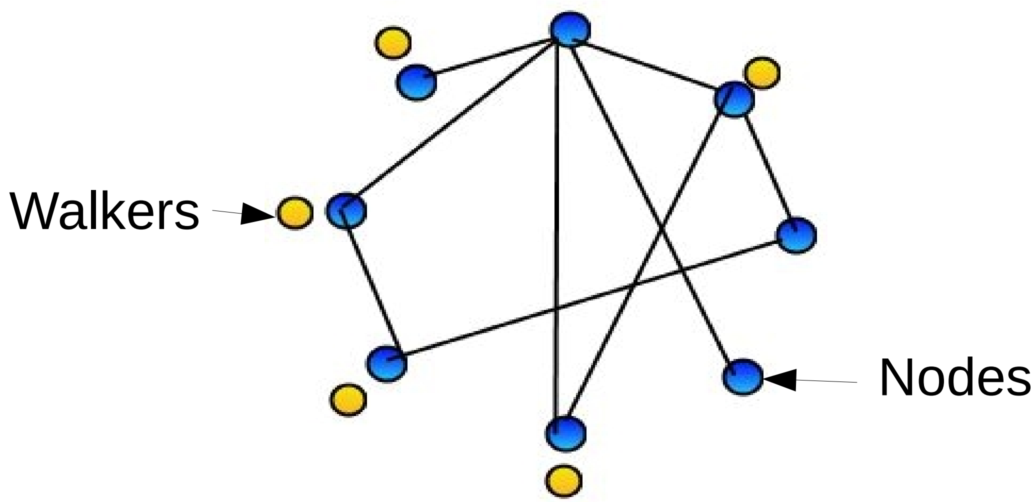
$\alpha = -1$

$\alpha = 0$

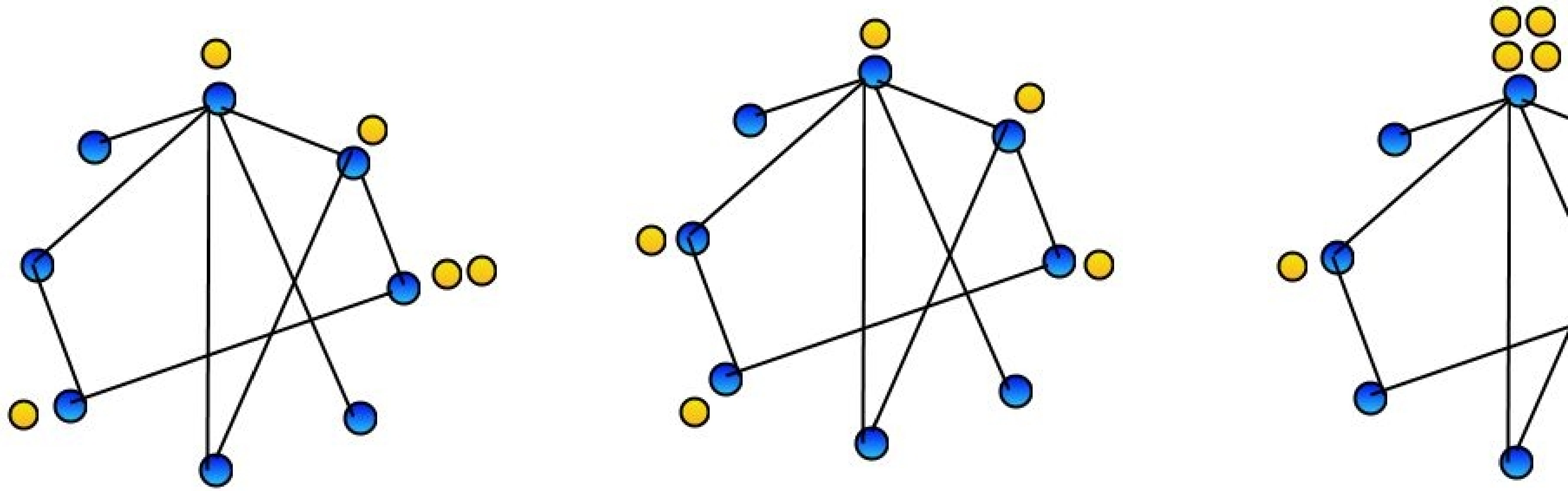
$\alpha = +1$

$\alpha = +2$

α as a parameter



t=0



t=1

$\alpha < 0$ $\alpha = 0$ $\alpha > 0$

Summary

We investigate the occurence of extreme events on complex networks using the generalized random walk model in which the walk is preferentially biased by the network topology. For a scale free network

- The generalized strength, depends on the degree of the node and that of its nearest neighbors, has been defined as a measure of the ability of a node to attract walkers.

- Nodes with lower strengths are more likely to experience extreme events than the ones with higher strengths.

- When walk is biased towards the hubs, extreme events can be of very large size.

- For the better functioning of a network, smaller strength nodes are very important.

In collaboration with...

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Extreme events arising due to inherent fluctuations will always take place and cannot be avoided, but one can be better prepared to meet the expected Extreme Events.

THANK YOU FOR YOUR ATTENTION!

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