A complex network perspective for time series analysis of dynamical systems

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SUMMARY

We review some recent results on two widely applicable transformations of dynamical systems to the complex network domain: recurrence networks (RNs) and visibility graphs (VGs). The structural characteristics of both types of networks can be interpreted in terms of distinct properties of the underlying dynamical system. Moreover, both can be understood as spatially embedded networks: RNs are random geometric graphs in the system's phase space, whereas VGs are ultimately associated with the one-dimensional time axis. The constructive (as well as potentially destructive) properties arising from this spatial embedding are discussed. As two prospective fields of application, we highlight the detection of hidden dynamical transitions from observational time series (RNs) and a possible surrogate data-free test for time-reversal asymmetry (VGs).

RECURRENCE NETWORKS

RNs encode the spatial backbone of recurrences in phase space and are defined by local proximity relationships between state vectors [R1-R3]:

$$A_{ij} = \Theta(\varepsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) - \delta_{ij}$$

Note that no time information is used, so that RNs are also applicable to data with non-uniform sampling [R4,R5]. However, constructing RNs from univariate time series requires time-delay embedding.

DETECTING DYNAMICAL TRANSITIONS

Global RN properties like average path length and transitivity are sensitive tracers of dynamical changes in complex systems such as

(1) Bifurcations in discrete and continuous systems (Fig. R1) [R1,R6]
(2) Transient bifurcations in discrete and continuous systems [R5]
(3) Regime shifts in geological records related to paleoclimate tipping points (Fig. R2) [R4,R5]

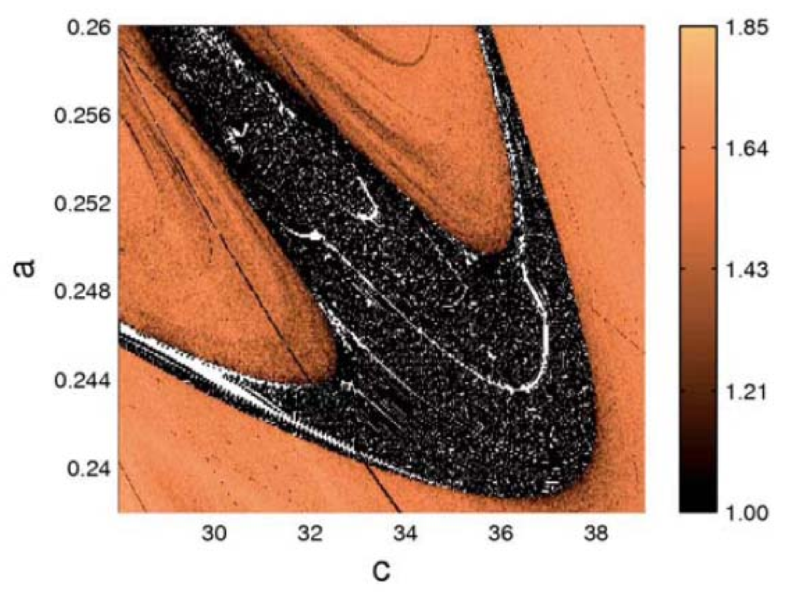

Fig. R1: Shrimp (complex periodic window in chaotic surrounding) in the two-dimensional parameter space (a=b) of the Rössler system encoded by the network transitivity dimension [R8].

EFFECTS OF SPATIAL EMBEDDING

RNs are random geometric graphs with a vertex density determined by the invariant density of the studied system [R7]. Their local and global properties can therefore be expressed analytically [R7,R8] and interpreted in terms of attractor properties like local and global dimensionality (clustering and transitivity dimensions [R8]). The parameter ε determines the scale of spatial resolution.

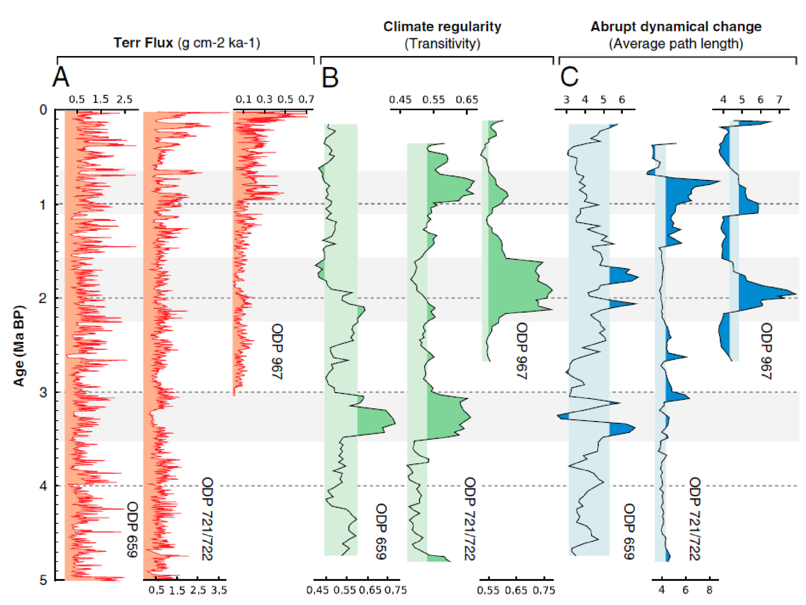

Fig. R2: Dynamical transitions in African dust flux records (A) identified by RN transitivity (B) and average path length (C) [R4].

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VISIBILITY GRAPHS

VGs are defined according to the local convexity properties of a time series: Two observations x_i, x_j (vertices) are linked *iff* they are "mutually visible" (see Fig. V1), resulting in the VG's adjacency matrix [V1]:

$$A_{ij}^{(VG)} = A_{ji}^{(VG)} = \prod_{k=i+1}^{j-1} \Theta\left(x_j + (x_i - x_j)\frac{t_j - t_k}{t_j - t_i} - x_k\right)$$

This does not require uniform sampling of the record [V2,V3].

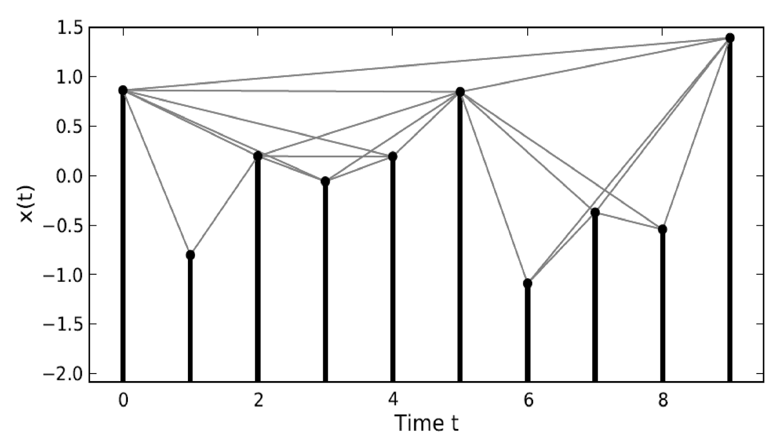

Fig. V1: Construction principle of a visibility graph.

TEST FOR TIME-REVERSAL ASYMMETRY

We define retarded (advanced) degrees and local clustering coefficients based on contributions from vertices exclusively from the past (future). The distributions of these characteristics are the same for linear processes, but display significant deviations for nonlinear systems (Fig. V3) (Kolmogorov-Smirnov test) [V3].

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EFFECTS OF SPATIAL EMBEDDING

By construction, VGs are embedded on the one-dimensional time axis. This strongly affects the resulting network properties [V4]:
(1) systematic downward bias of vertex properties towards the ends of a time series (Fig. V2)
(2) distributions of vertex characteristics may look qualitatively the same even for very different data sets

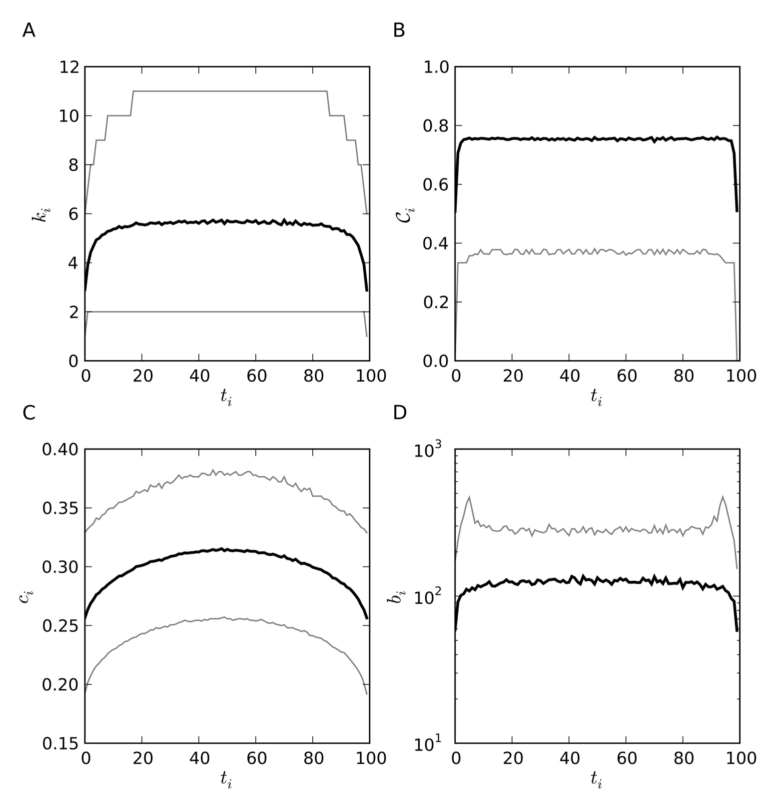

Fig. V2: Vertex properties (mean values and 10/90% confidence levels) for VGs constructed from 10,000 independent realizations of a Gaussian white noise time series (N=100): (A) degree, (B) local clustering coefficient, (c) closeness, (d) betweenness.

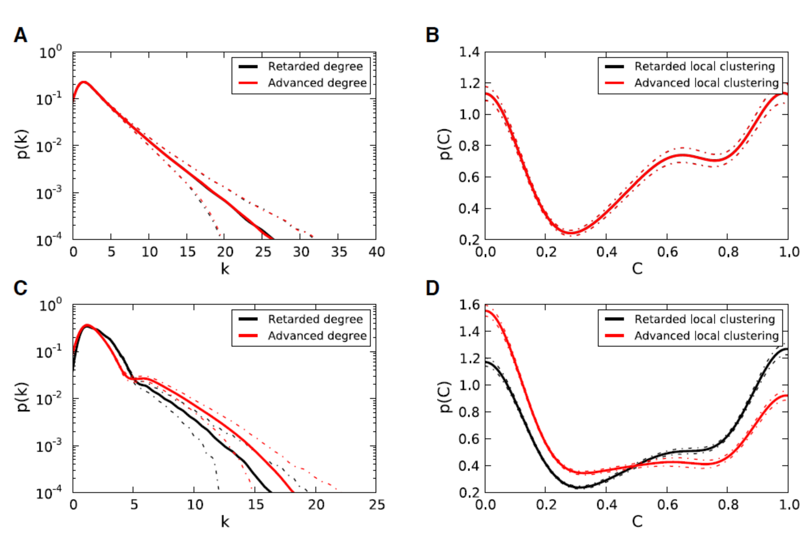

Fig. V3: Distributions of retarded/advanced (A,C) degree and (B,D) local clustering coefficient for (A,B) AR[1] processes and (C,D) first component of the Henon map (N=500). Mean (solid) and standard deviations from 1,000 realizations.

Leibniz Gemeinschaft

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ECONS Evolving Complex Networks