Patterns of Cooperation
(in a simple model)

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MAPCON
MPI-PKS Dresden, 2012-05-15

Collaboration Networks

Wide degree distribution

Strong clustering

Cliques

...

Modeling: Snowdrift Game

Snowdrift

Modeling: Snowdrift Game

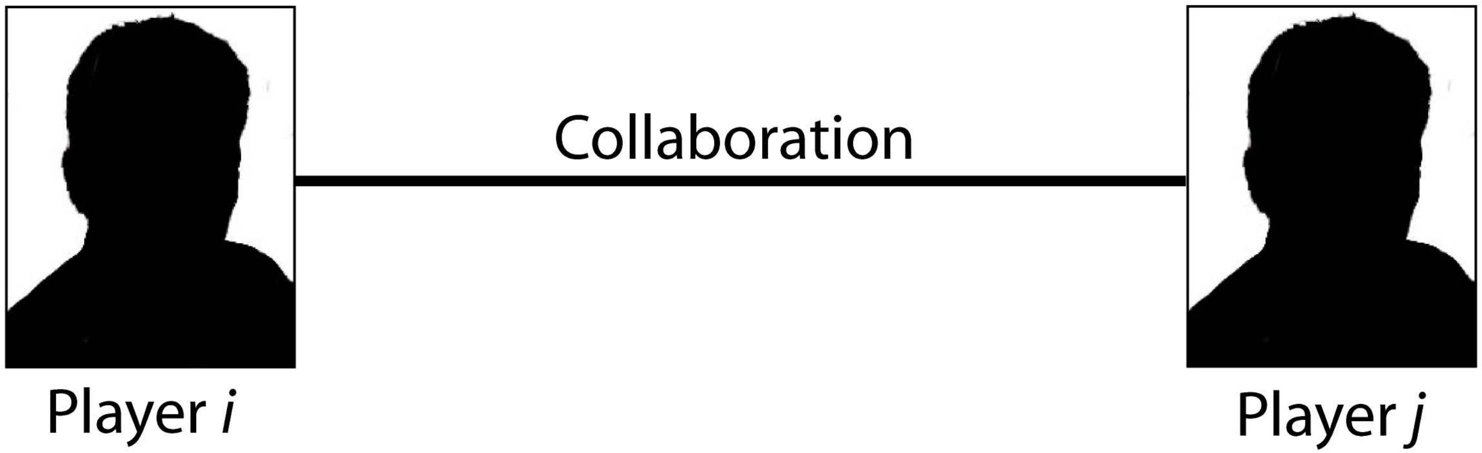
Modeling: Snowdrift Game

Modeling: Snowdrift Game

Modeling: Patterns of Collaboration

Anne-Ly Do, Lars Rudolf, Thilo Gross:
Patterns of Cooperation
NJP **12**, 063023, 2010

Modeling: Patterns of Collaboration



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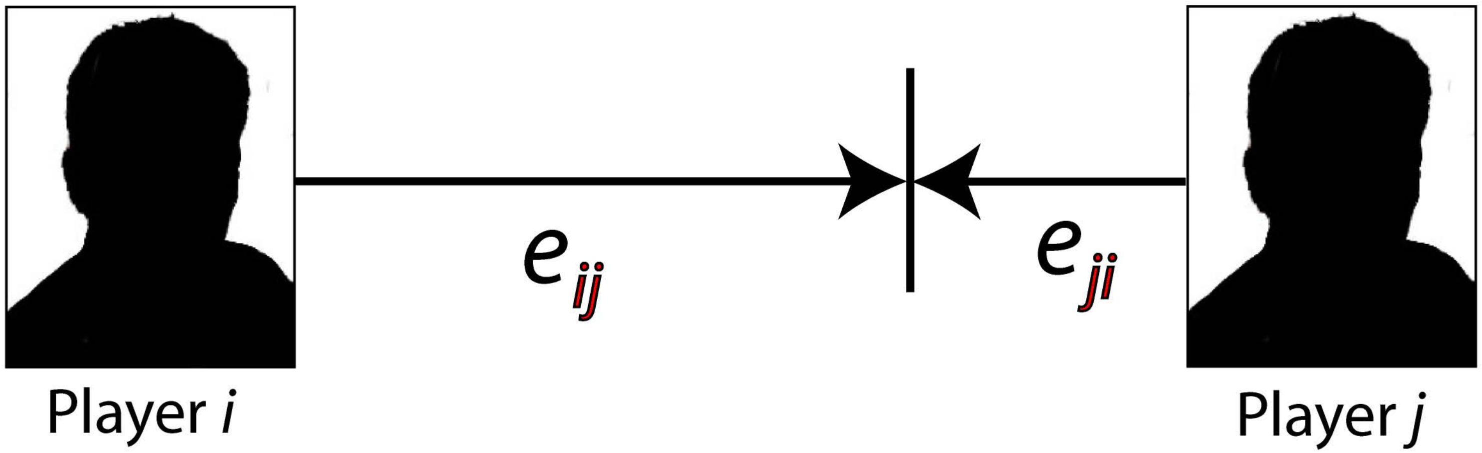

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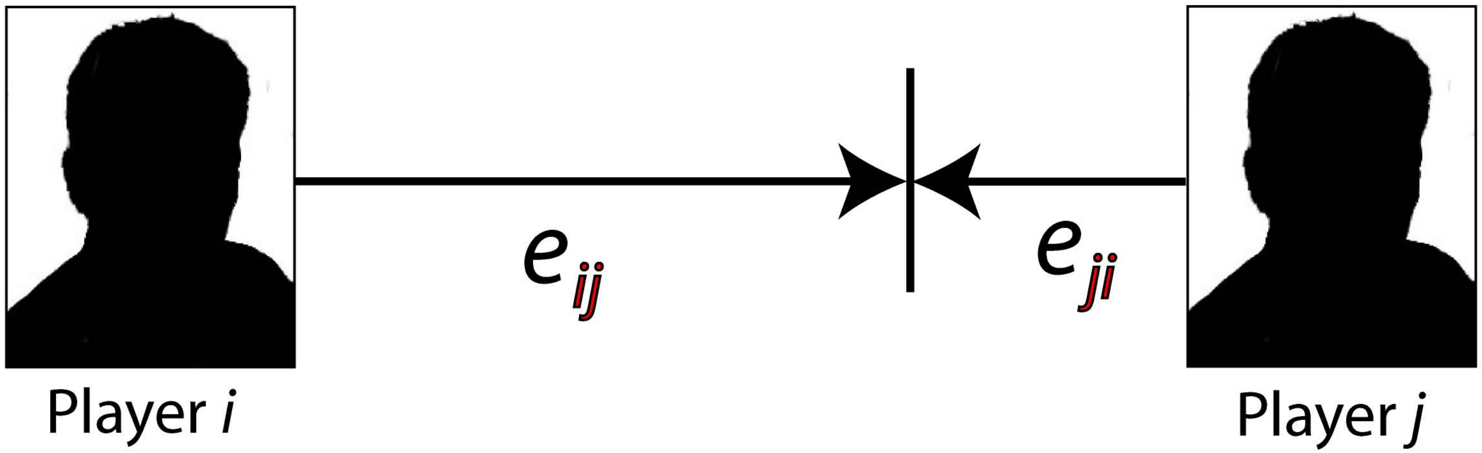
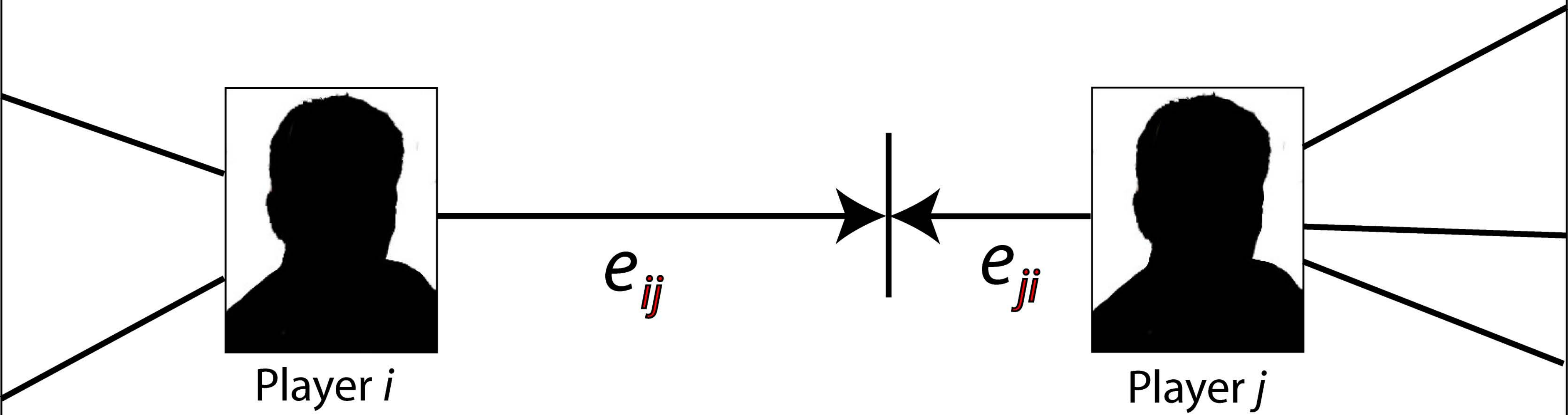


$$\text{Payoff } i = \text{Benefit}(e_{ij} + e_{ji}) - \text{Cost}(e_{ij})$$

Modeling: Patterns of Collaboration



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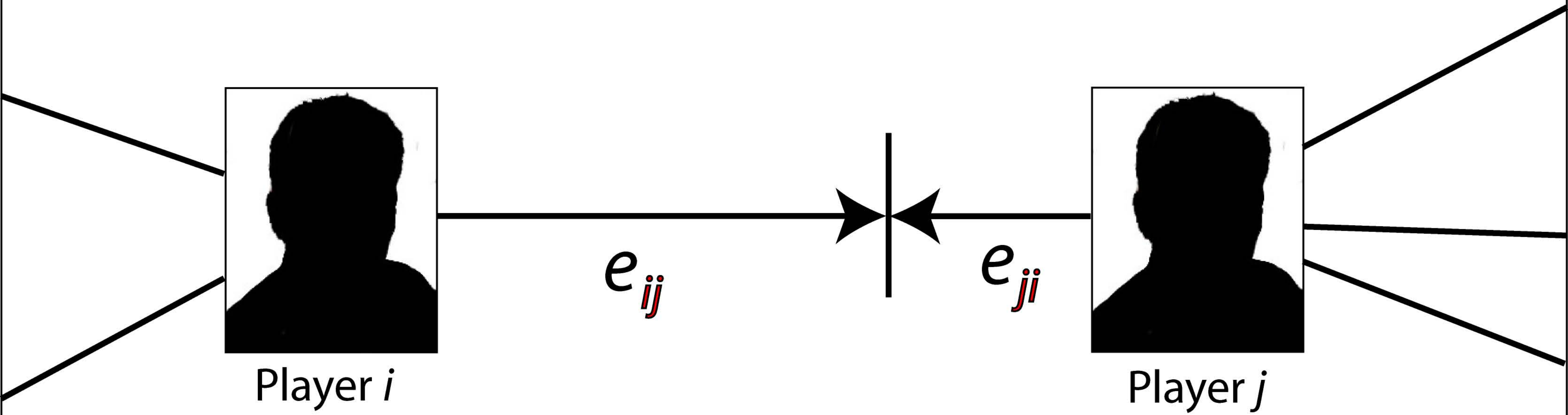
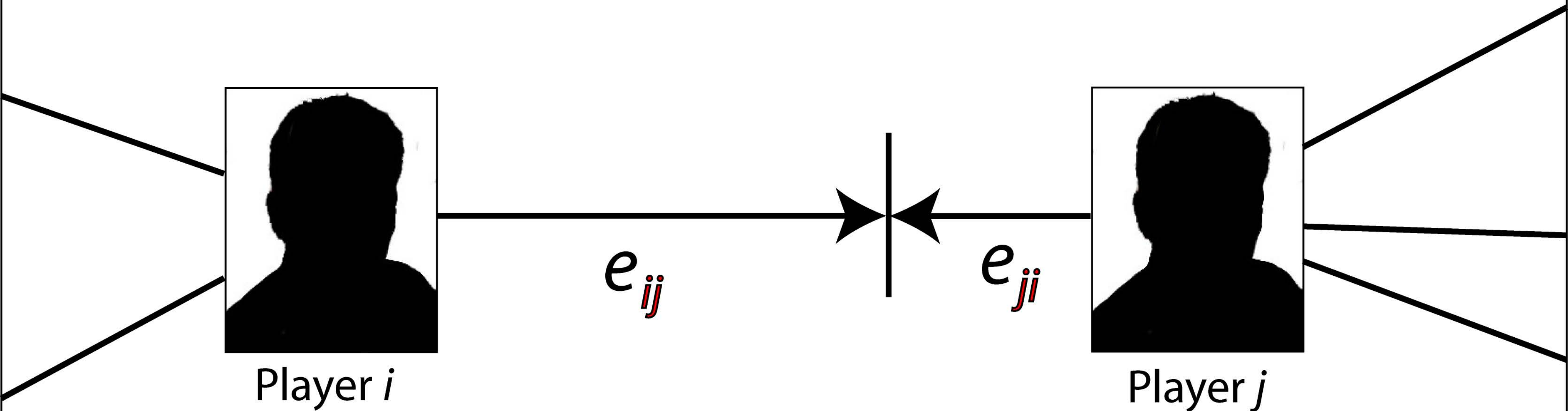


$$p_i = \left(\sum_j b\left(e_{ij} + e_{ji}\right)\right) - c\left(\sum_k e_{ik}\right)$$

Modeling: Patterns of Collaboration



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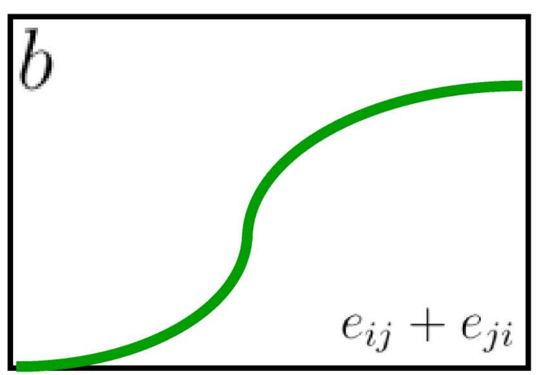
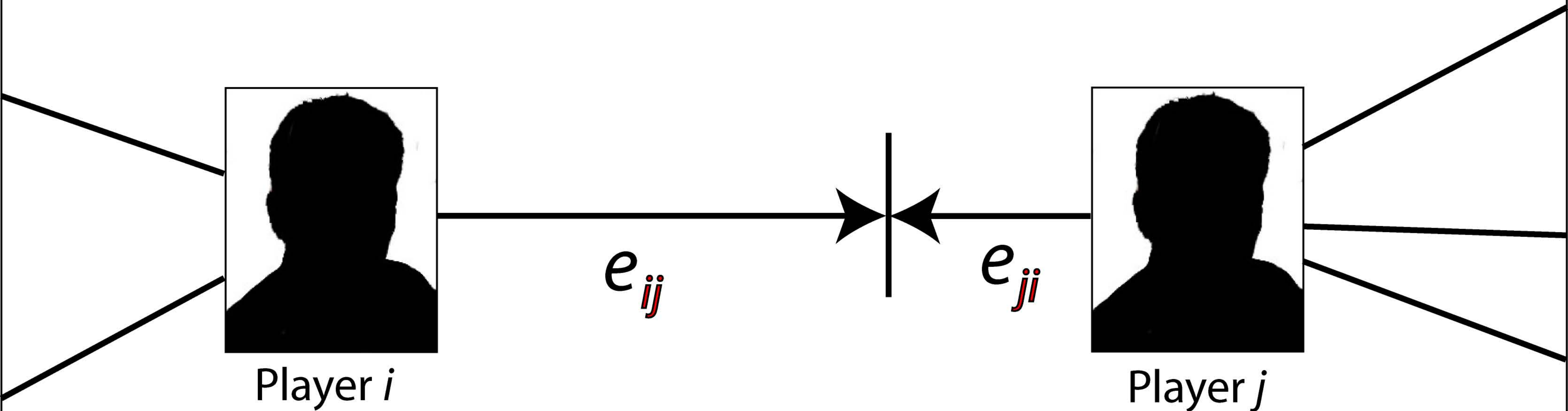


Modeling: Patterns of Collaboration



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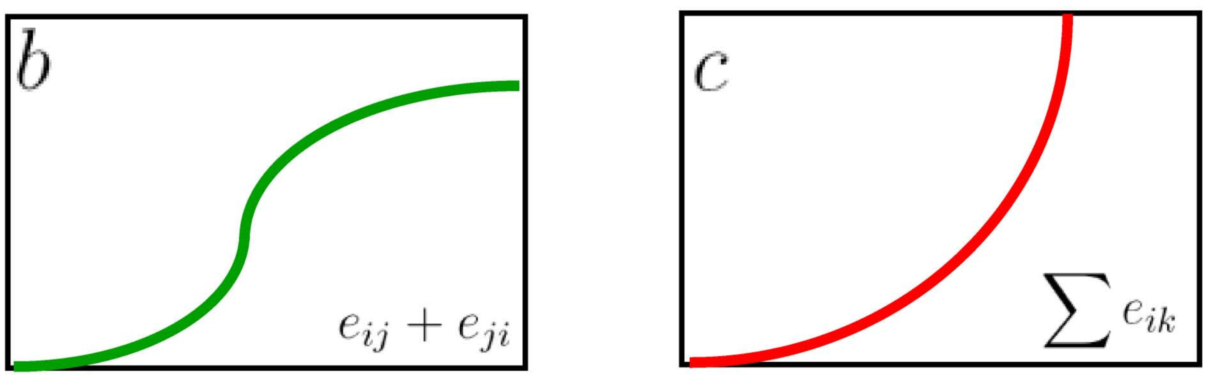
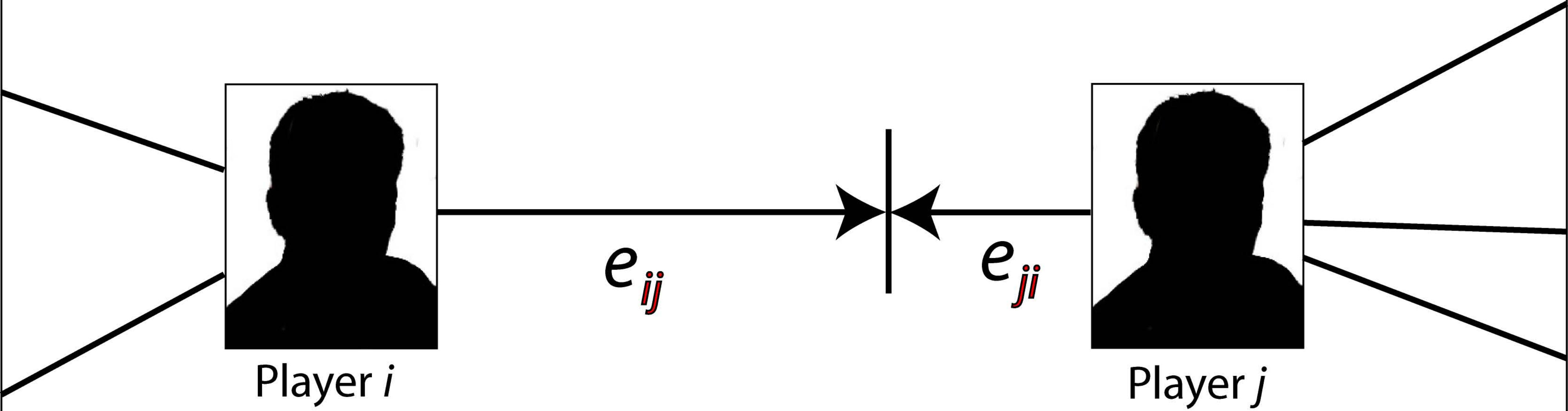


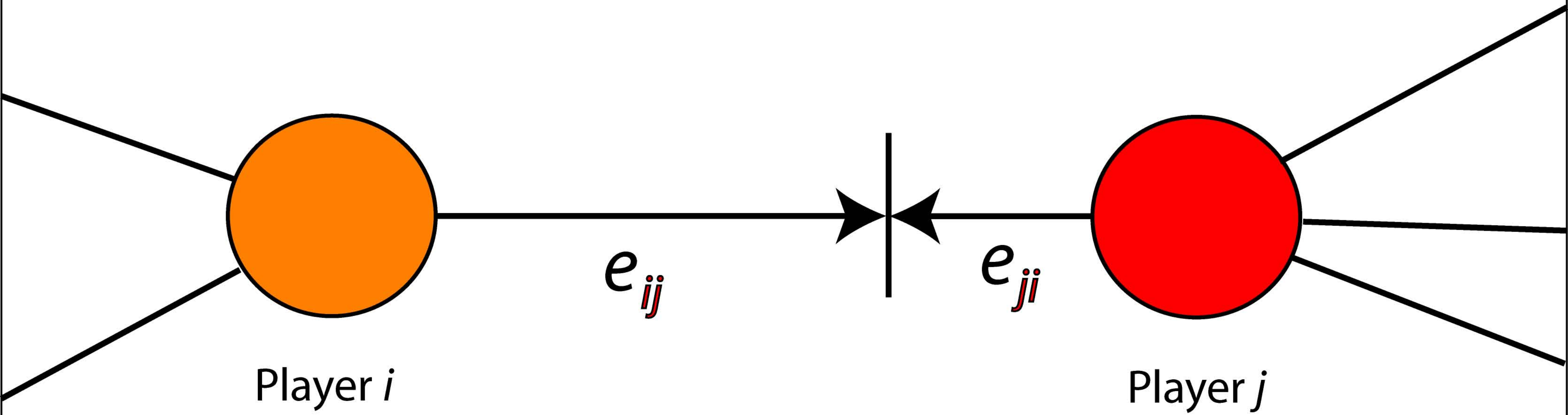
Modeling: Patterns of Collaboration



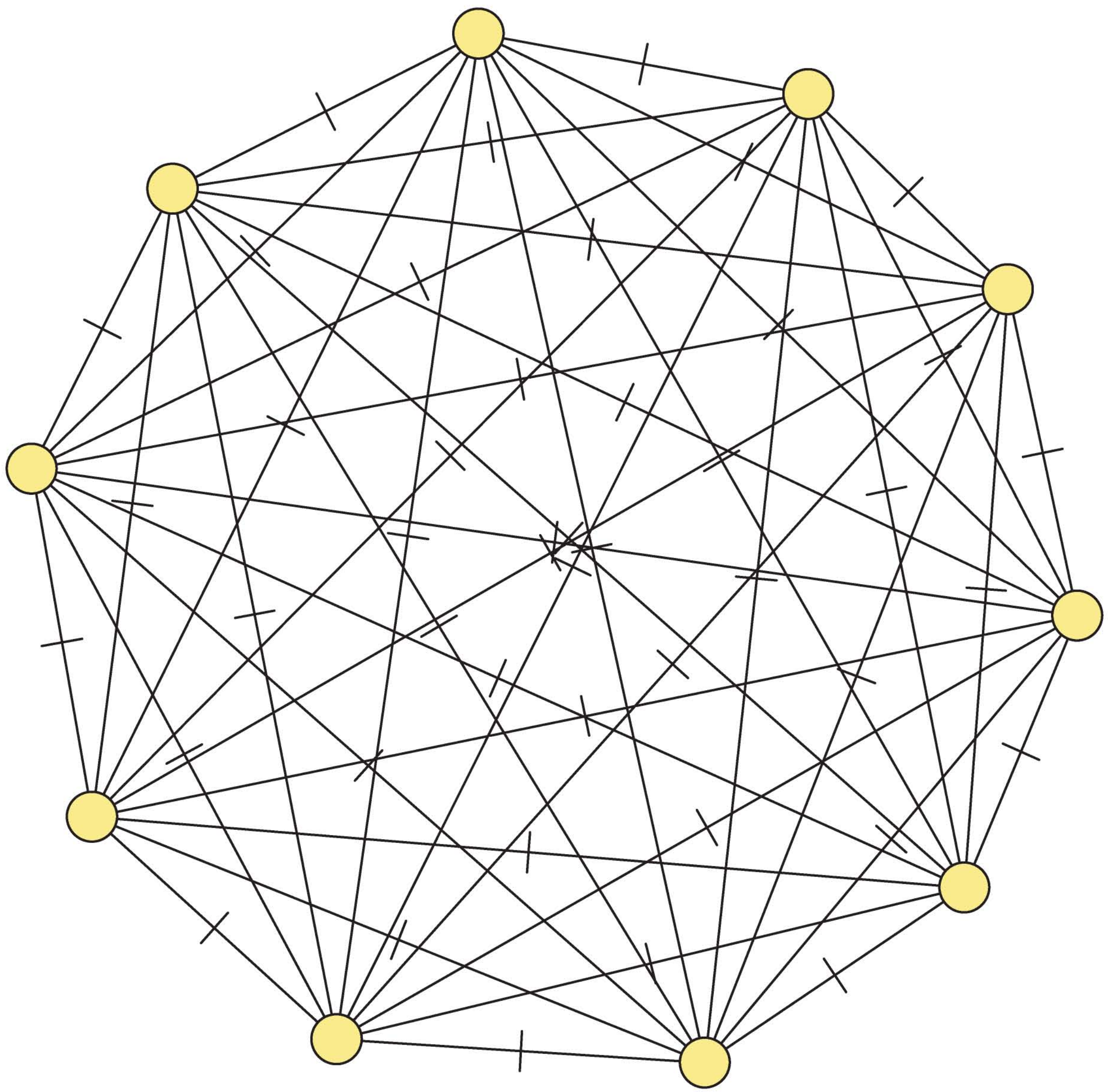
$$\frac{\partial}{\partial t}\, e_{ij} = \frac{\partial}{\partial e_{ij}}\, p_i$$

(Local optimization of payoffs - Players are selfish)

Modeling: Patterns of Collaboration



Results: Simulation



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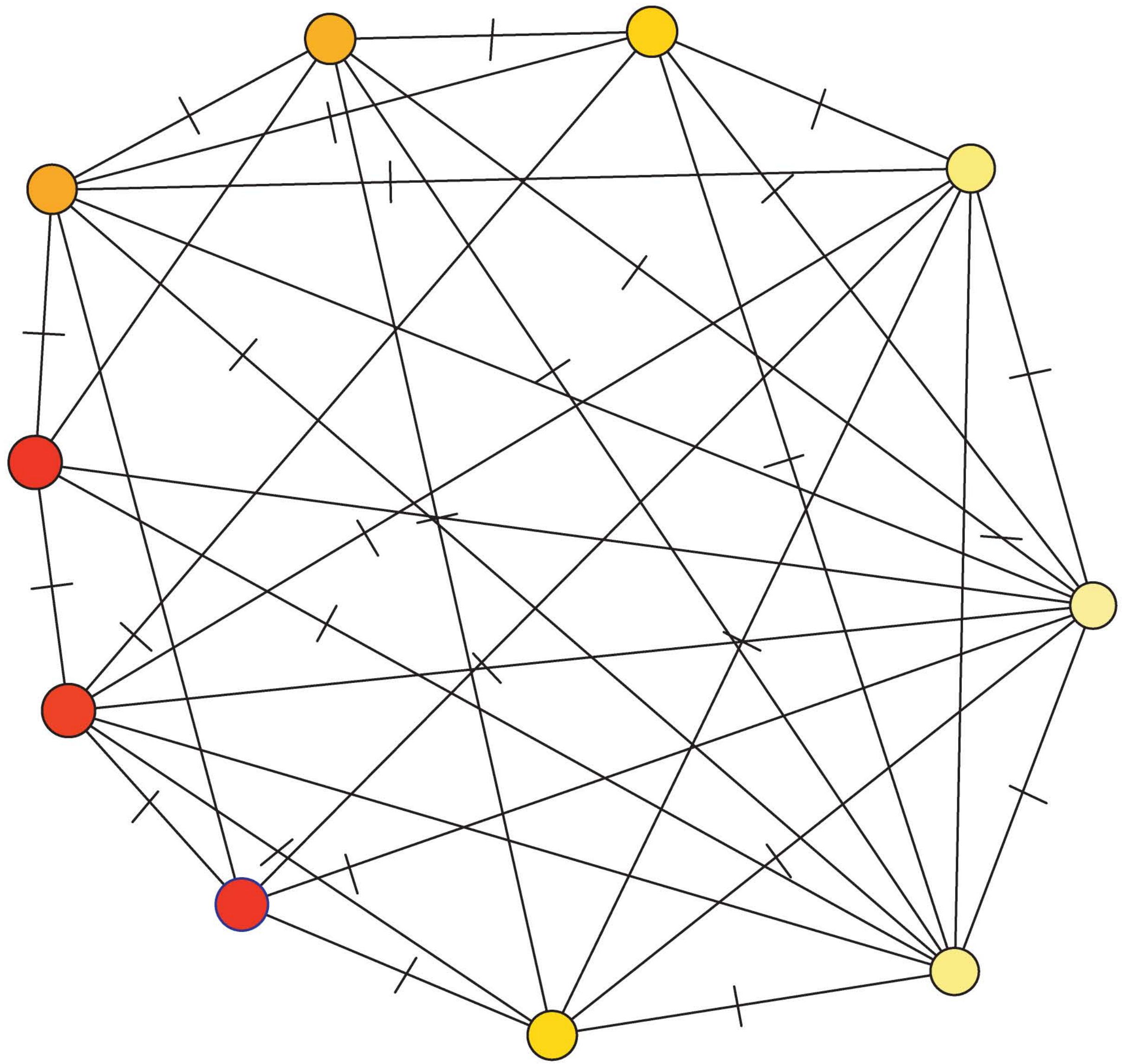

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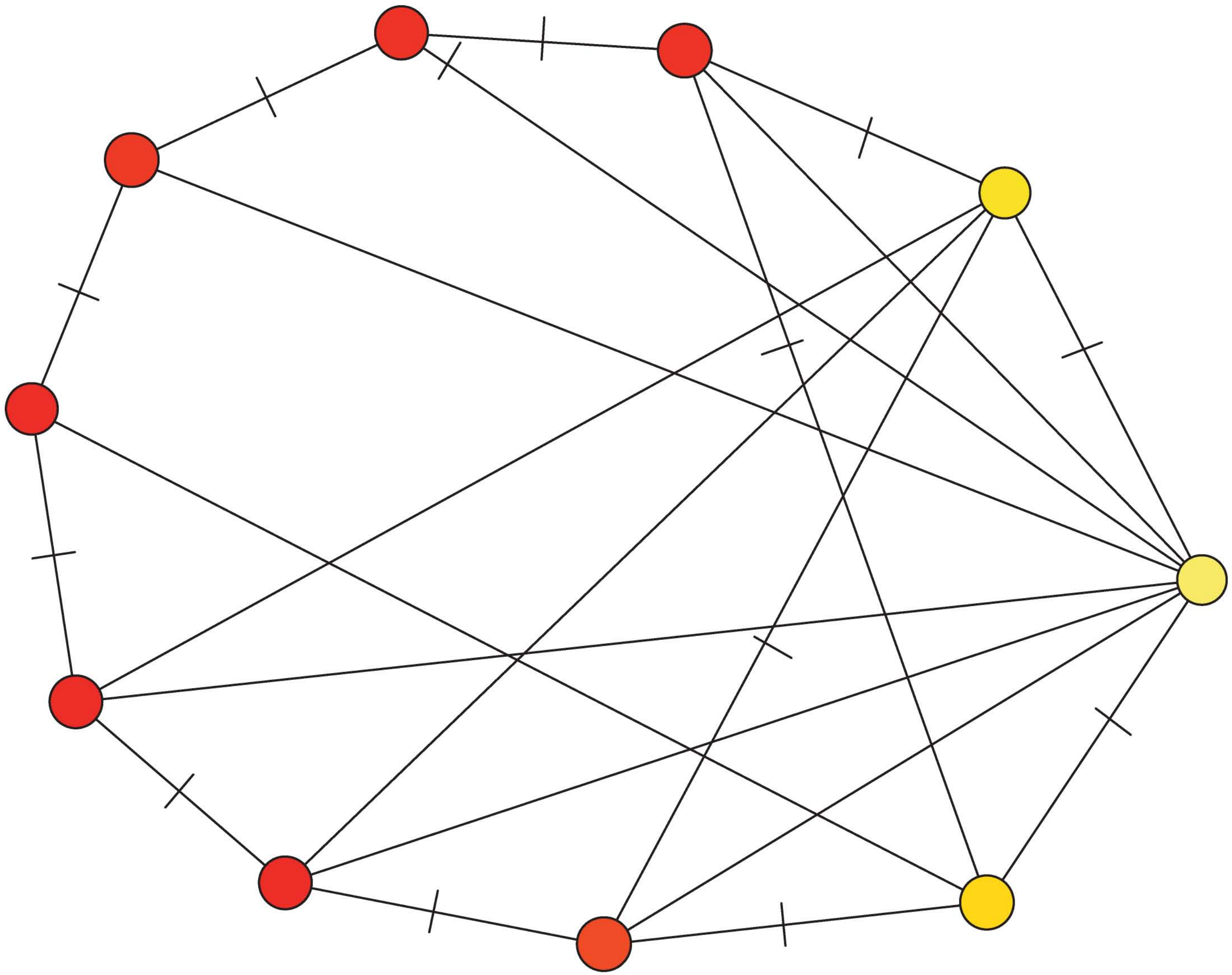

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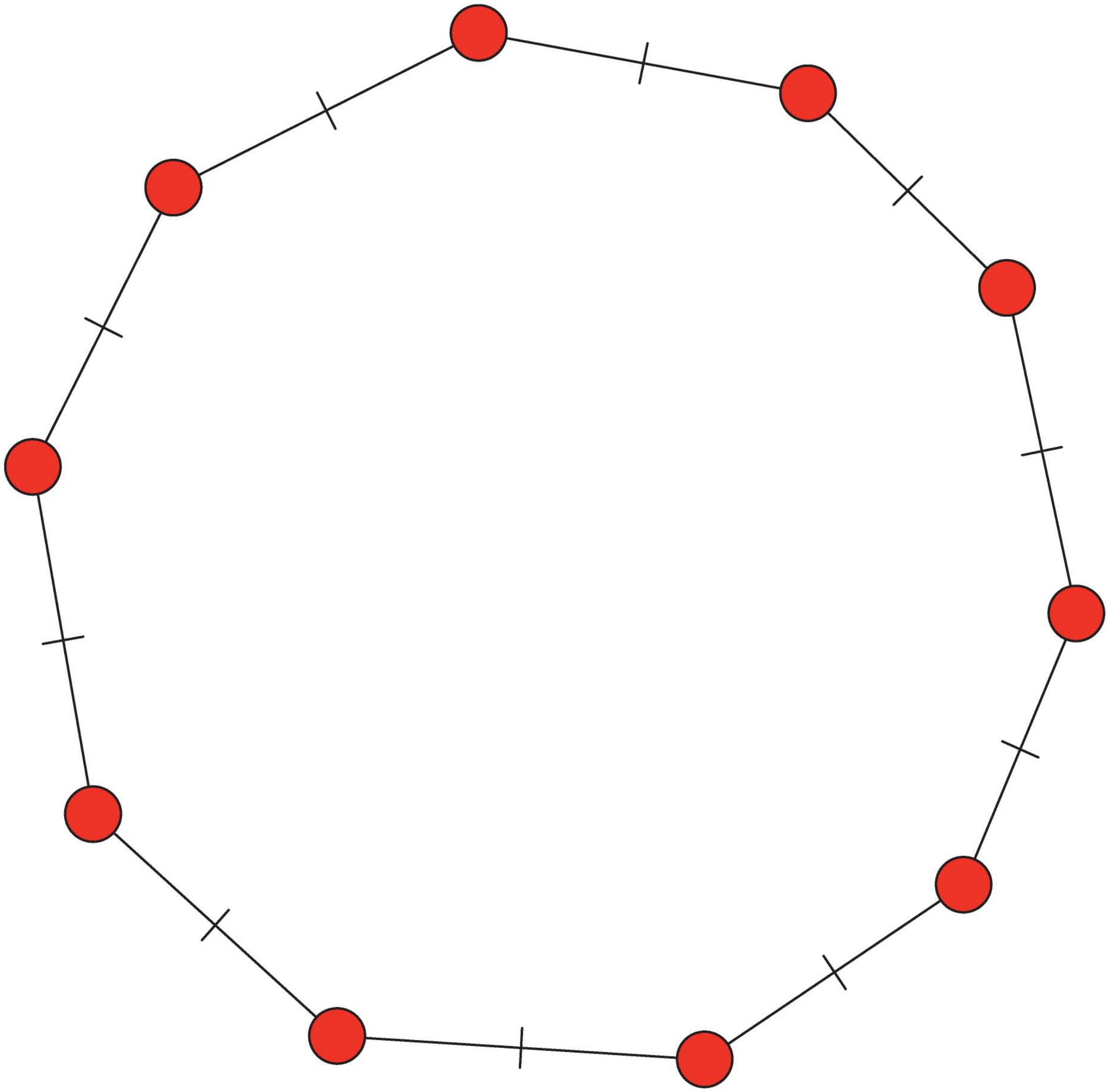

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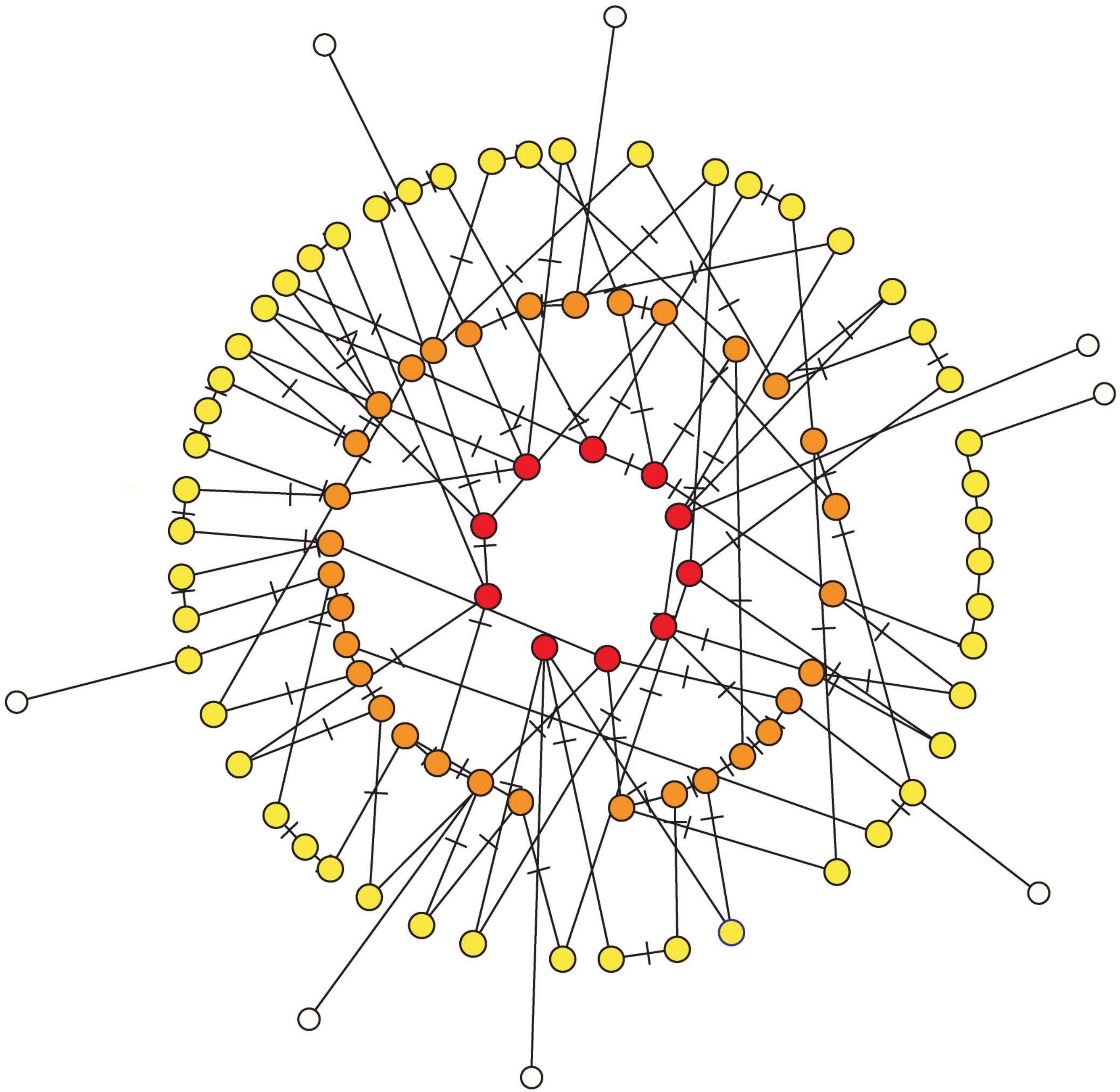
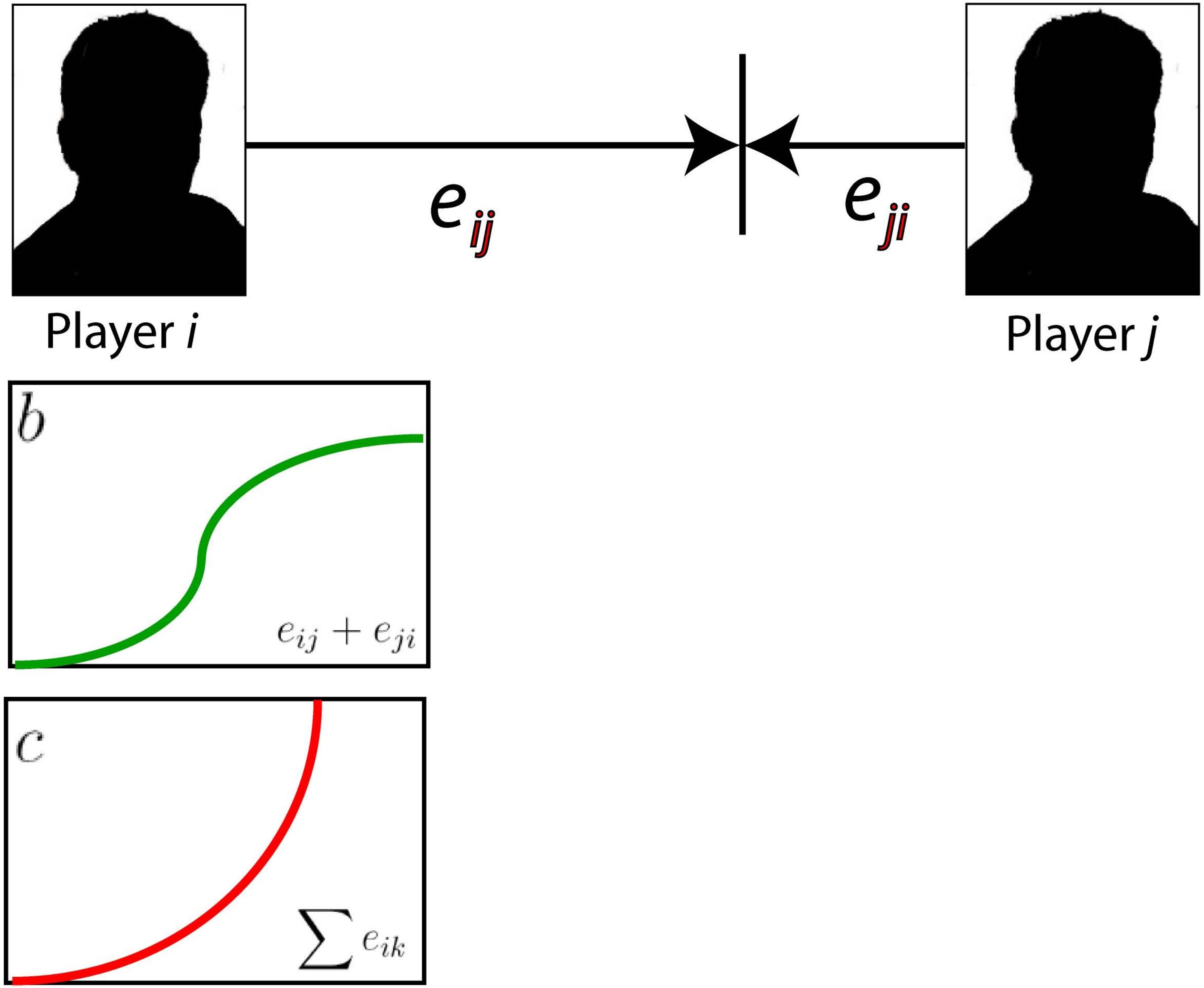


Results: Coordination



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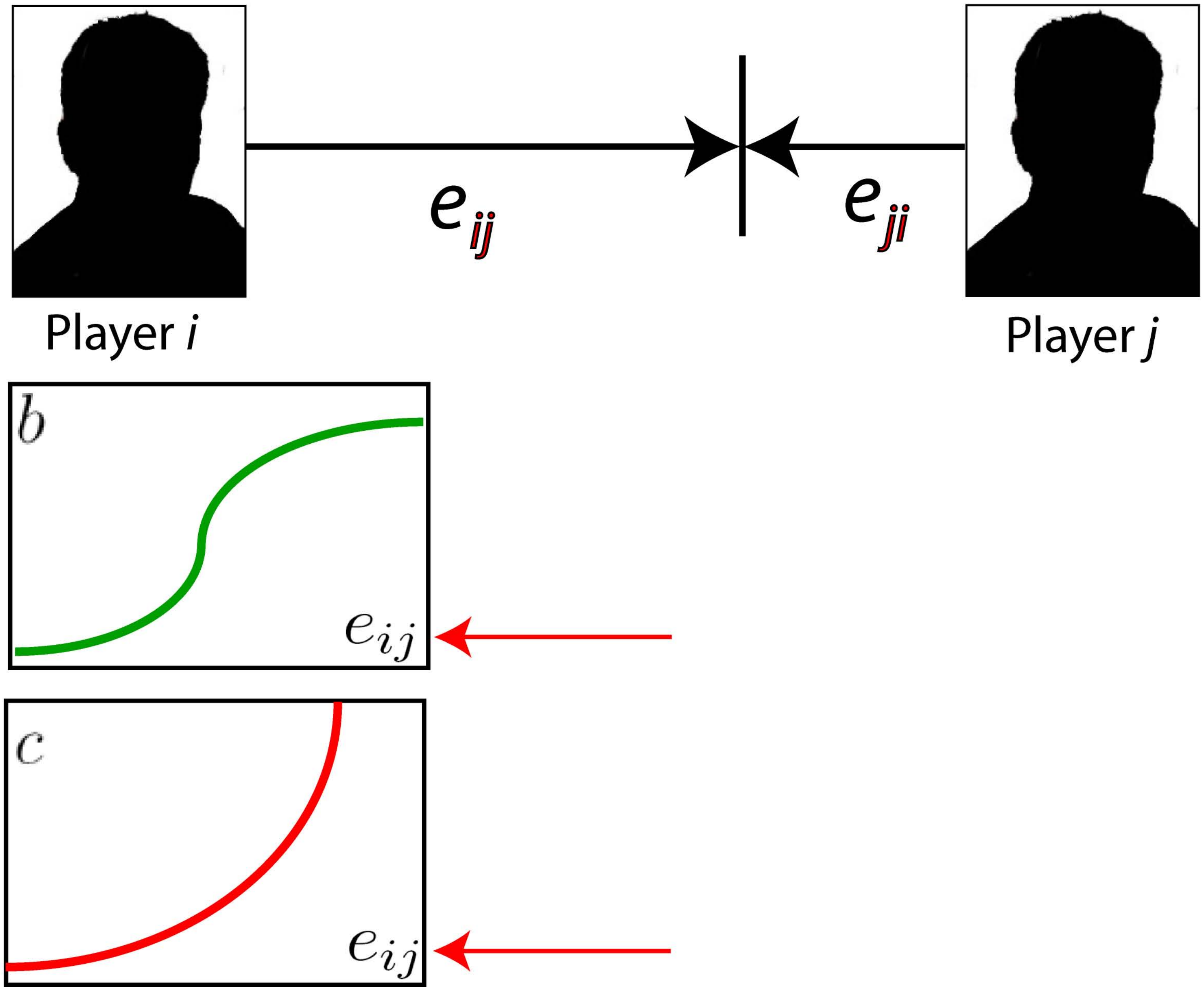

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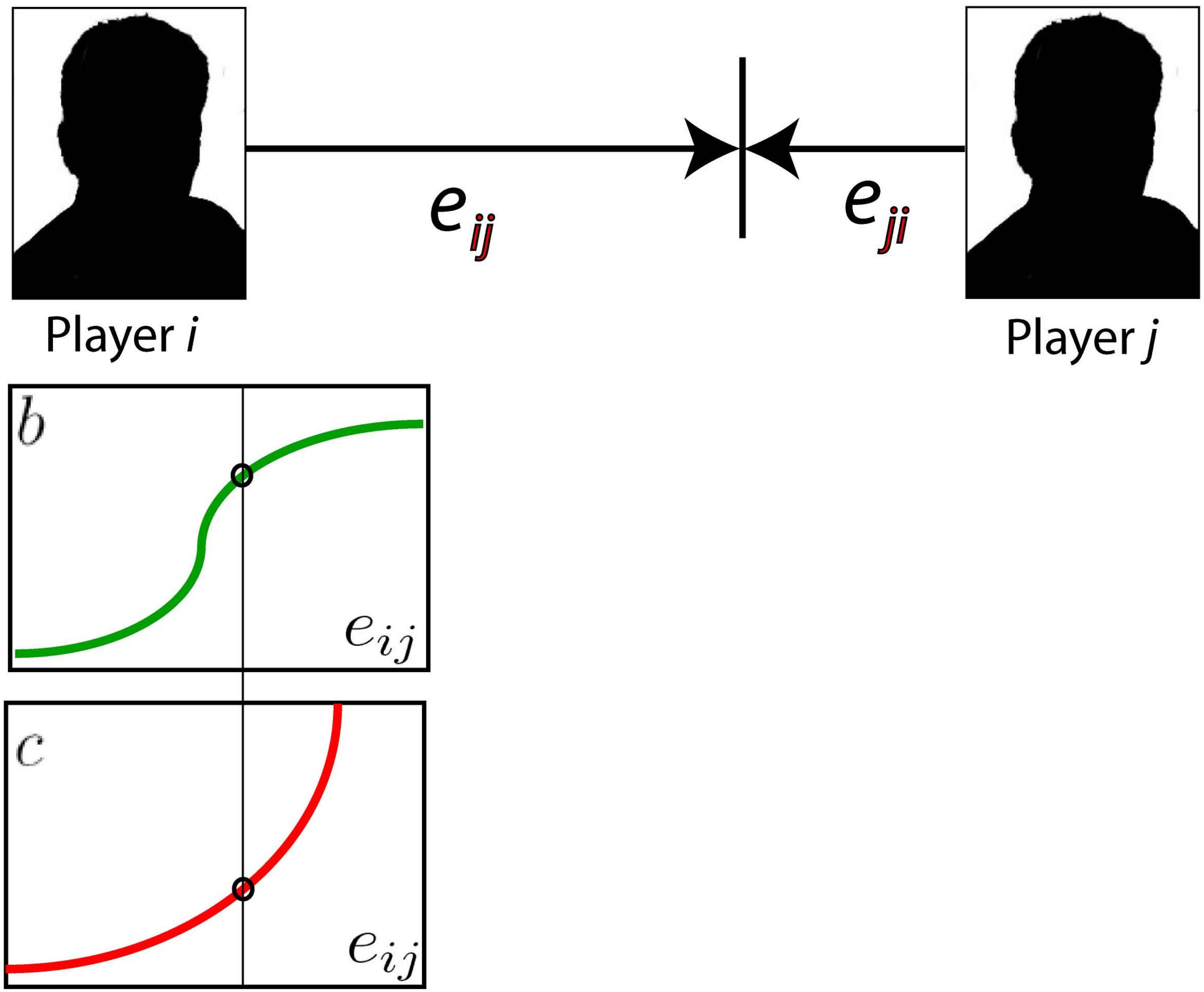

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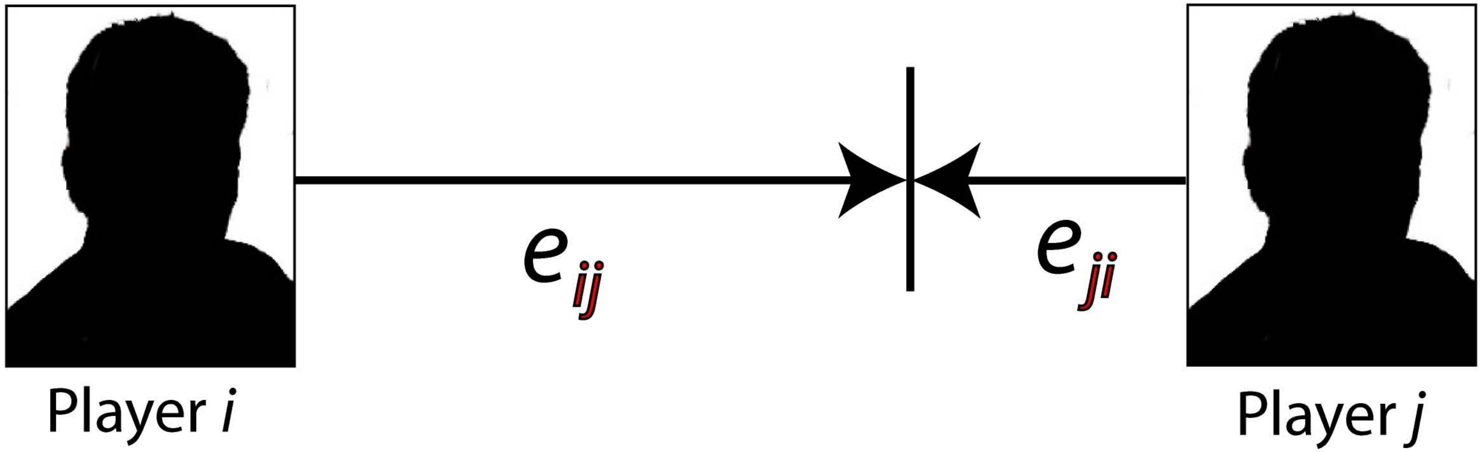
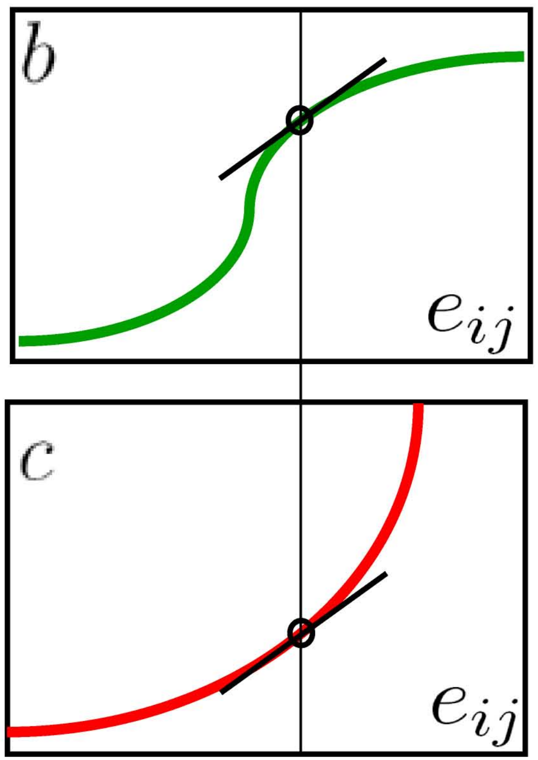
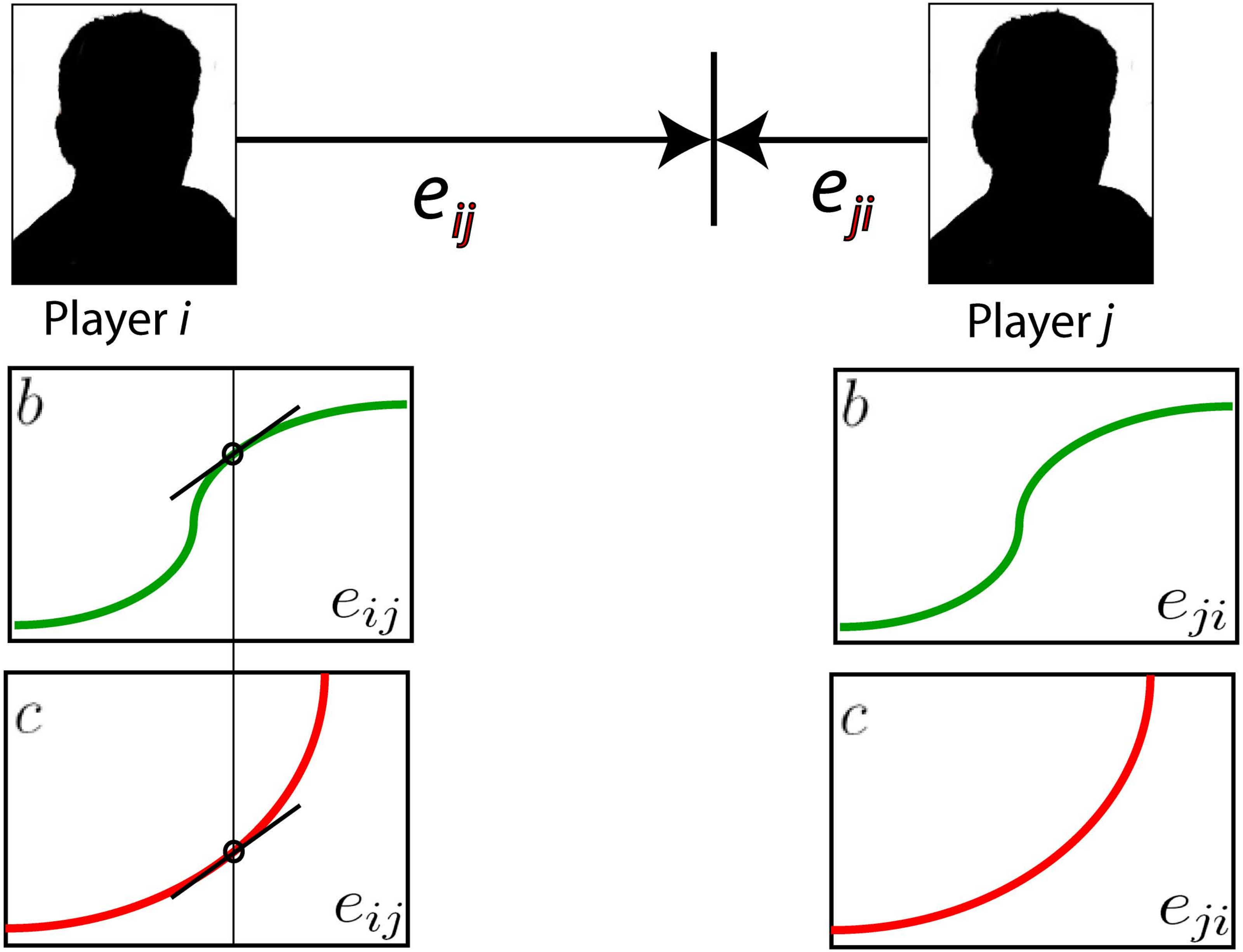




Stationarity requires identical slopes

Results: Coordination



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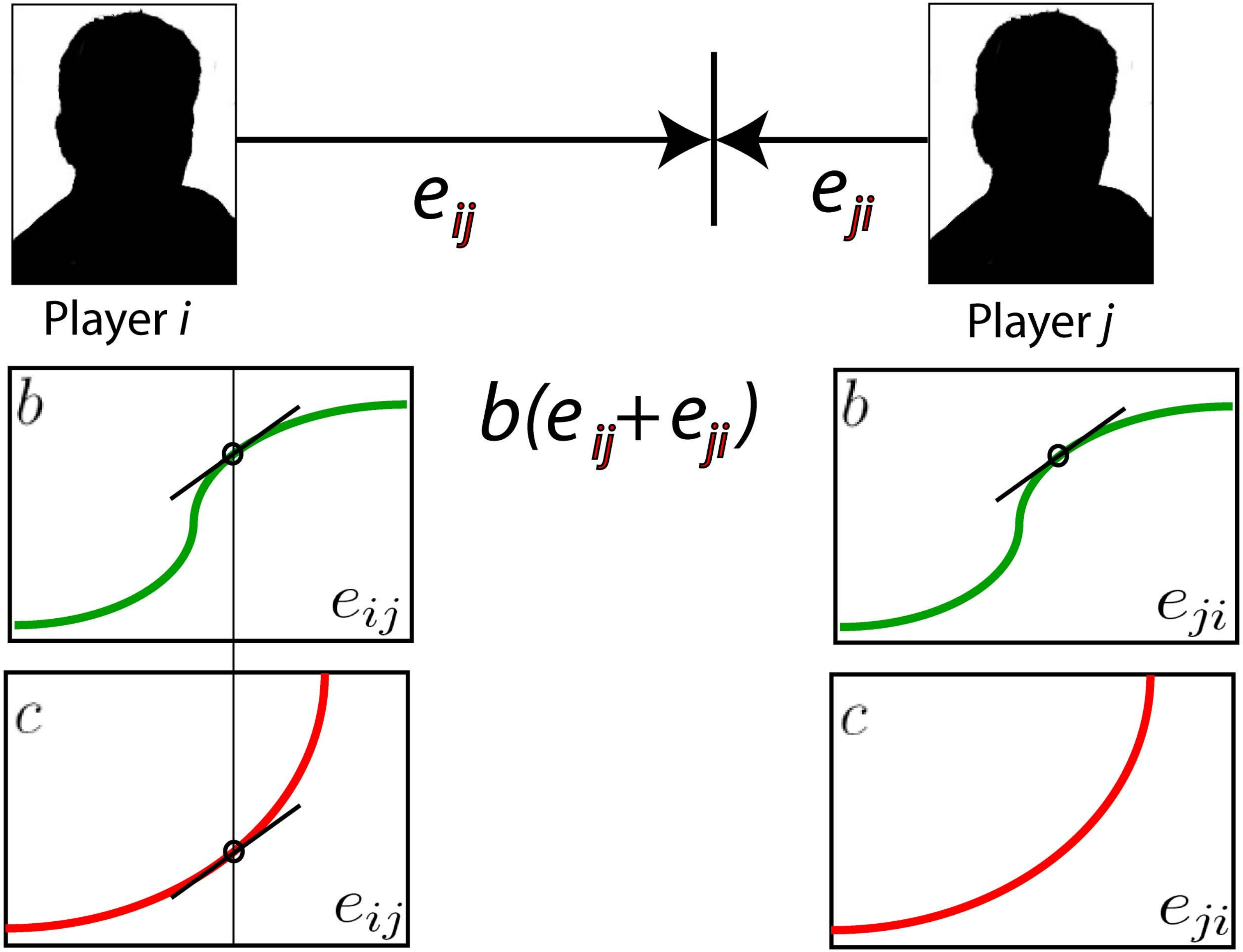

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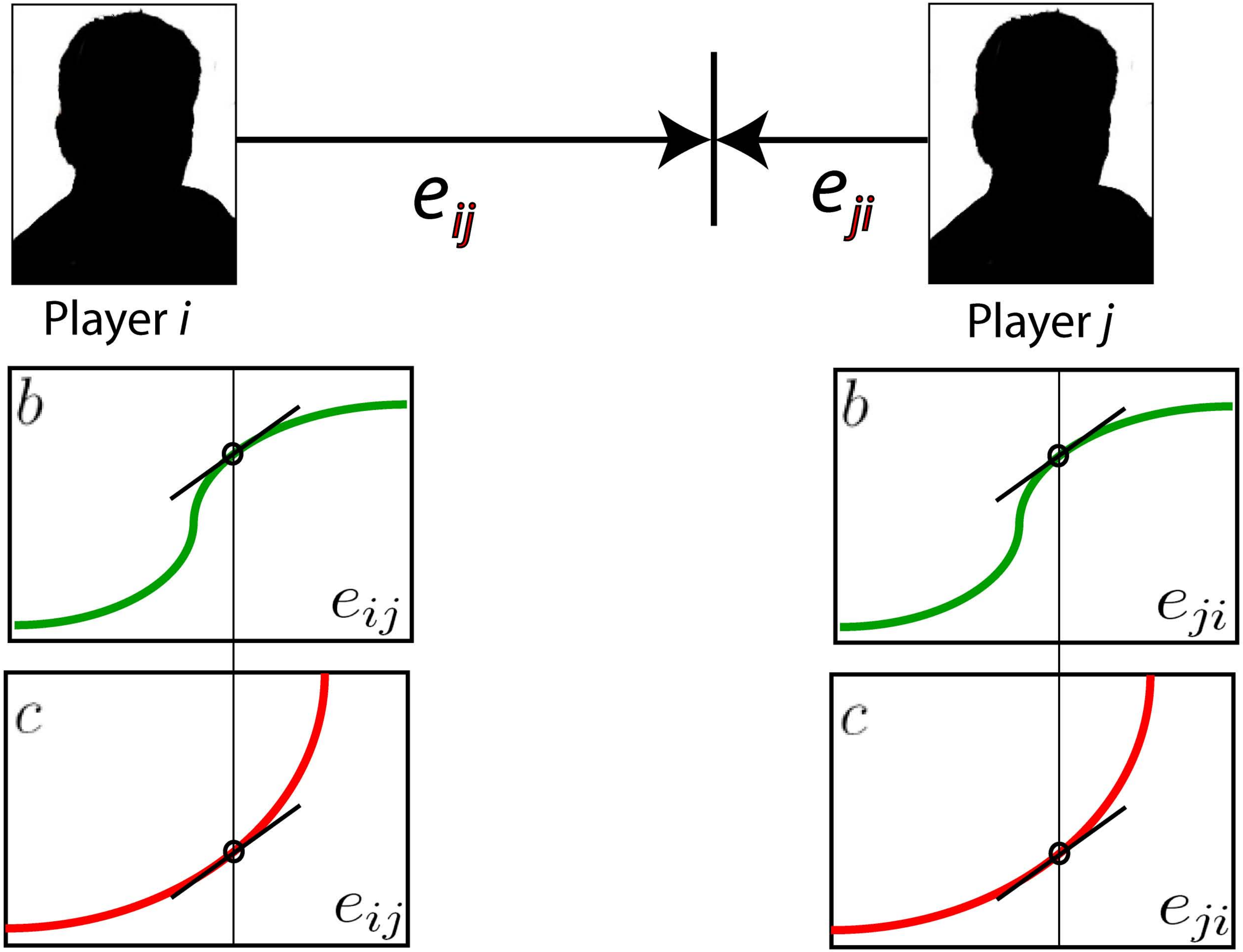

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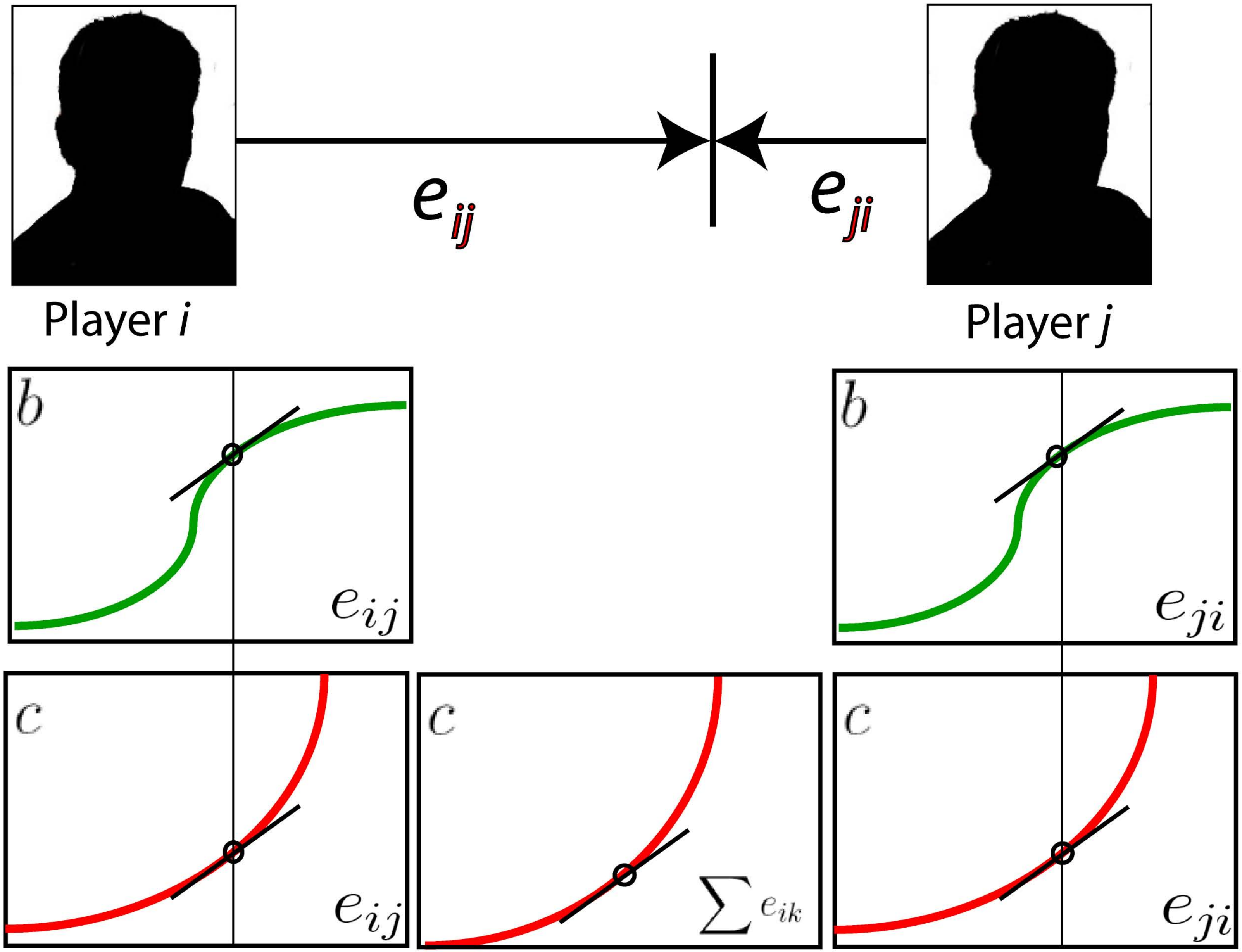
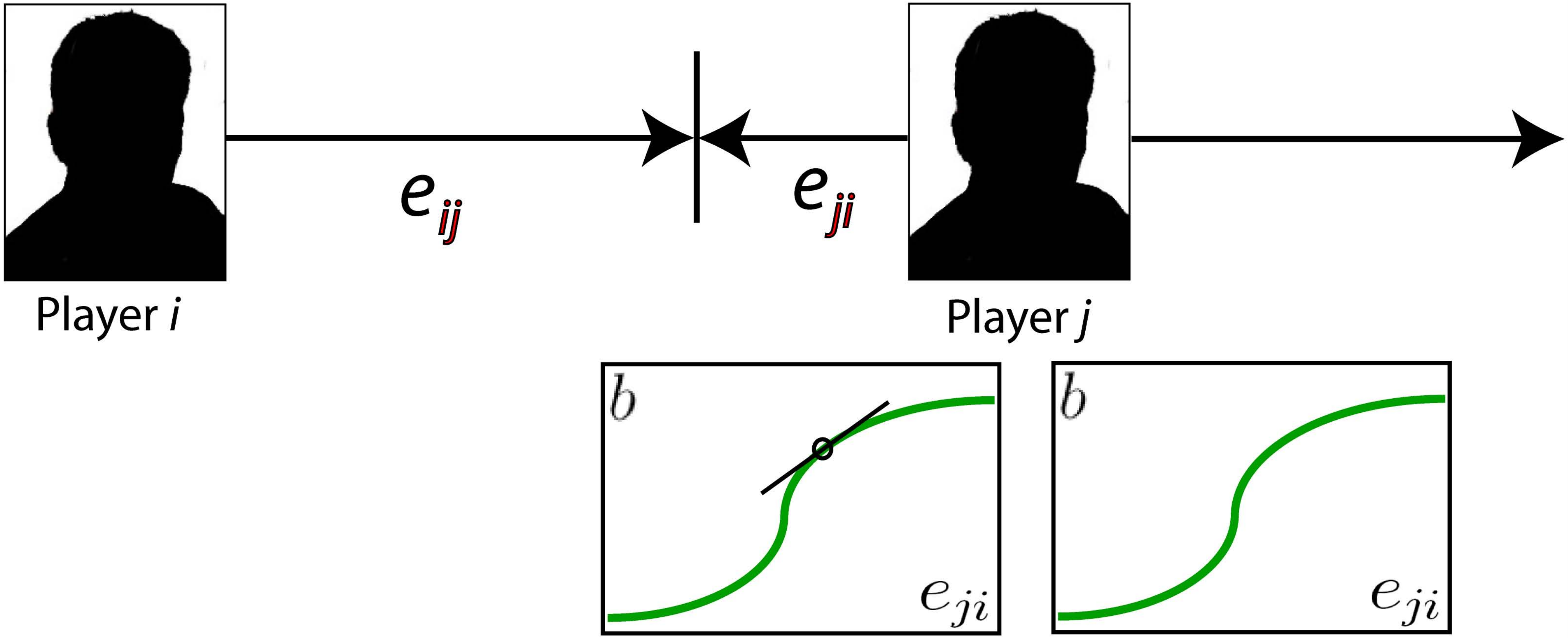


Results: Coordination



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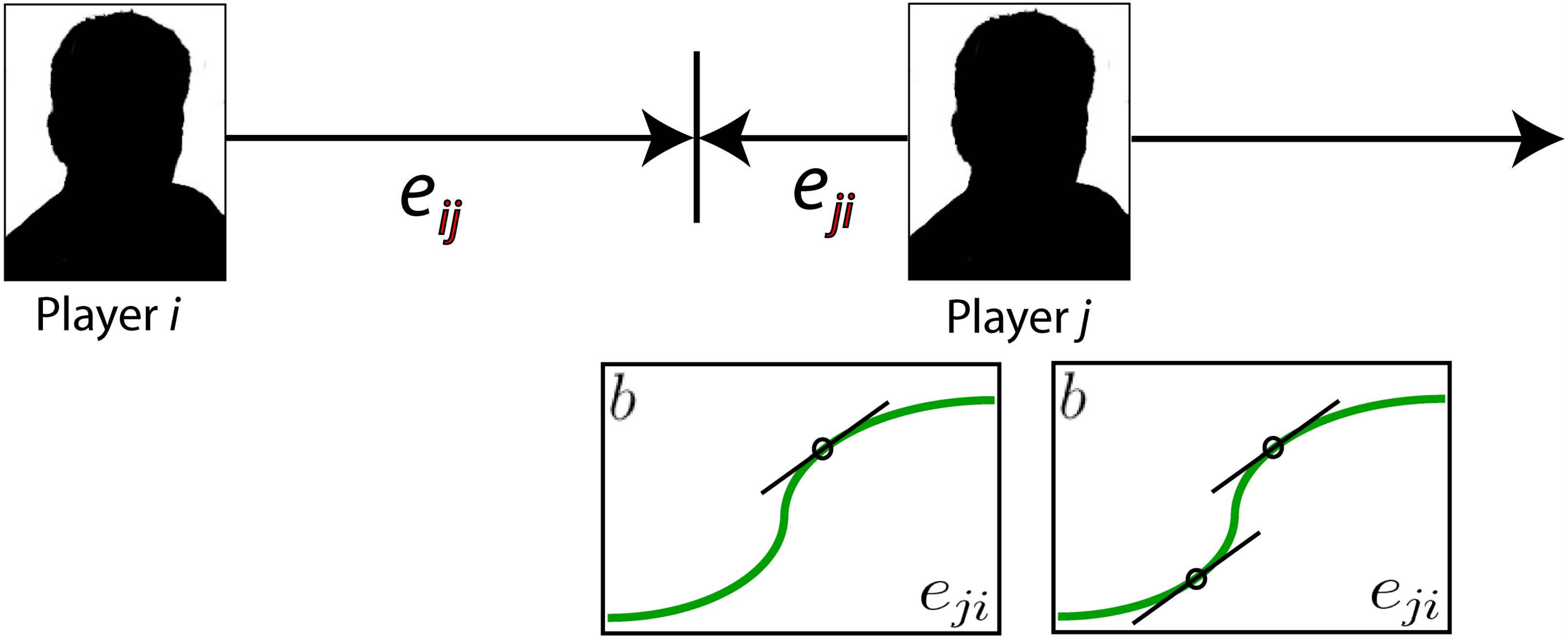
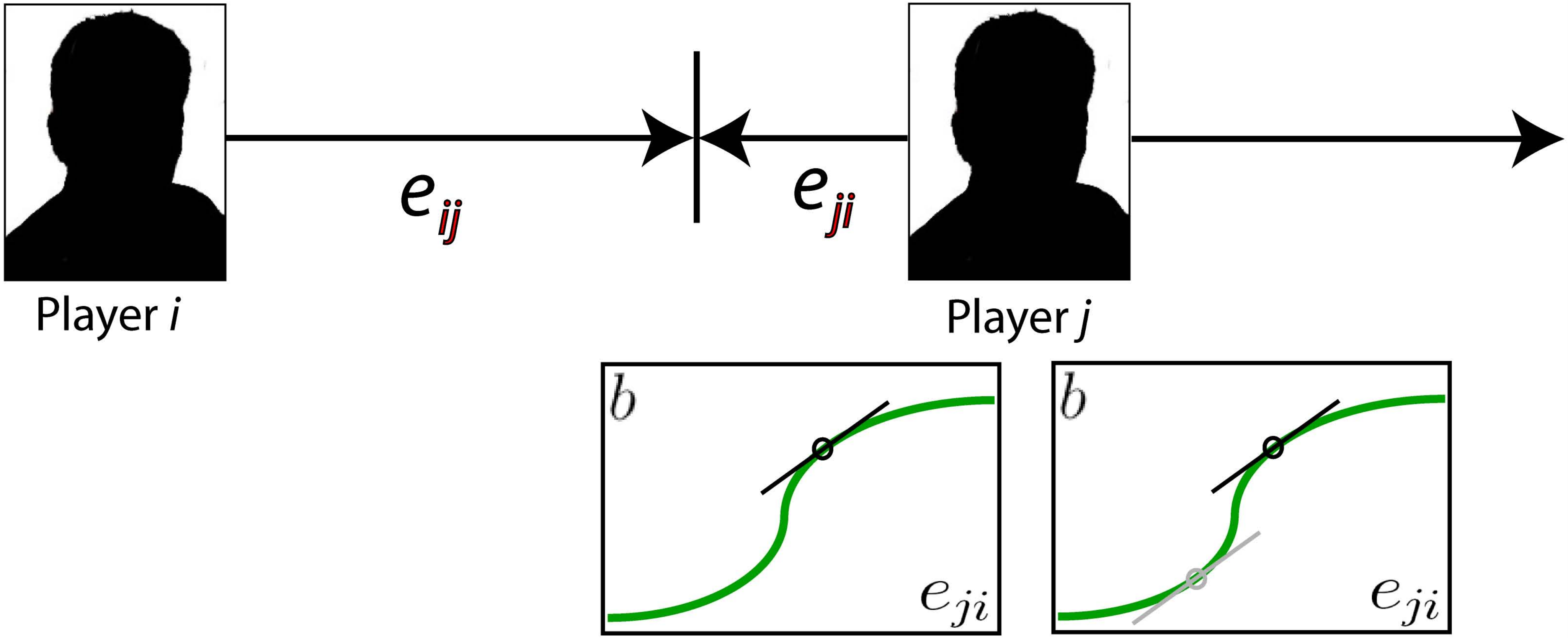


Stationarity implies:

Links can provide only two different levels of benefit

Results: Coordination

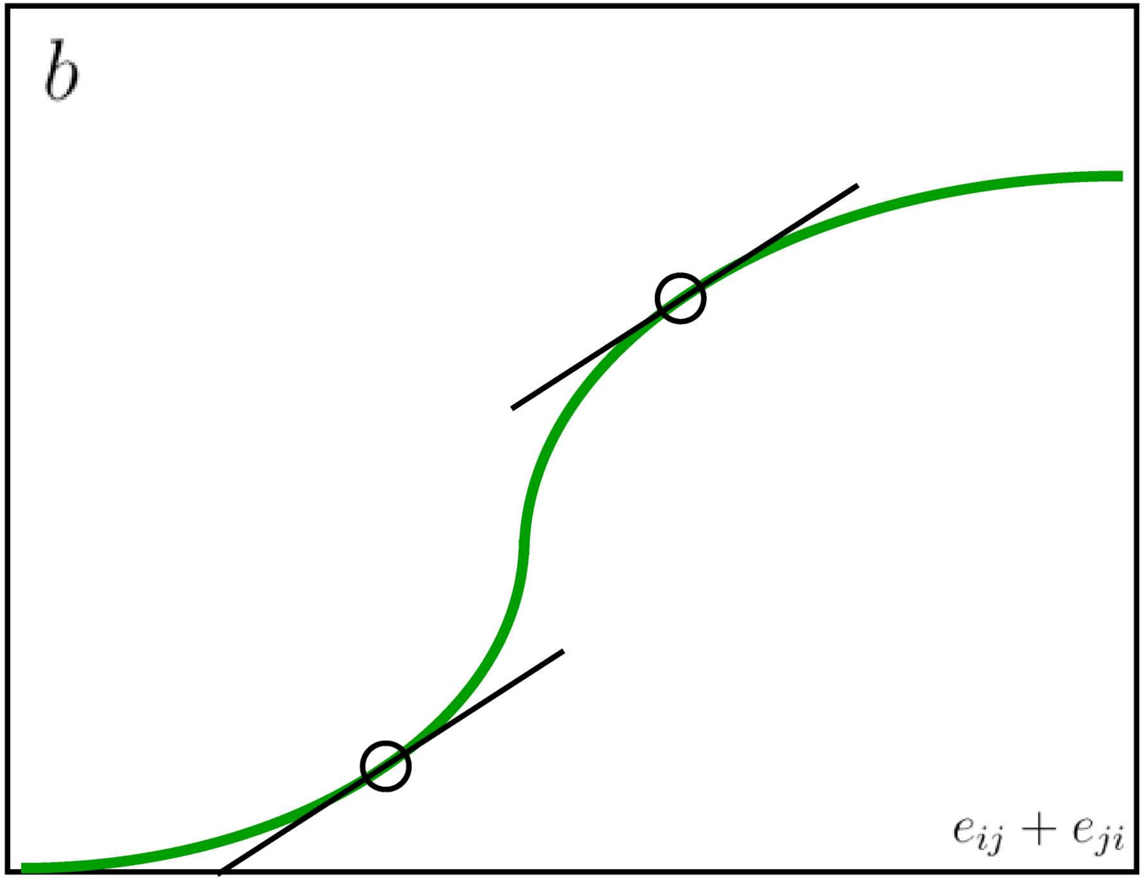


Stationarity implies:

Links can provide only two different levels of benefit
(one turns out to be dynamically unstable)

Results: Stability (hand-wavy explanation)

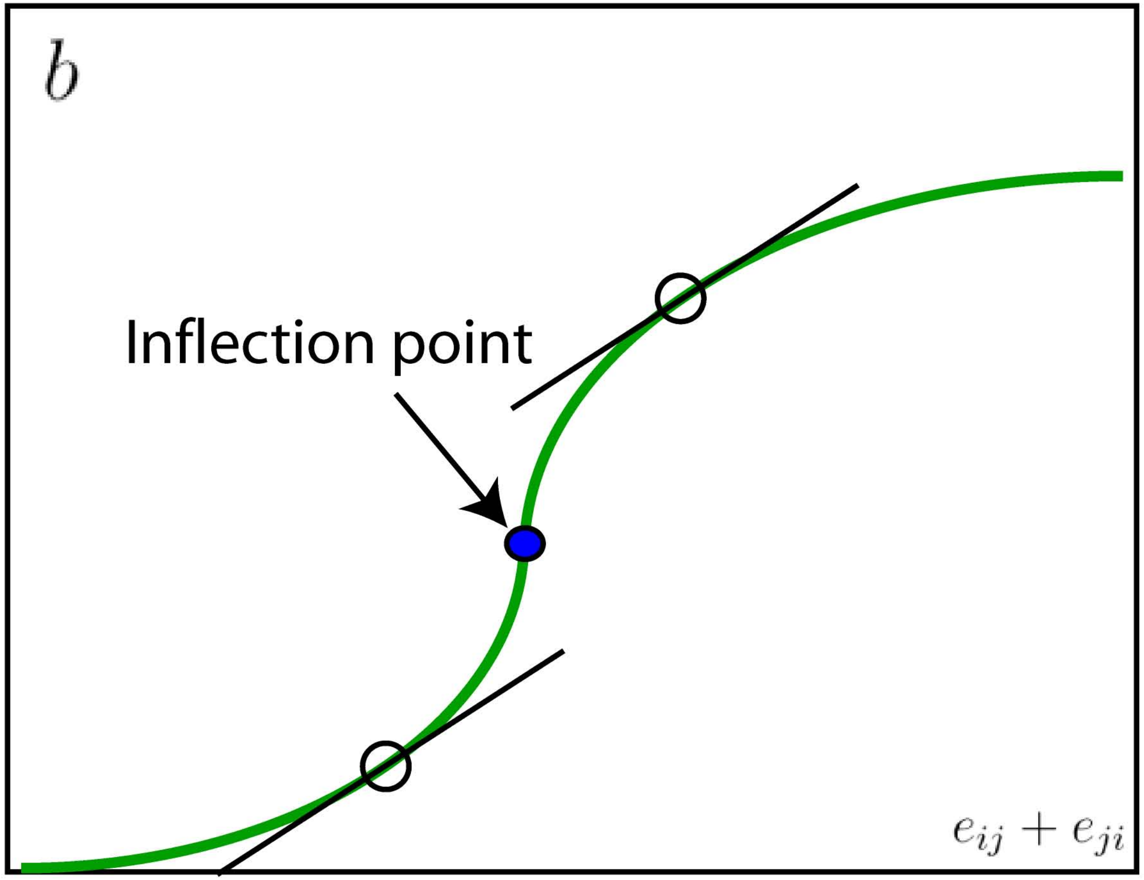
Player *i*



Player *j*

Results: Stability (hand-wavy explanation)

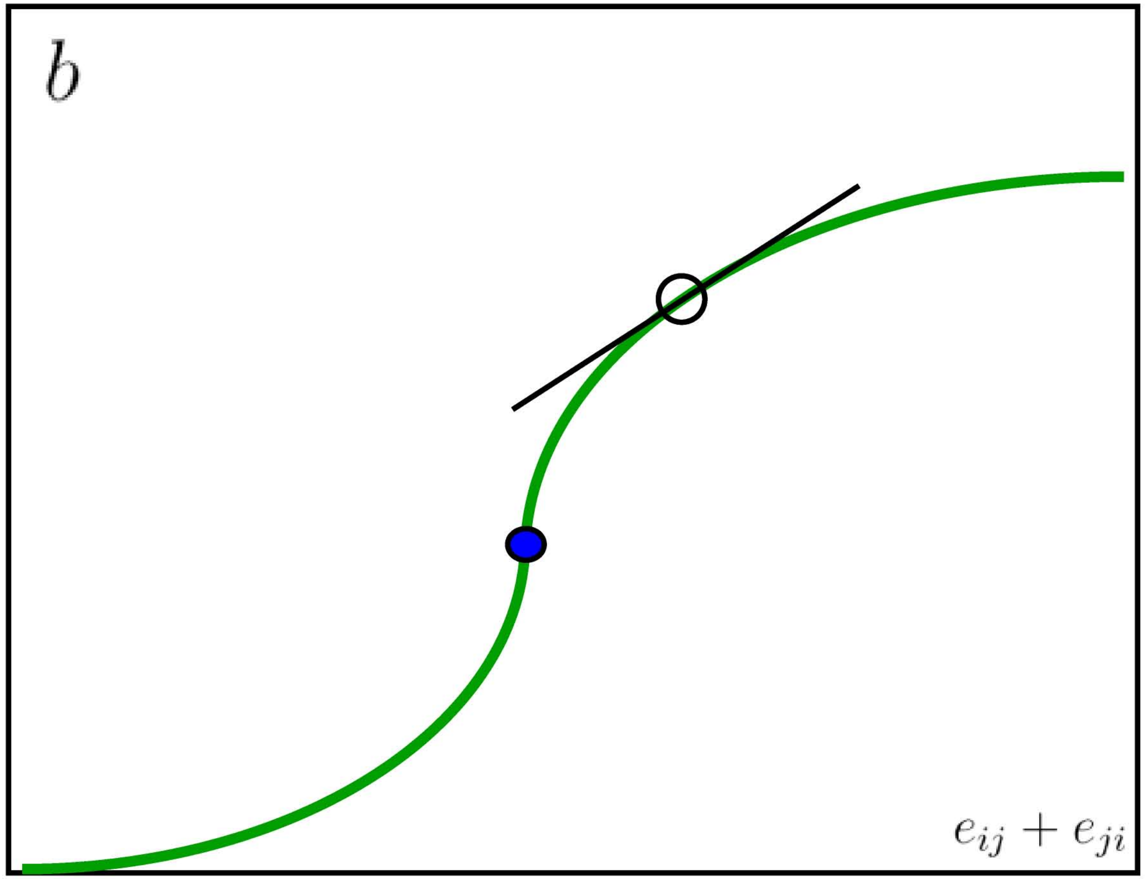
Player i

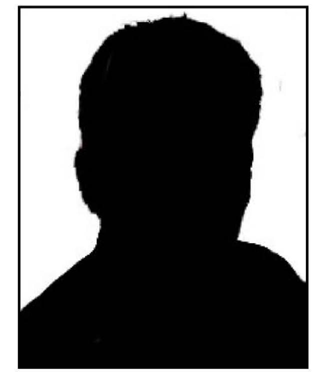


Player j

Results: Stability (hand-wavy explanation)

Player i

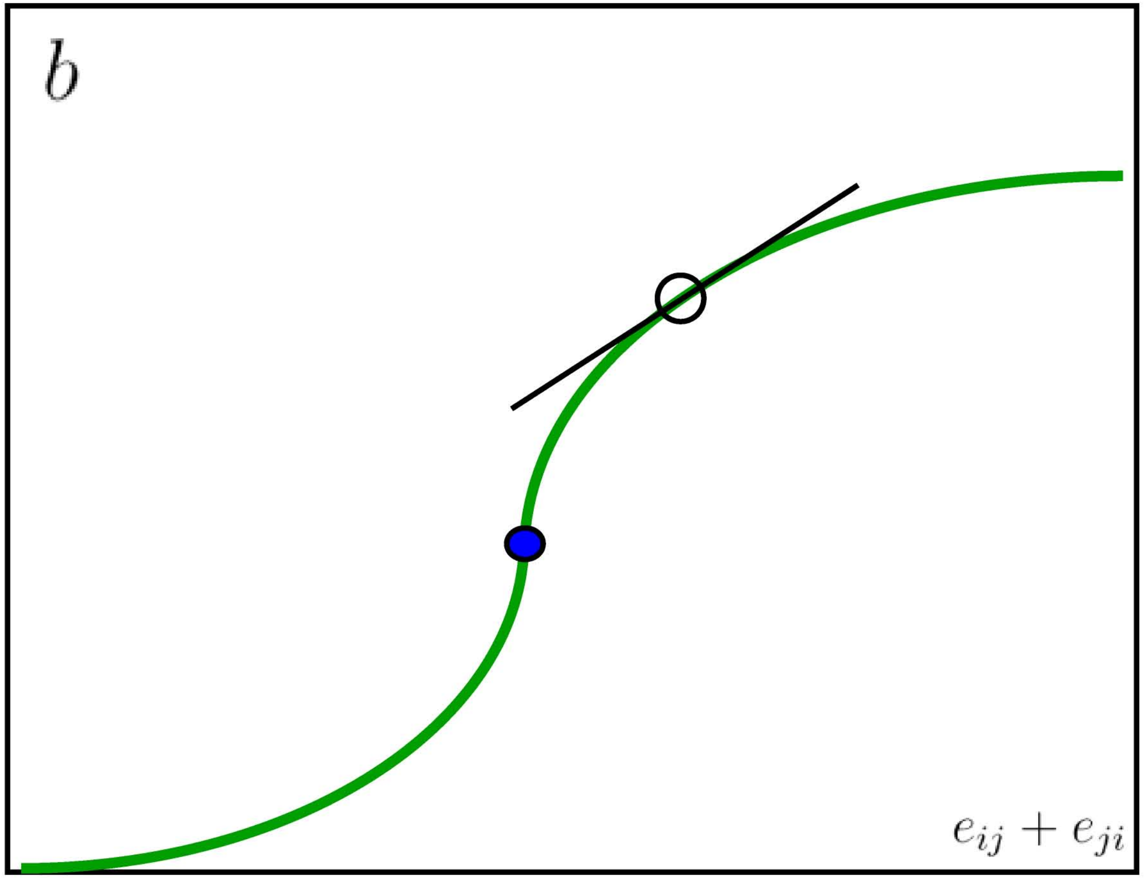


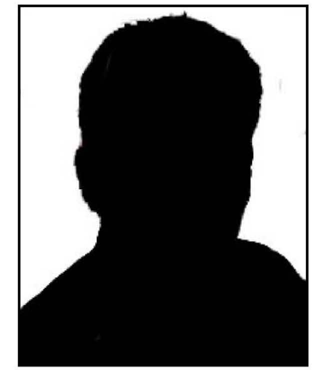


Player j

Results: Stability (hand-wavy explanation)

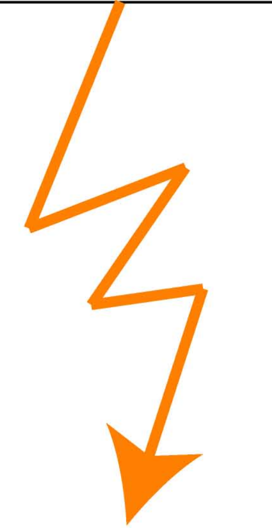
Player i

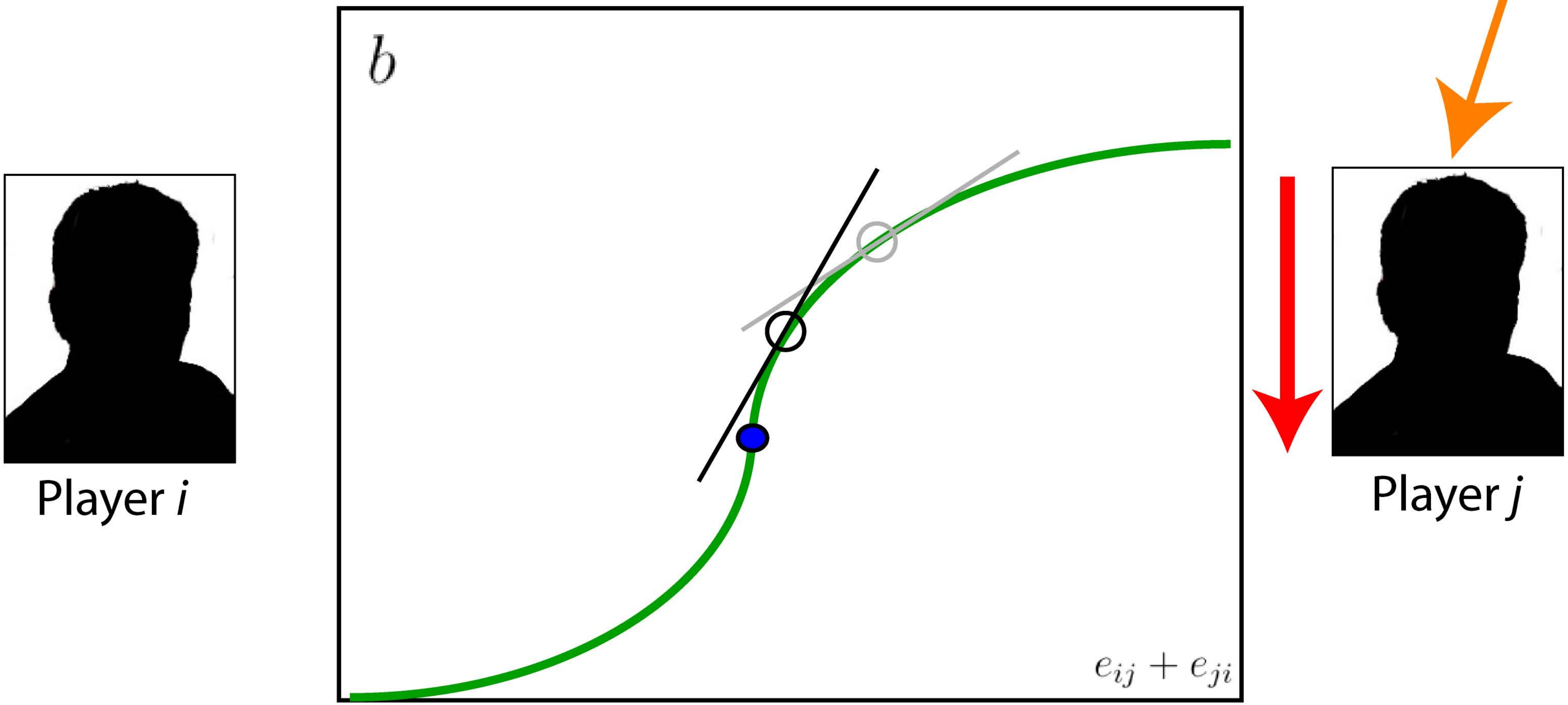




Player j

Results: Stability (hand-wavy explanation)





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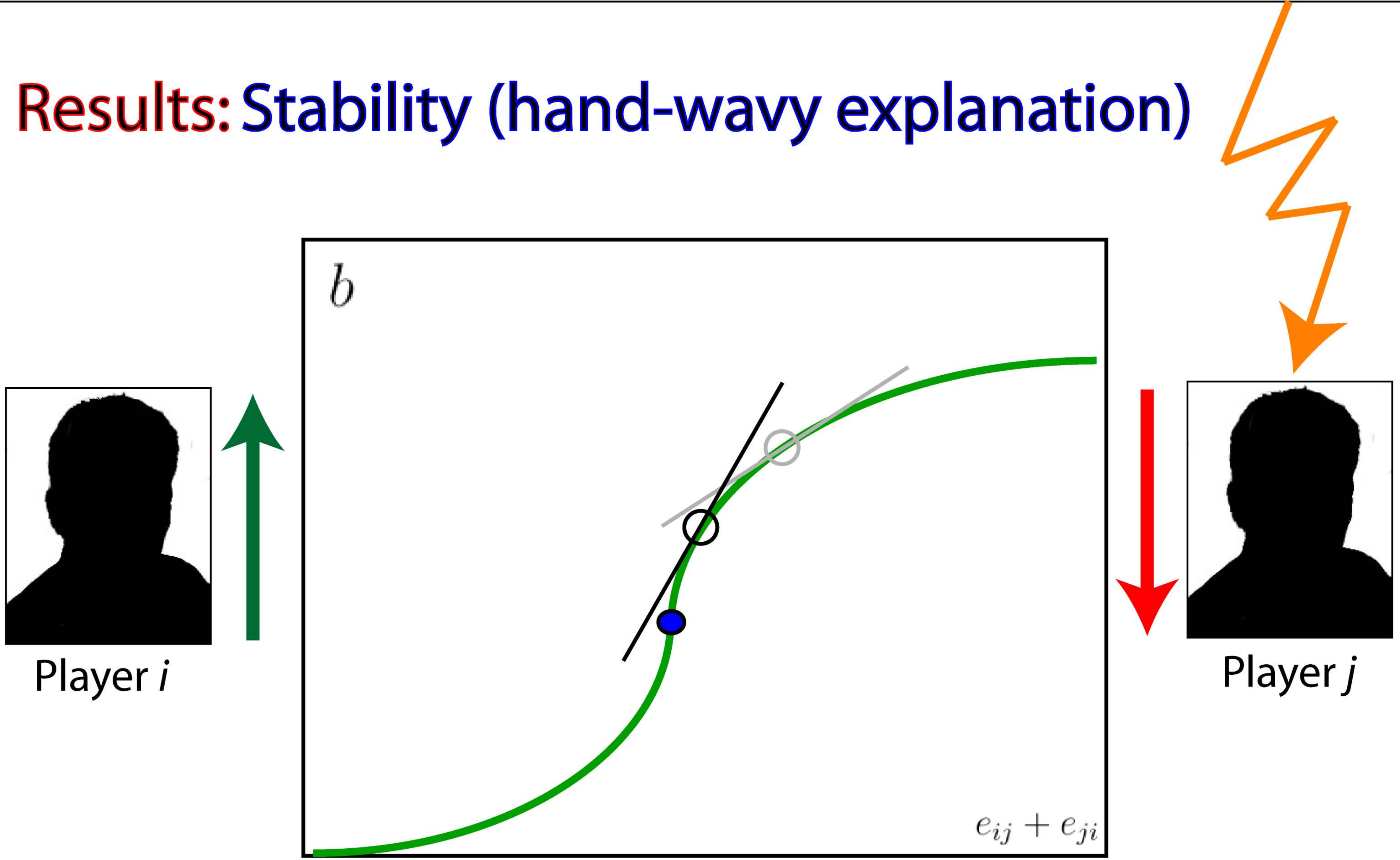

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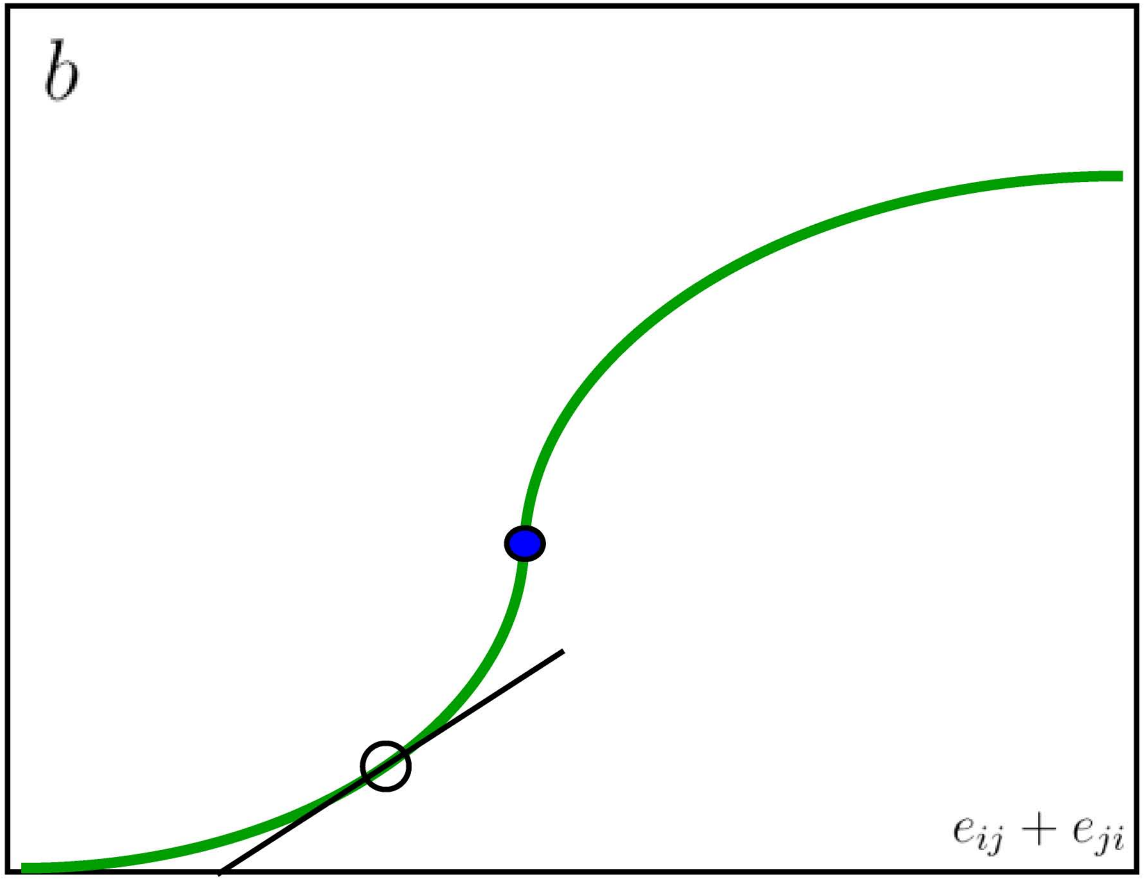
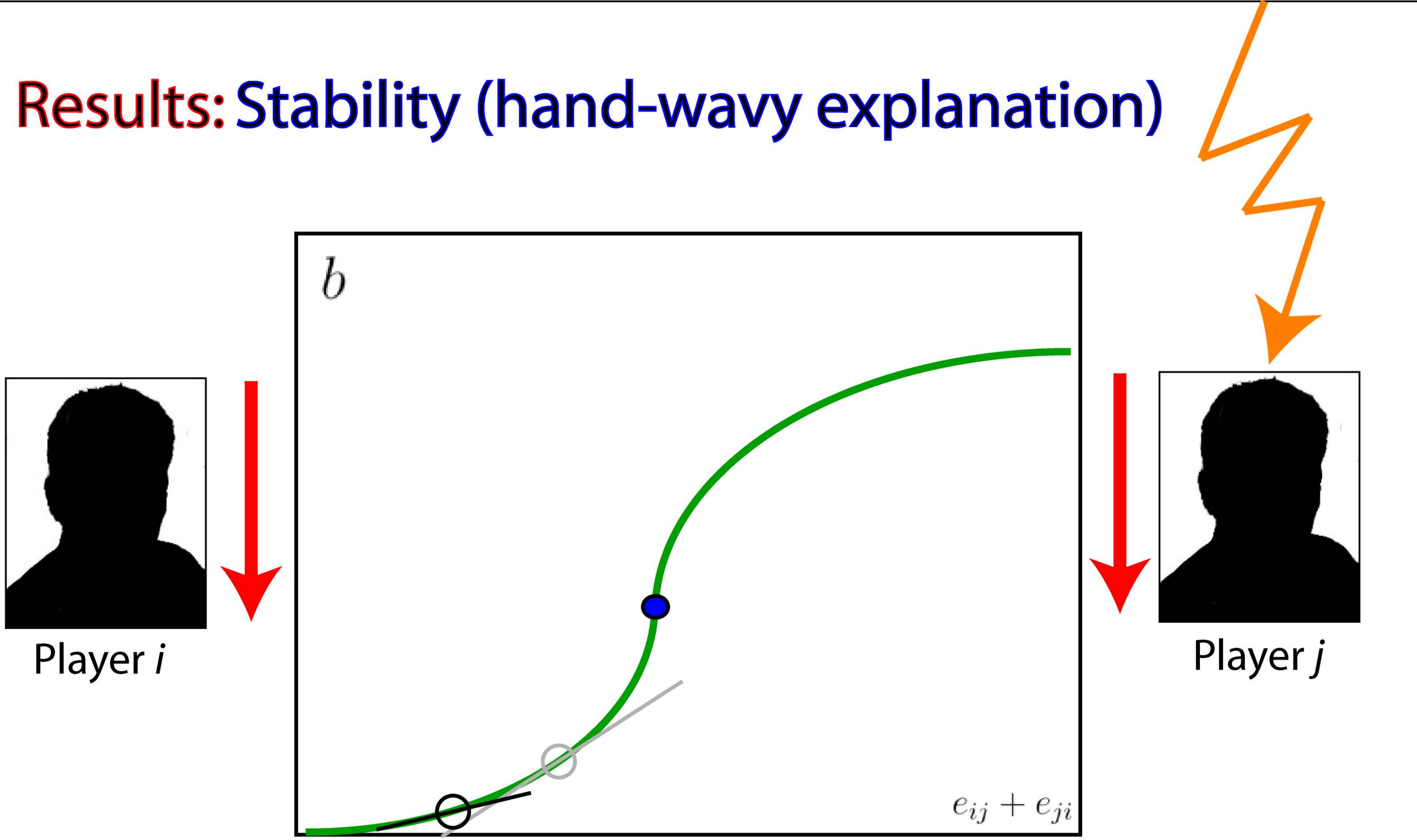


Player j

Results: Stability (hand-wavy explanation)



Results: Stability (rigorous)

Jacobi Signature Criterion:

For a symmetric matrix, the number of sign changes in the sequence of sub-determinants equals the number of negative eigenvalues.

$$\mathbf{J} = \begin{pmatrix} J_{11} & J_{12} & J_{13} & \cdots \\ J_{12} & J_{22} & J_{23} & \cdots \\ J_{13} & J_{32} & J_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad D_1,\ D_2,\ \boxed{D_3},\ \ldots$$

Do, Boccaletti, Gross, PRL 108,194102, 2012.

Results: Coordination

A bidirectionally connected network component (generally) approaches a steady state in which:

- all agents invest the same amount of resources
- all collaborations yield the same benefit

Results: Coordination

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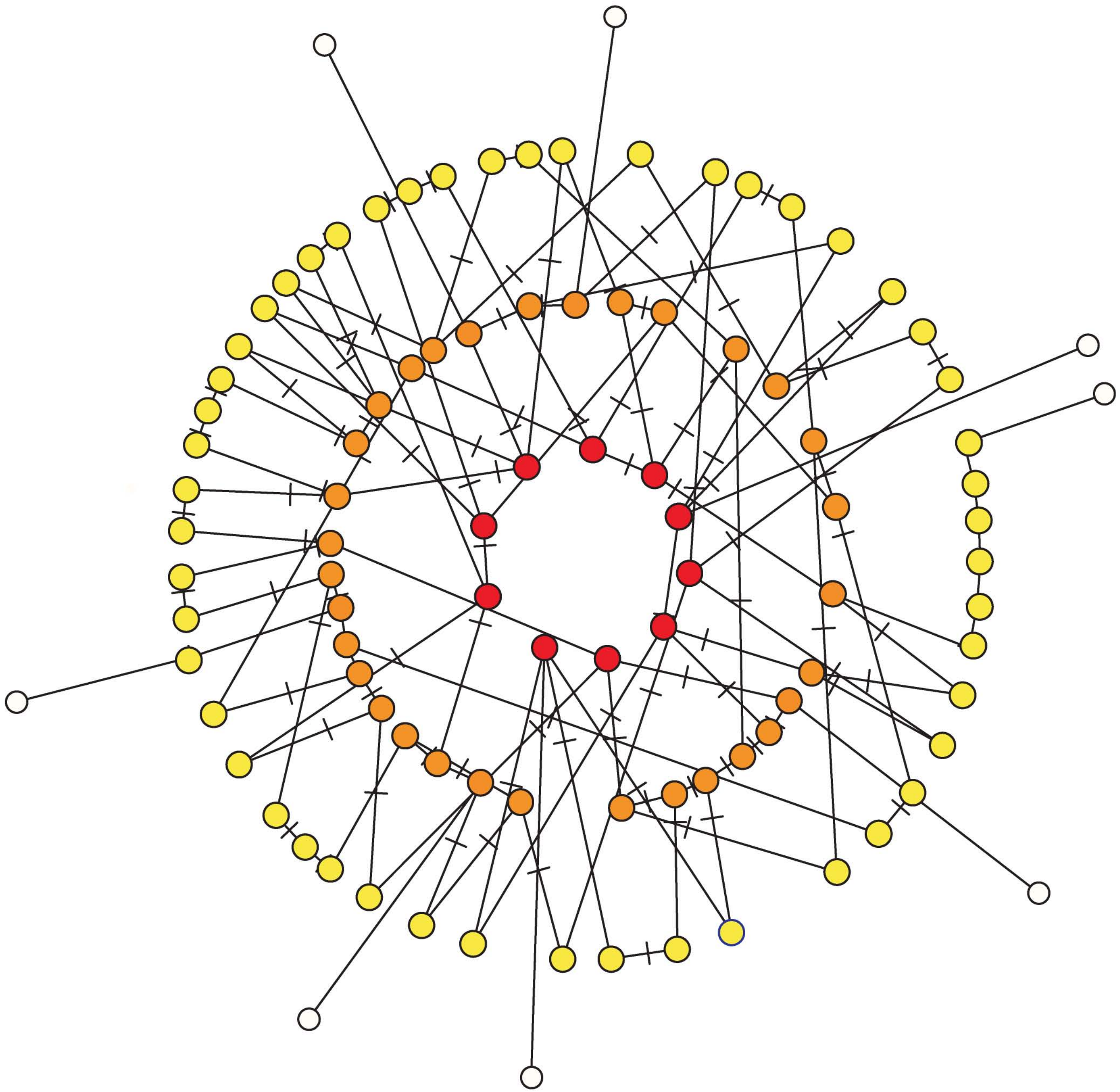
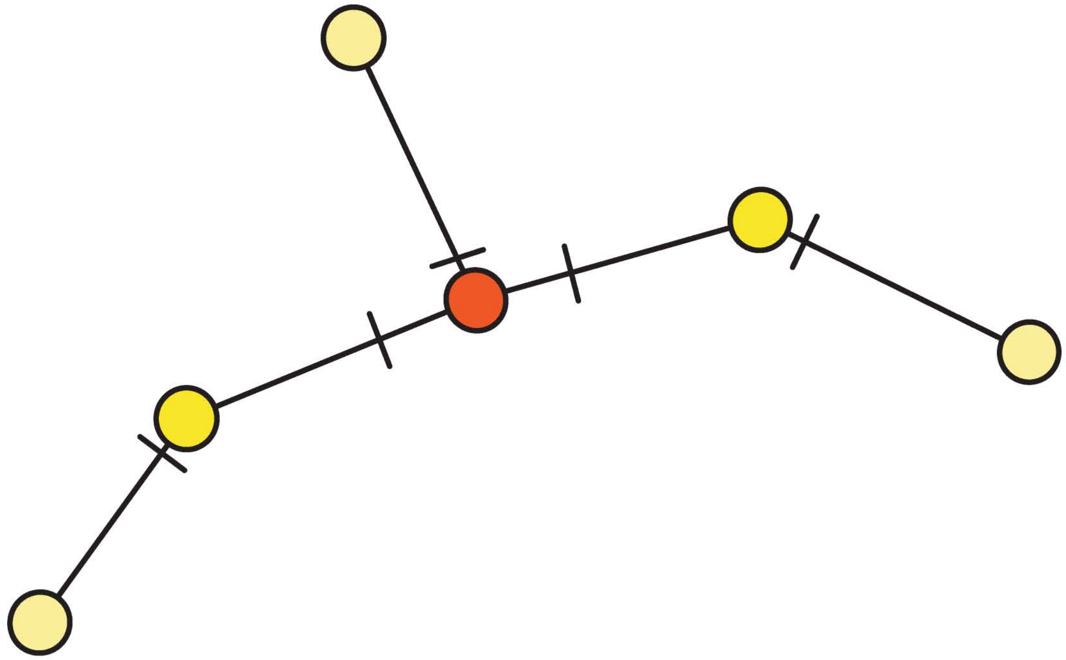
- all collaborations yield the same benefit

IS THIS FAIR ?

Results: Simulation



Results: Emergence of Leaders



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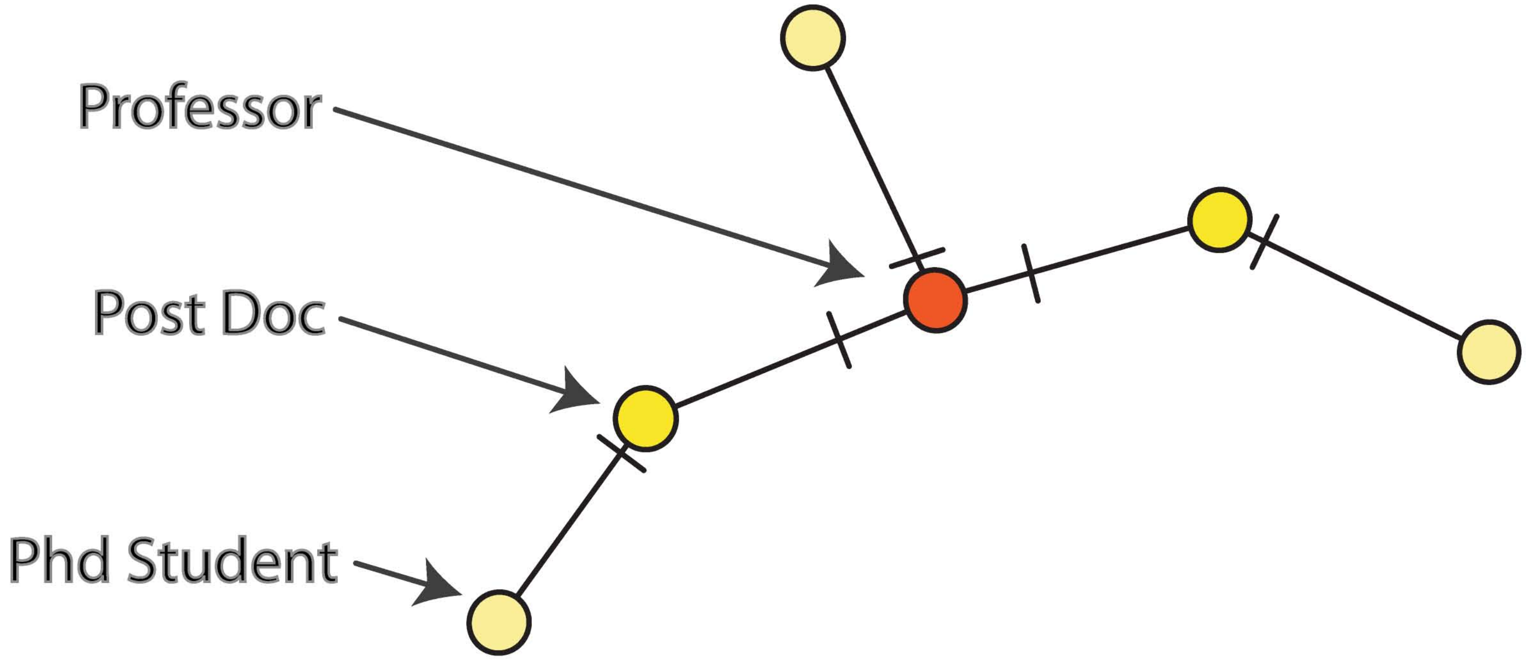

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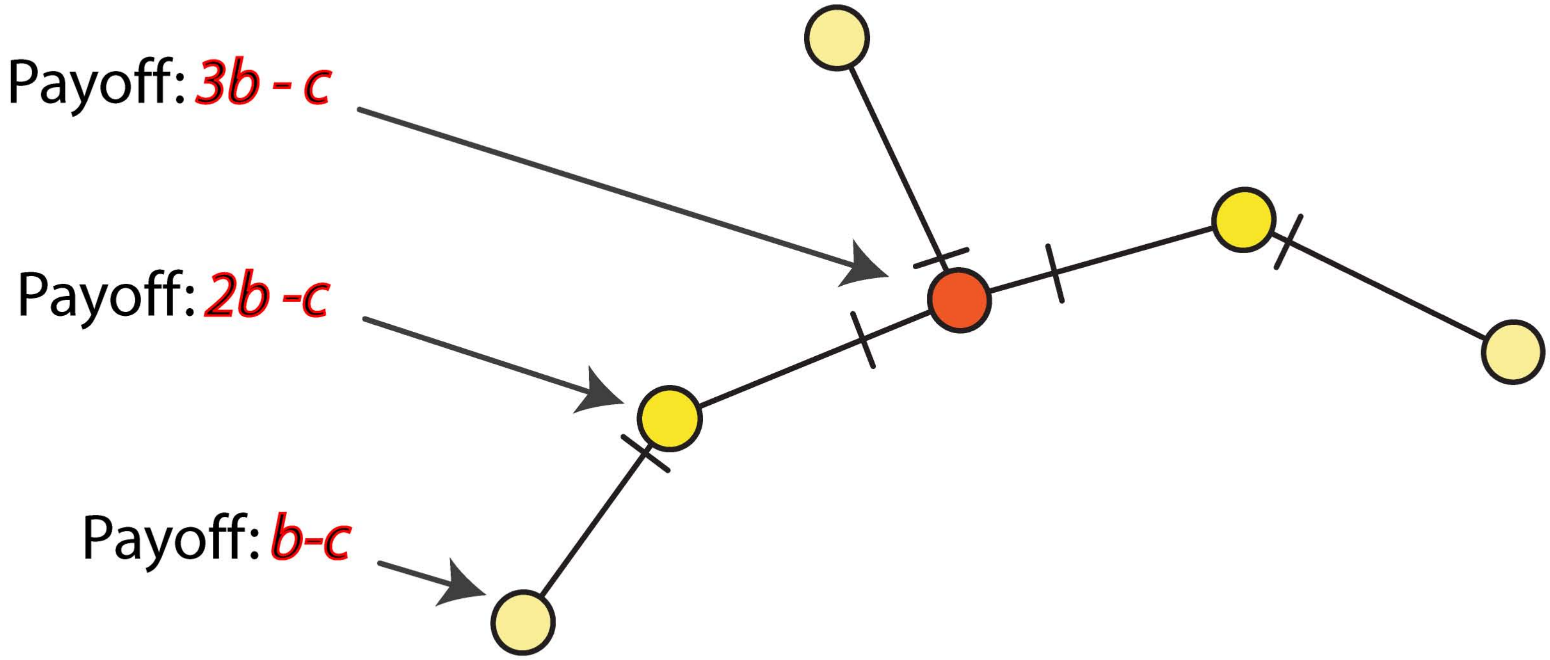

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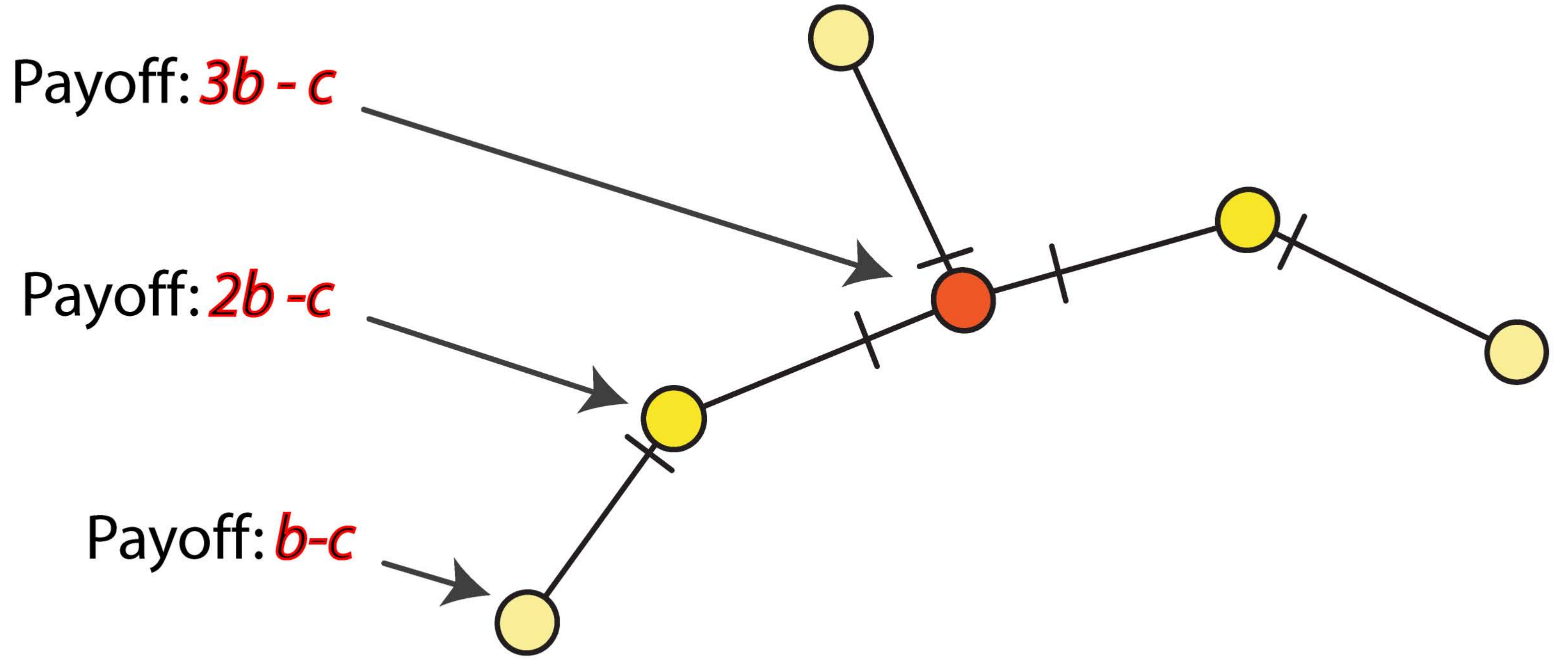
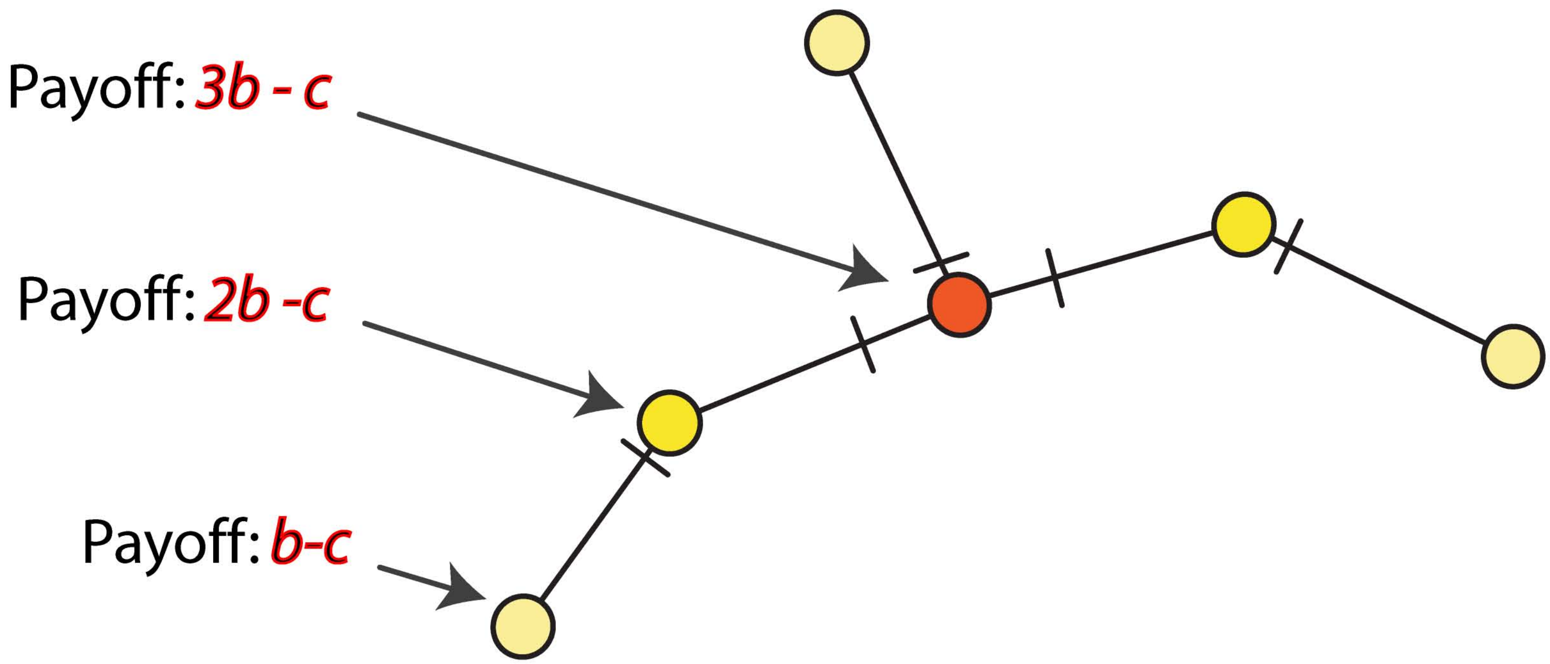


Agents holding previleged positions of high degree centrality extract significantly more payoff than average nodes.

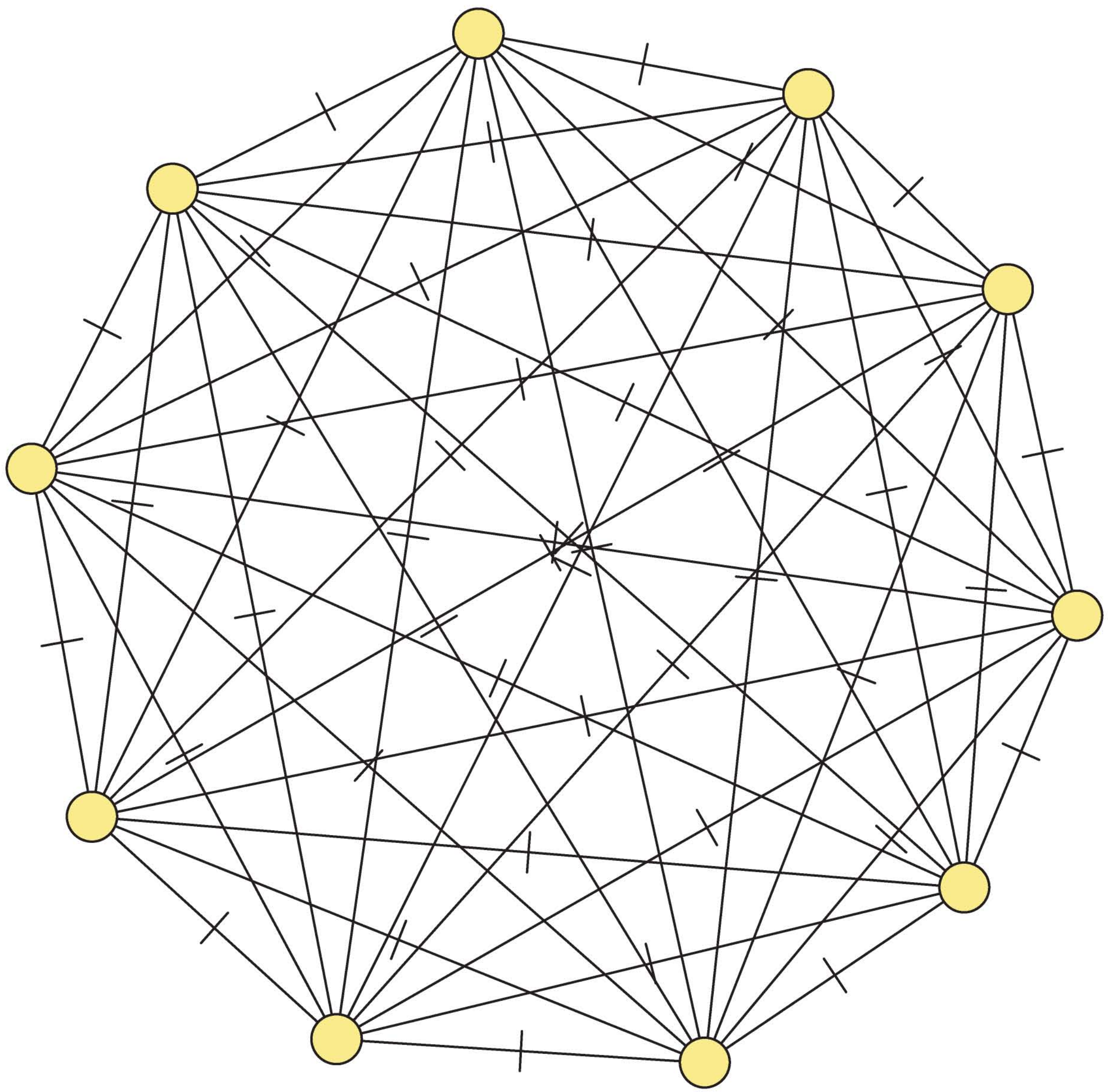
Results: Emergence of Leaders



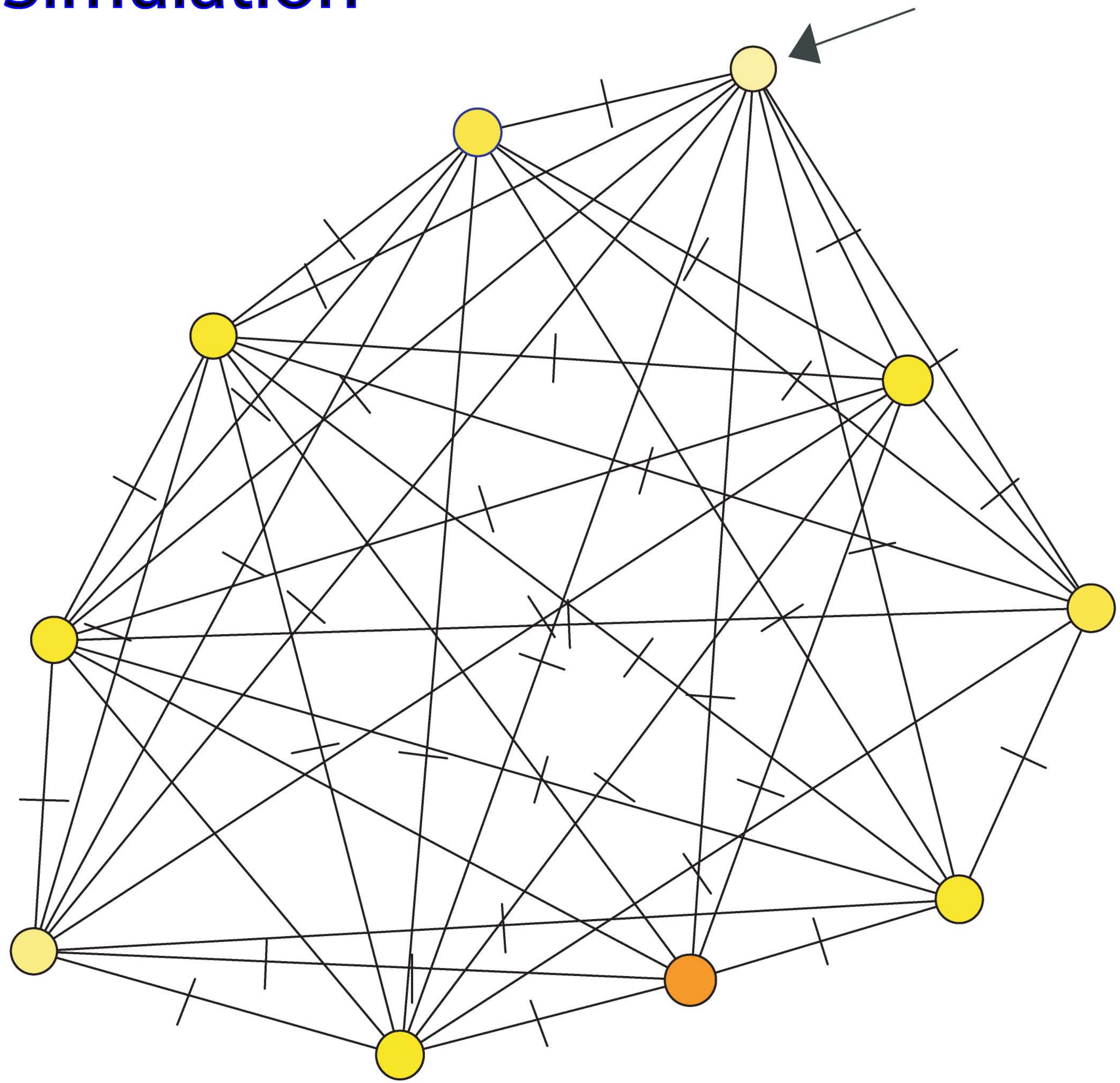
Agents holding previleged positions of high degree centrality extract significantly more payoff than average nodes.

Investments flow toward agents of high degree.

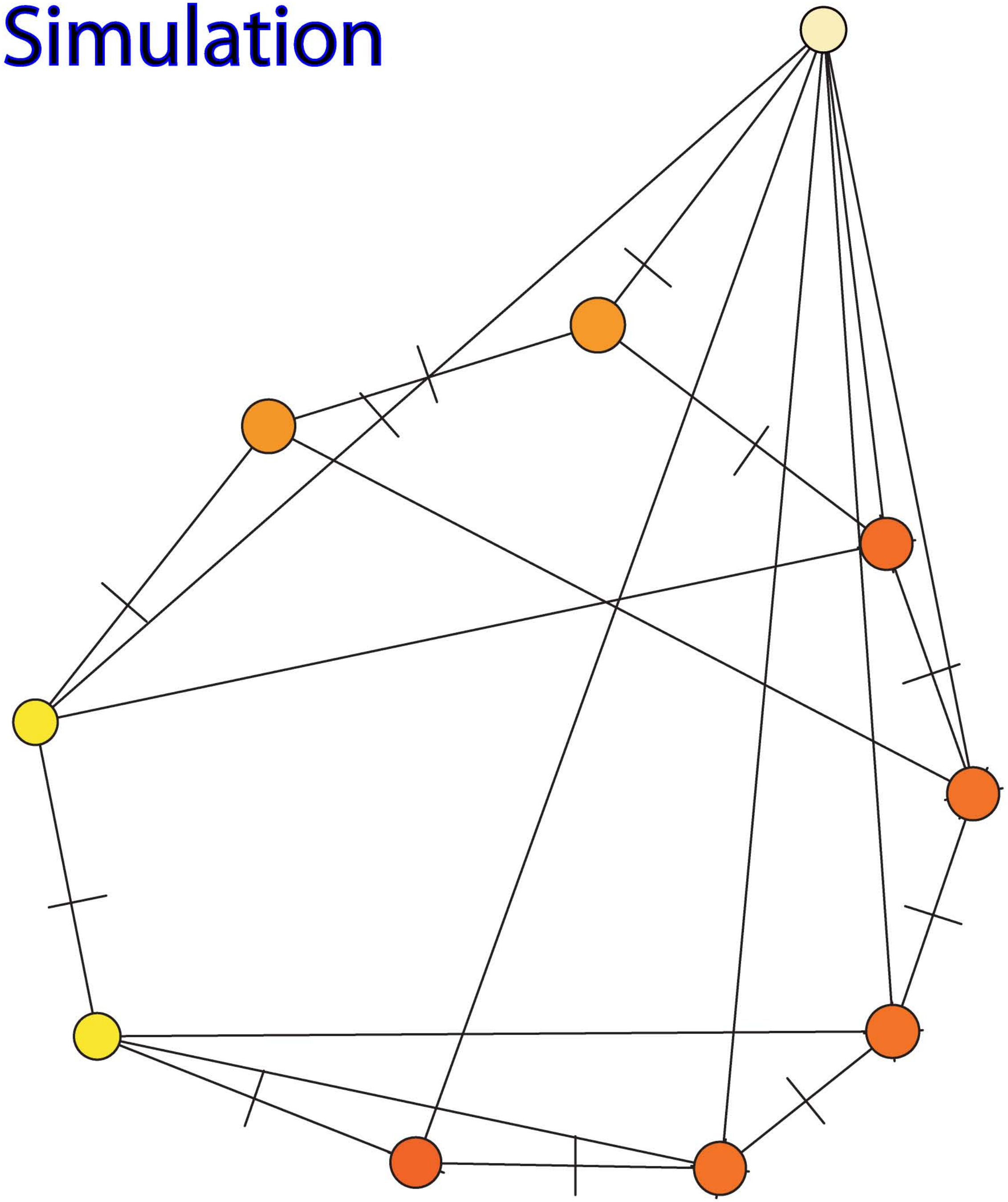
Results: Simulation



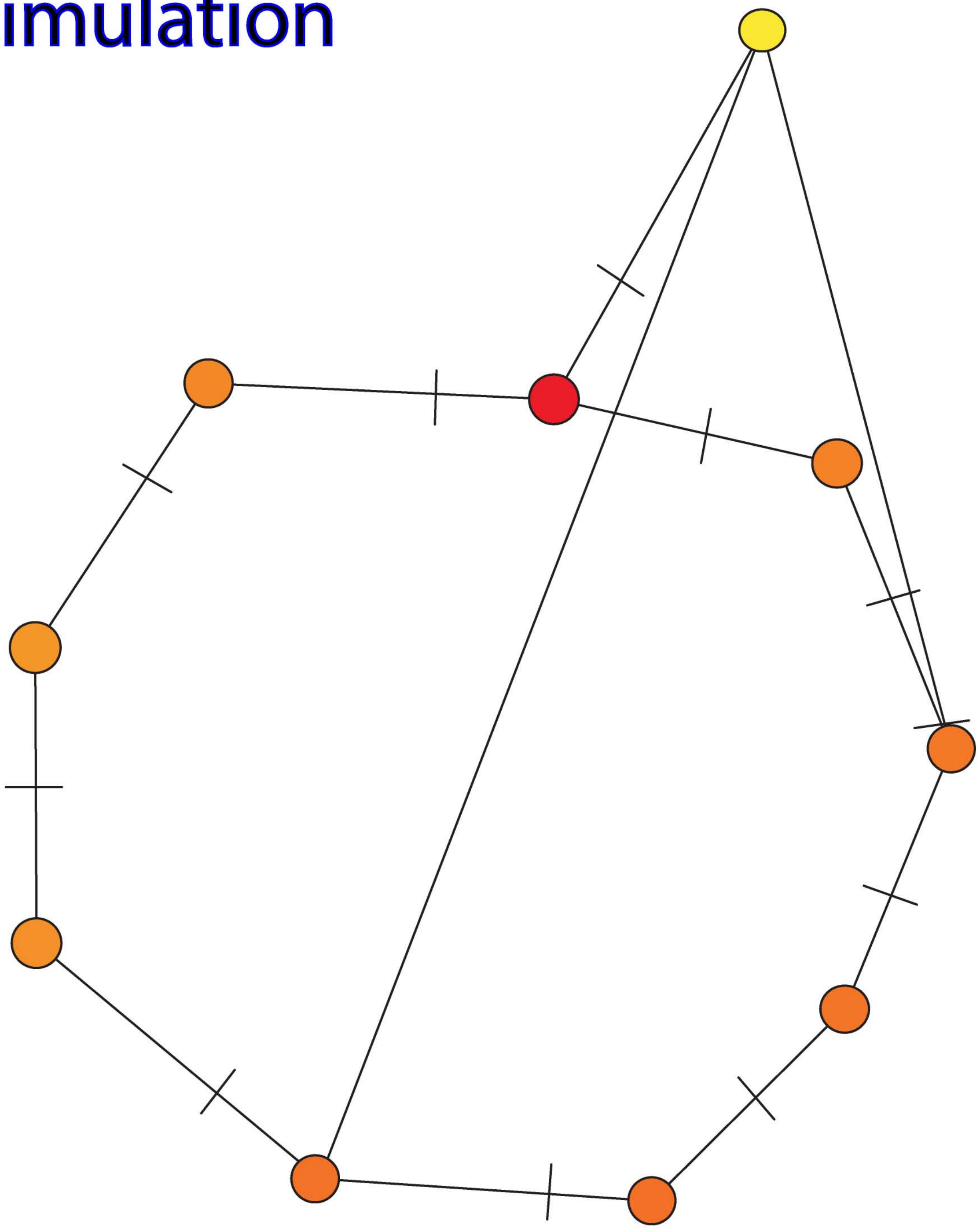
Results: Simulation



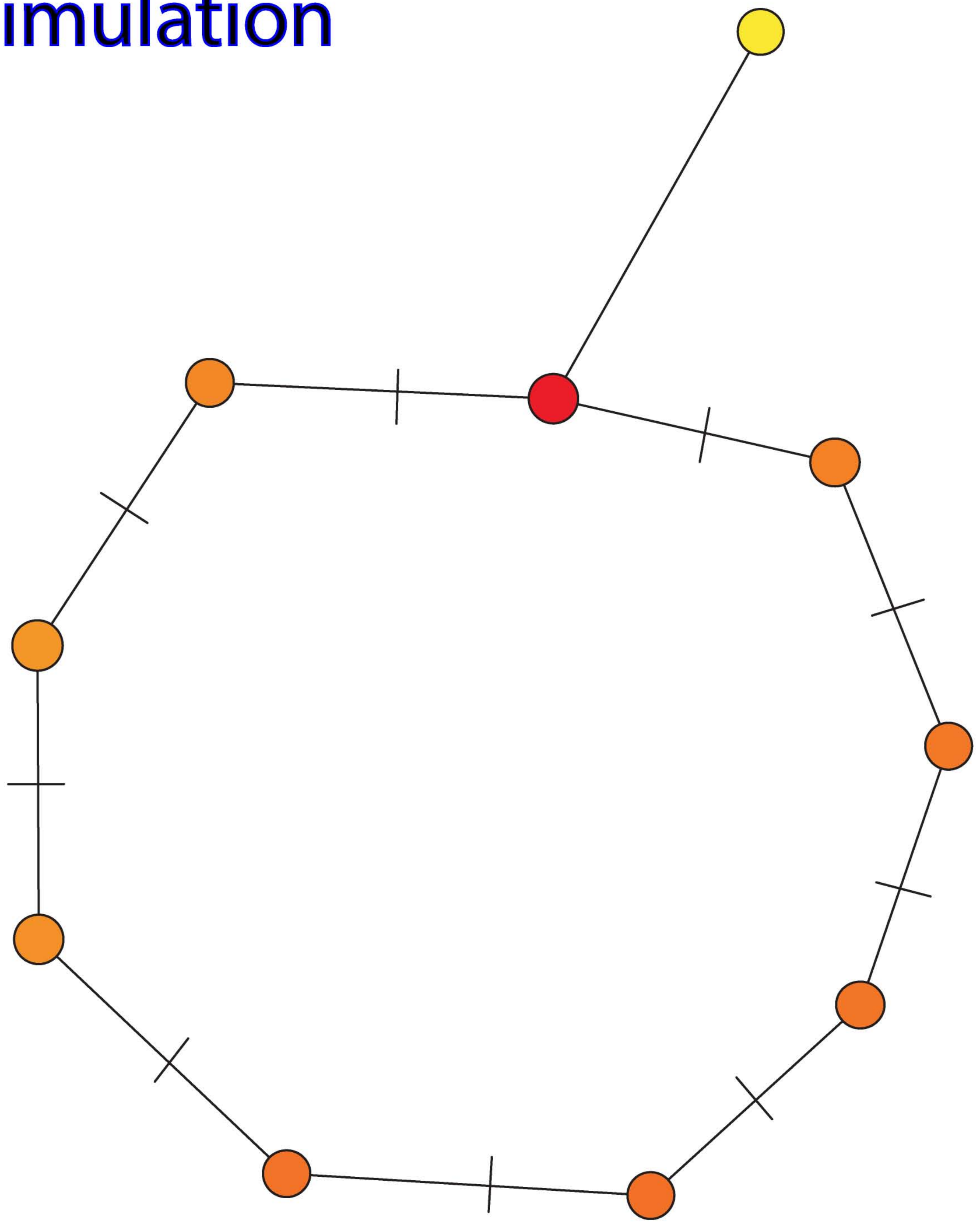
Results: Simulation



Results: Simulation



Results: Simulation



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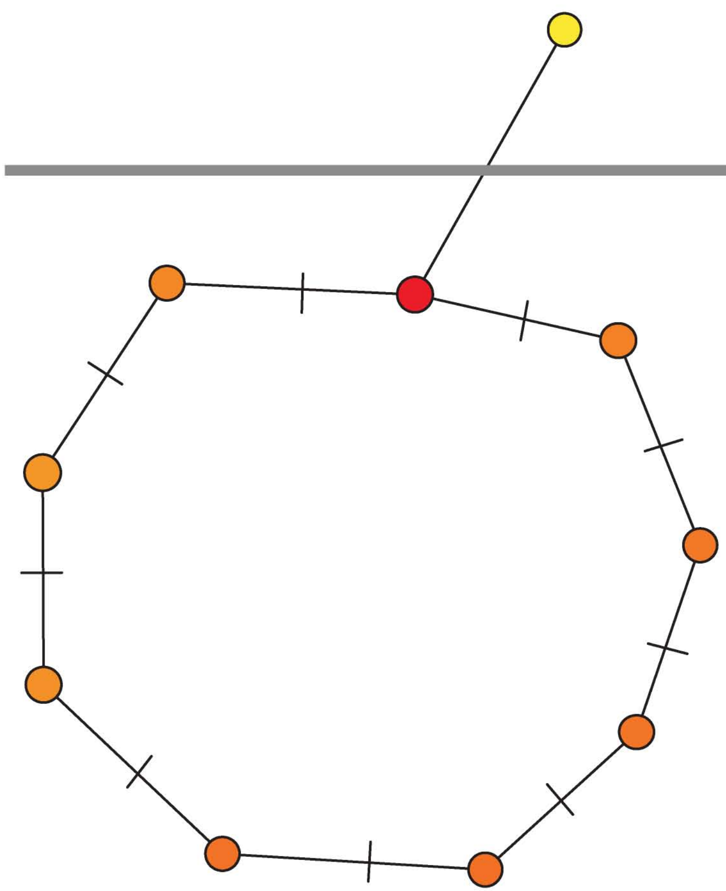


Benefit:	1	(1/20)
Cost:	1	(1/10)

Benefit:	19
Cost:	9

Collaboration Networks

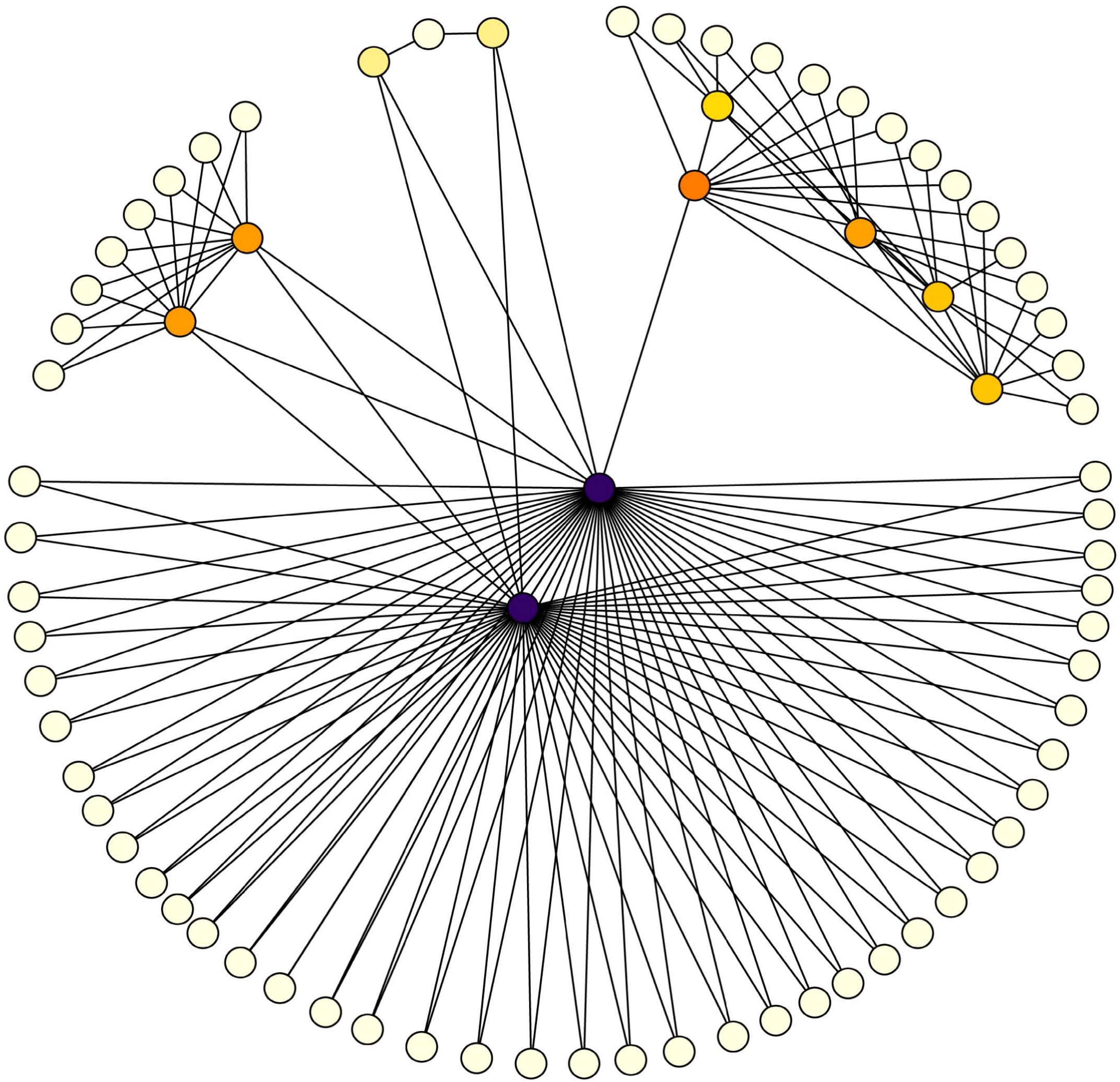
Wide degree distribution

Strong clustering

Cliques

...

Results: Growing Networks



Results: Growing Networks

Benefit: 0

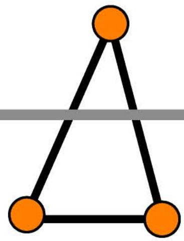
Cost: 0

Results: Growing Networks

Benefit: 1 (1/2)
Cost: 1 (1/2)

Benefit: 1
Cost: 1

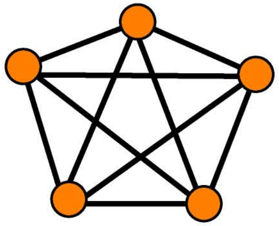
Results: Growing Networks



Benefit:	1	(1/3)
Cost:	1	(1/3)
Benefit:	2	
Cost:	2	

Results: Growing Networks

Benefit: 0
Cost: 0

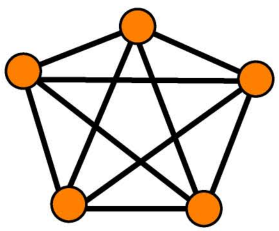


Benefit: 20
Cost: 5

Results: Growing Networks

4 Links ?

Benefit:	4
Cost:	1

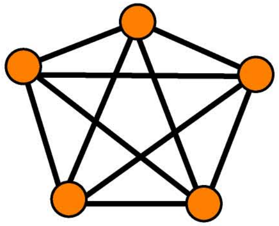


Benefit:	24
Cost:	5

Results: Growing Networks

3 Links ?

Benefit:	3	(9/78)
Cost:	1	(13/78)

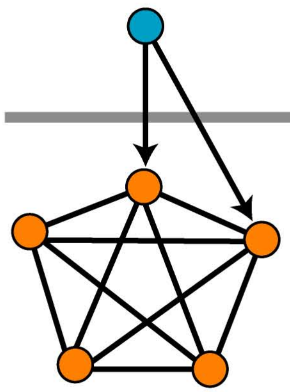


Benefit:	23
Cost:	5

(13/9 investment per link)

Results: Growing Networks

2 Links ?



Benefit:	2	(1/12)
Cost:	1	(1/6)

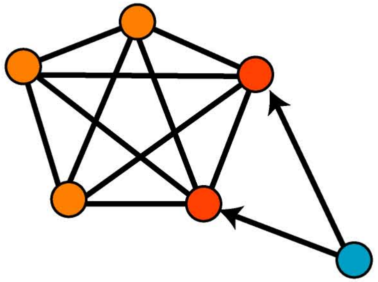
Benefit:	22
Cost:	5

(2 investment per link)

Results: Growing Networks

More of the same ?

Benefit: ?
Cost: ?

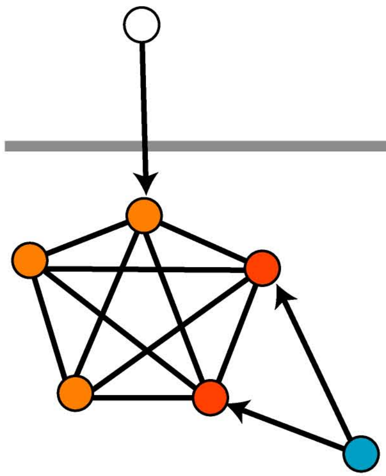


Benefit: 20
Cost: 5

(2 investment per link)

Results: Growing Networks

1 Link

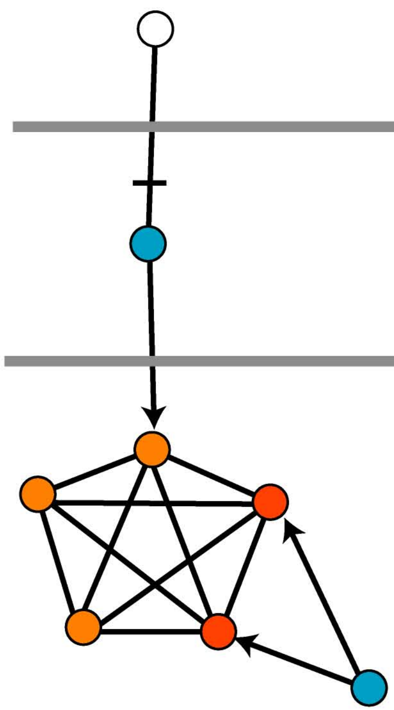


Benefit: <1
Cost: <1

Benefit: 20
Cost: 5

(2 investment per link)

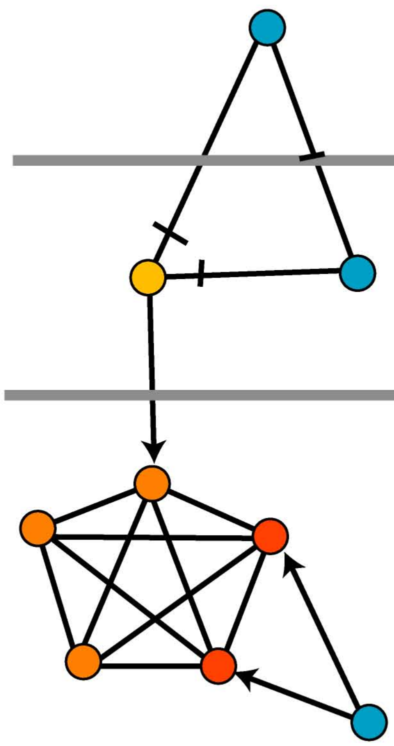
Results: Growing Networks



Benefit:	1	(1/3)
Cost:	1	(1/2)

Benefit:	2
Cost:	1

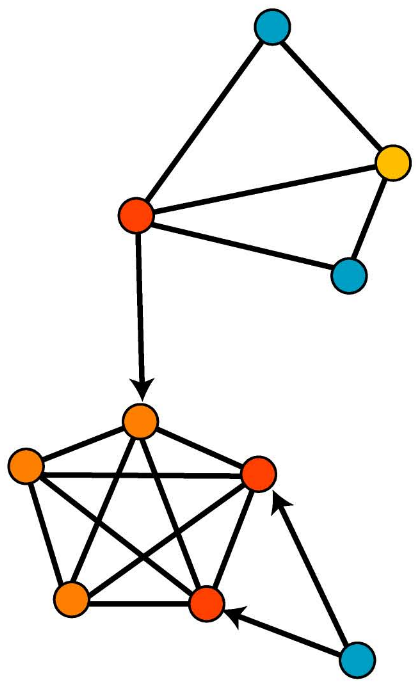
Results: Growing Networks



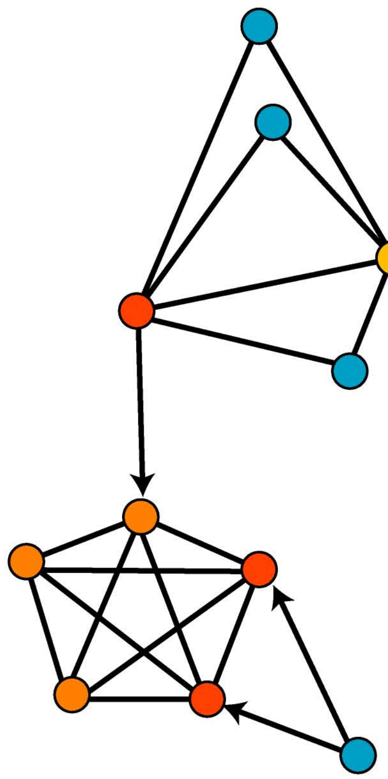
Benefit: 2 (6/21)
Cost: 1 (7/21)

Benefit: 5
Cost: 2

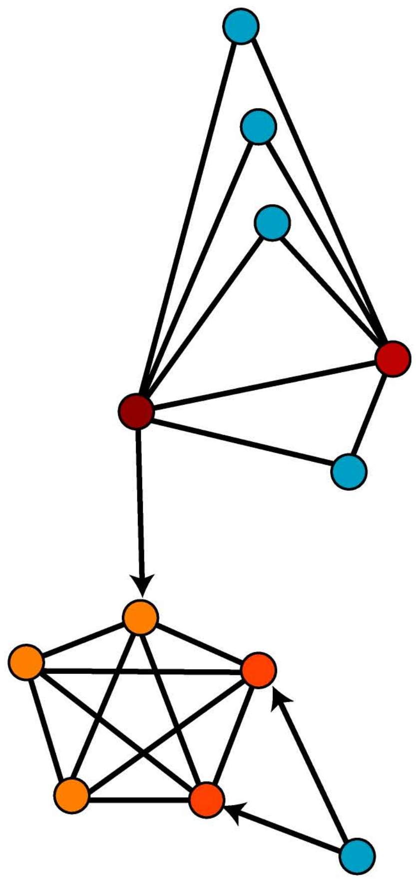
Results: Growing Networks



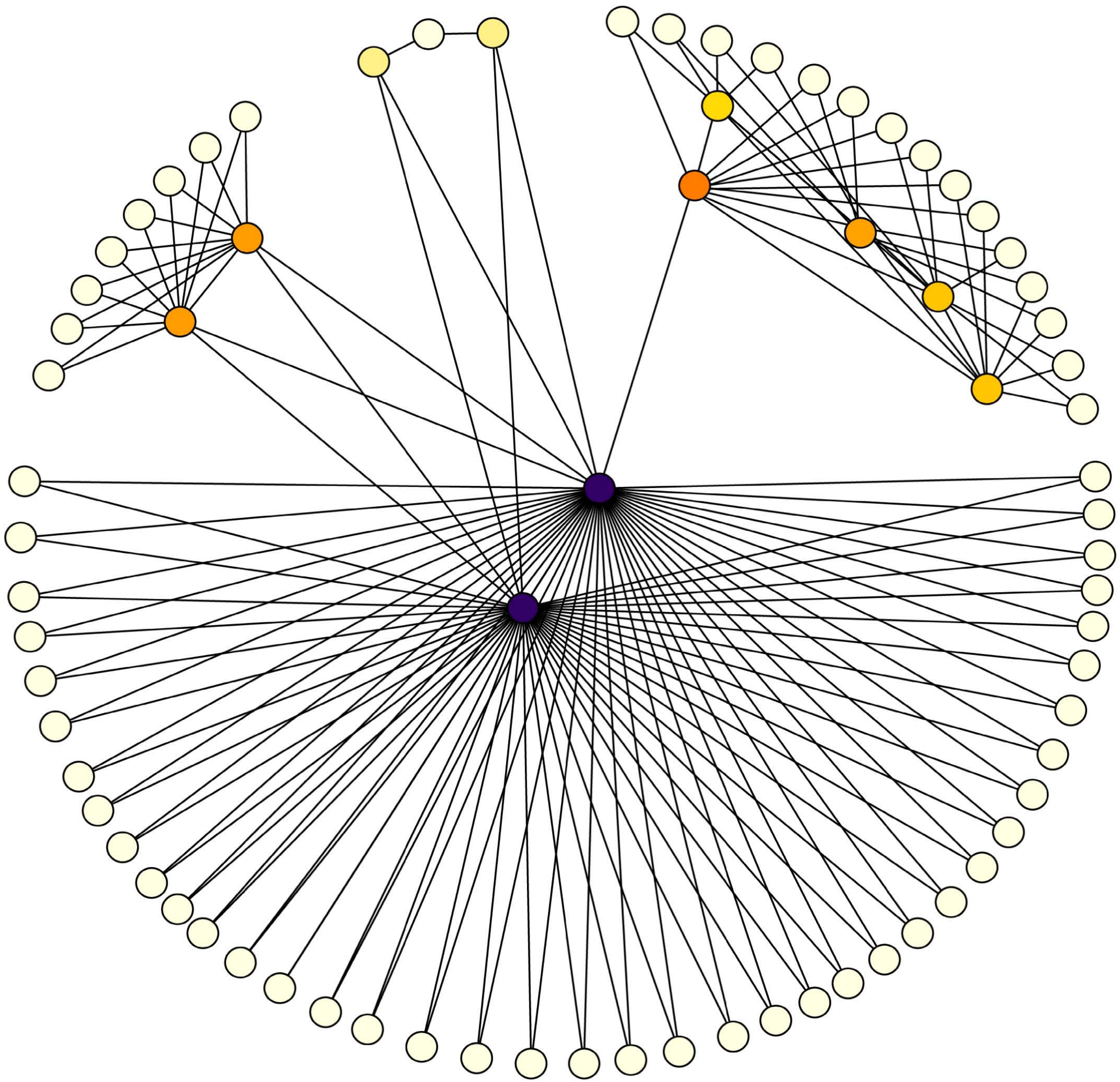
Results: Growing Networks



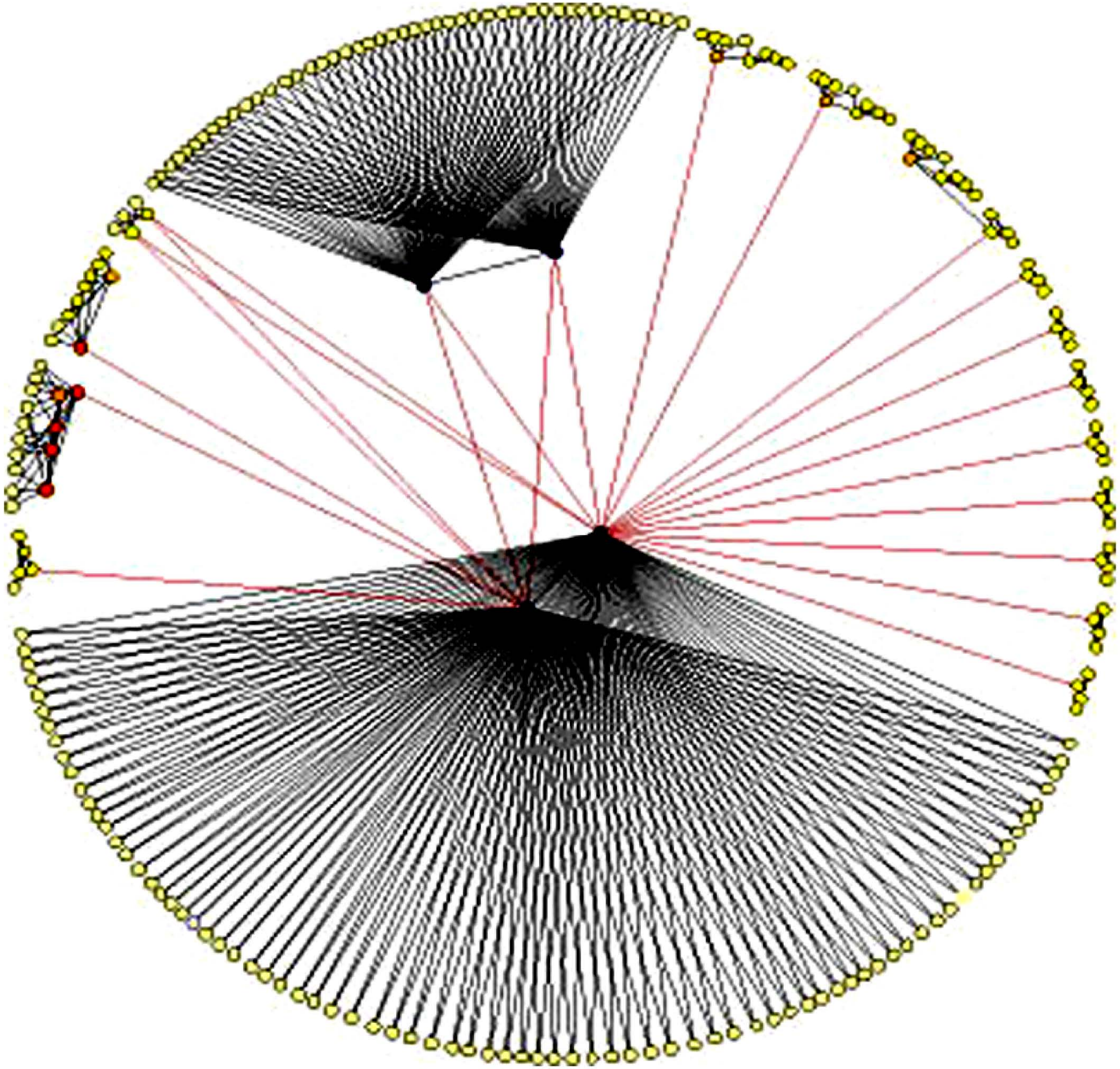
Results: Growing Networks



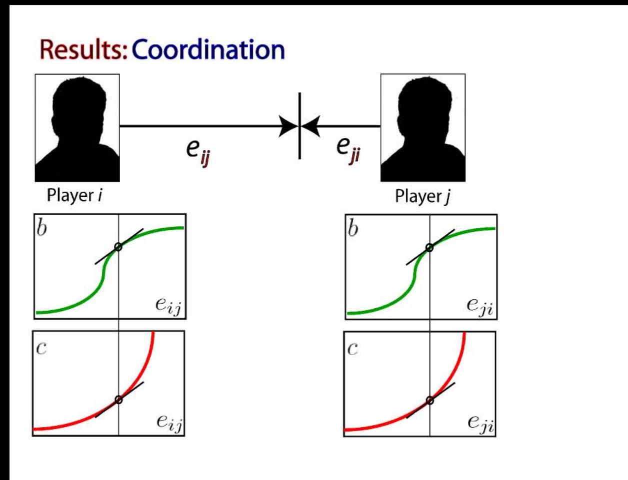
Results: Growing Networks

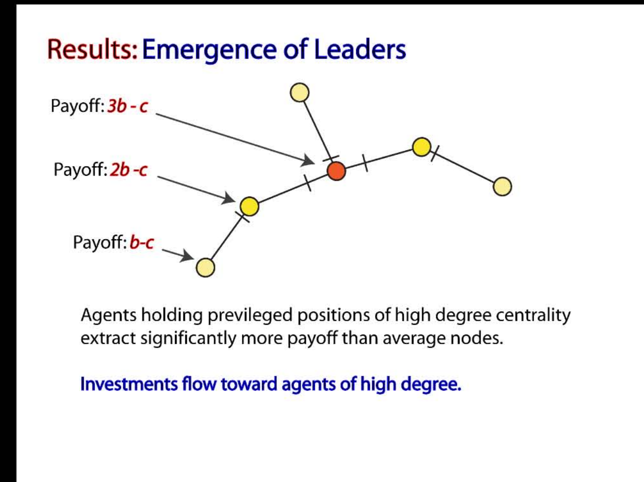


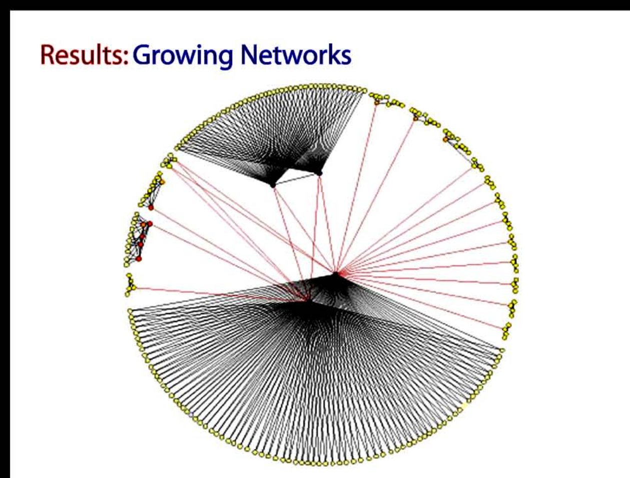
Results: Growing Networks

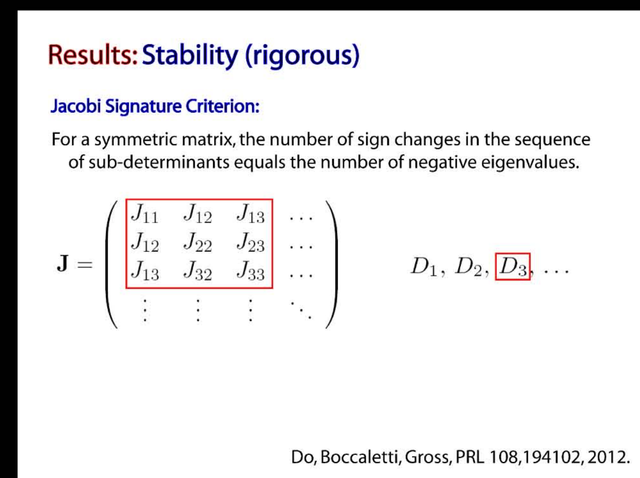


Conclusions









Do, Rudolf, Gross, NJP **12**, 063023, 2010

Do, Rudolf, Gross, Games **3**, 30, 2012

Do, Boccaletti, Gross, PRL **108**, 194102, 2012

Thank you very much for your attention !