Synchronization of mobile oscillators

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Universitat de Barcelona

Complex networks

- Complex networks everywhere
- Nodes and links. Real or virtual.
- Something more
- New paradigms of complex networks

Multidimensional networks

- Social networks:
  - kinship networks
  - friendship
  - professional

Interconnected networks

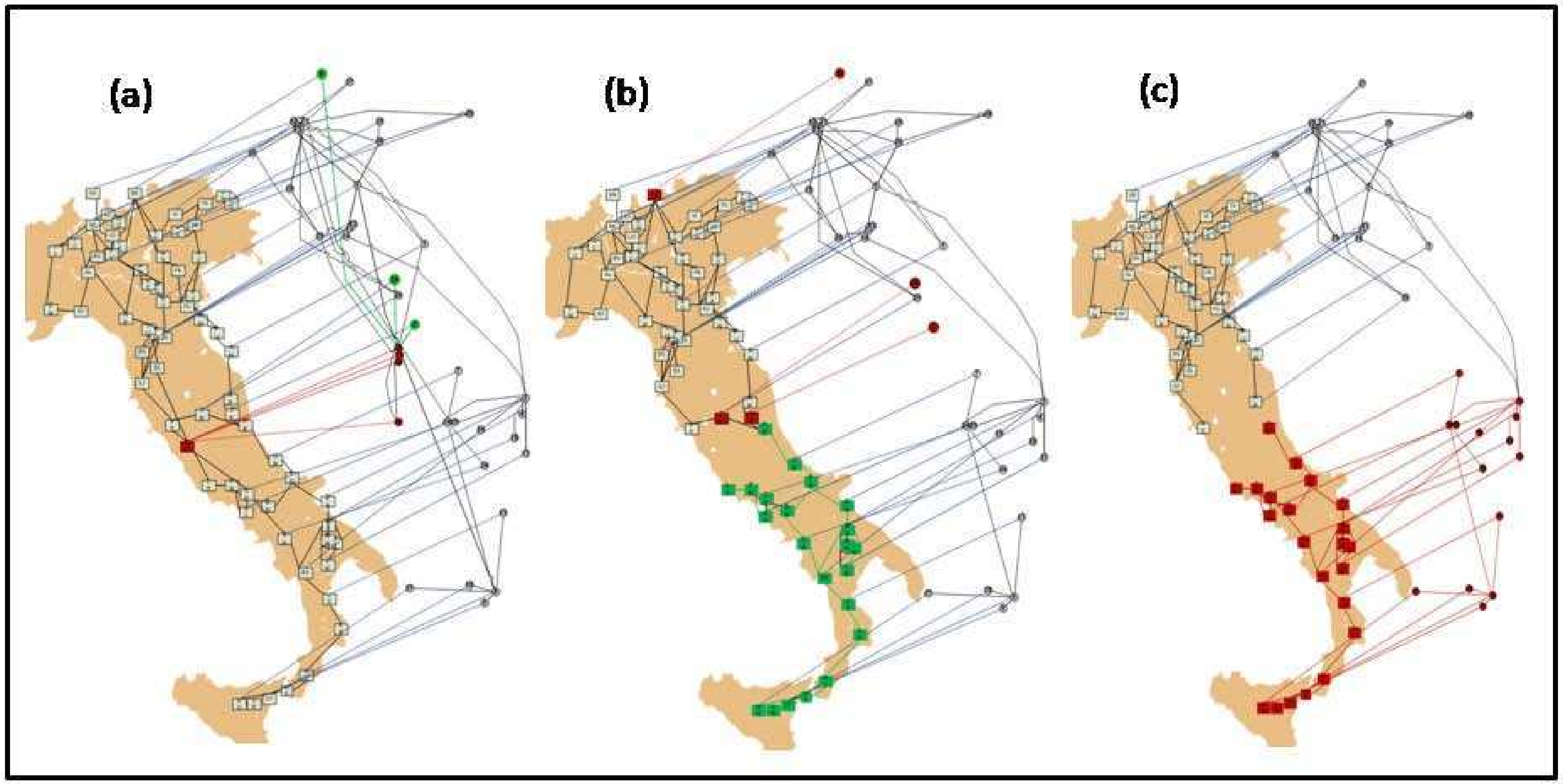
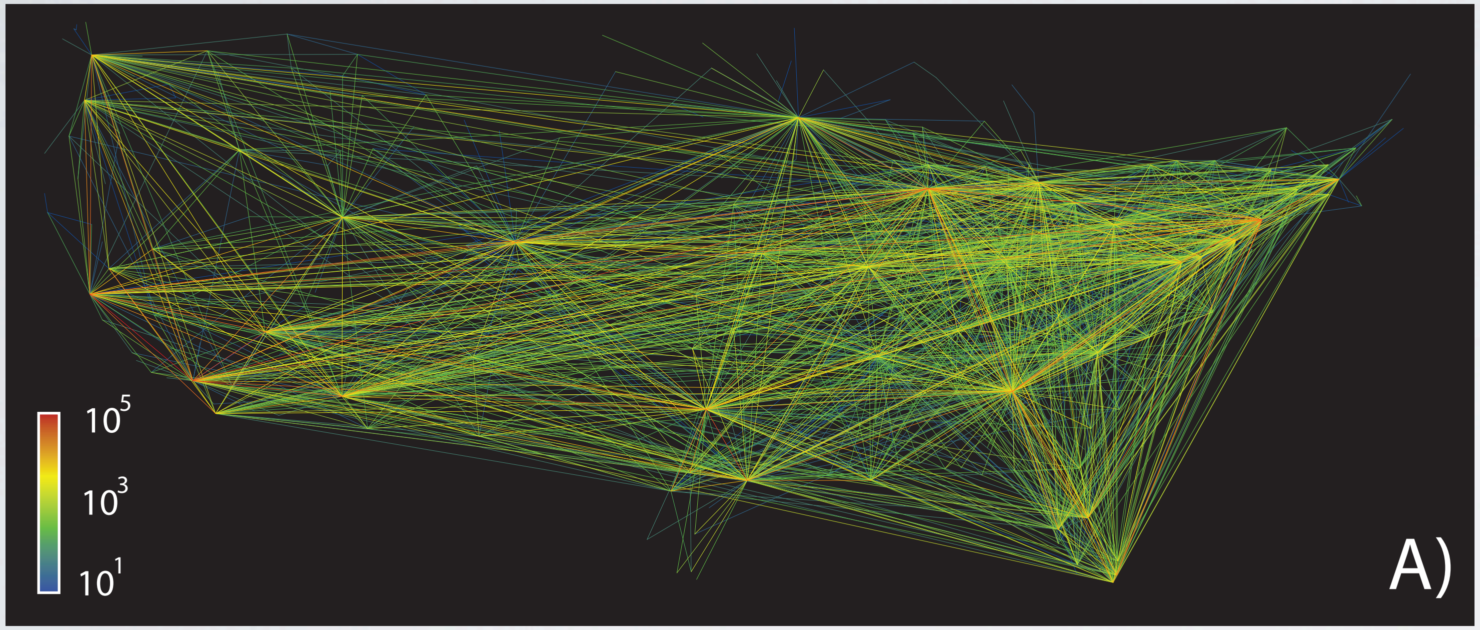


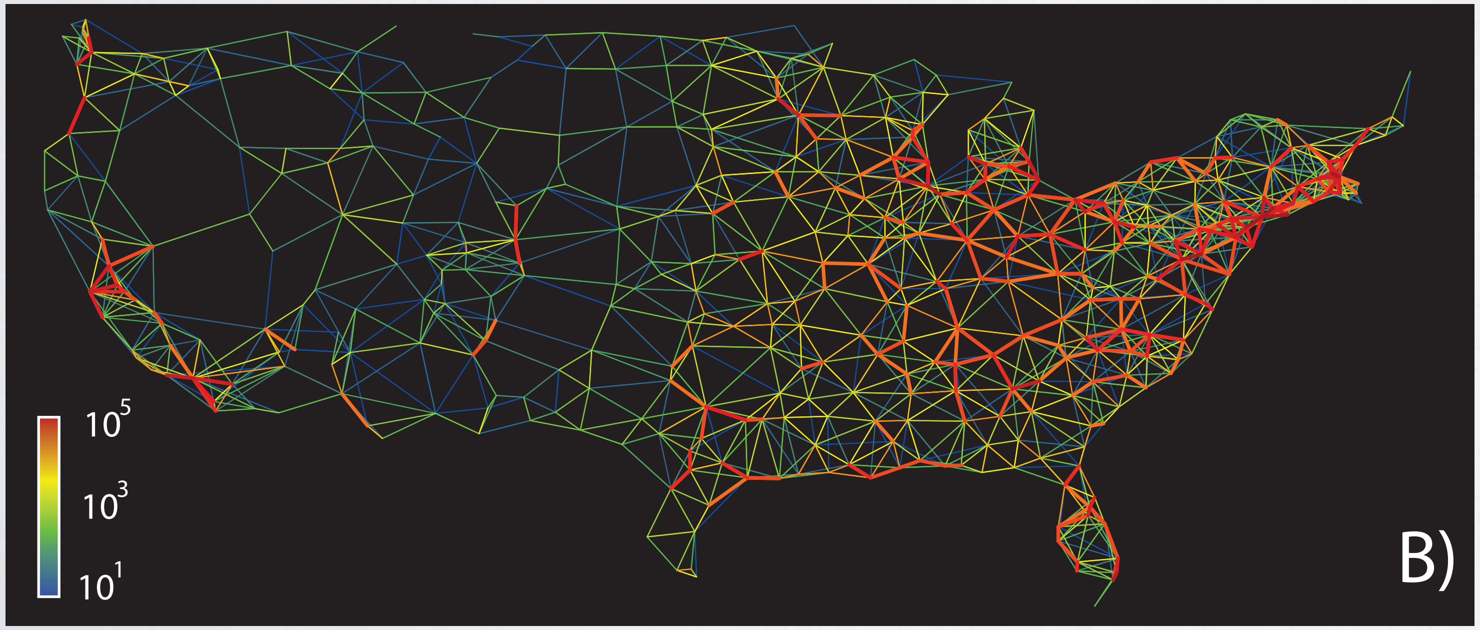
Fig. 2. – Cartoon of a typical cascade obtained by implementing the described model on the real coupled system in Italy. Over the map is the network of the Italian power network and, slightly shifted to the top, is the communication network. Every server was considered to be connected to the geographically nearest power station. (After Buldyrev *et al.* [15])

Buldyrev et al., Nature 464, 1025 (2010)

Network of networks



airline transportation network



commuting network

Balcan et al., PNAS 196, 21484 (2009)

Dynamics **OF** complex networks

- S.H. Strogatz, “Exploring complex networks”, Nature (2001) 410, 268

Strogatz 2001

- But networks are inherently difficult to understand, as the following list of possible complications illustrates.

1. **Structural complexity**: the wiring diagram could be an intricate tangle.
2. **Network evolution**: the wiring diagram could change over time. On the World-Wide Web, pages and links are created and lost every minute.
3. **Connection diversity**: the links between nodes could have different weights, directions and signs. Synapses in the nervous system can be strong or weak, inhibitory or excitatory.
4. **Dynamical complexity**: the nodes could be nonlinear dynamical systems. In a gene network or a Josephson junction array, the state of each node can vary in time in complicated ways.
5. **Node diversity**: there could be many different kinds of nodes. The biochemical network that controls cell division in mammals consists of a bewildering variety of substrates and enzymes.
6. **Meta-complication**: the various complications can influence each other. For example, the present layout of a power grid depends on how it has grown over the years — a case where network evolution (2) affects topology (1). When coupled neurons fire together repeatedly, the connection between them is strengthened; this is the basis of memory and learning. Here nodal dynamics (4) affect connection weights (3).

Complex networks with time dependent topology

- **Many examples of changing topology network in real systems**

  ・**social network**: J.-P. Onnela et al., PNAS 104, 7332 (2007)

  ・**brain network**: M. Valencia et al., Phys. Rev. E 77, 050905R (2008)

  ・**human mobility**: M.C. González et al., Nature 453, 779(2008);L. Isella et al. PLoS ONE 6 (2011)e17144

Complex networks with time dependent topology

- **Synchronization in time dependent networks is important**
  - **mobile devices** (e.g. bluetooth): M Maróti et al., Proc. 2^{nd} ACM Conf, 39(2004)
  - **consensus**: R. Olfati-Saber, J. A. Fax, R. M. Murray, Proceedings IEEE 95, 215 (2007)

Complex networks with time dependent topology

- **Spreading in communication networks**: M. Karsai et al., Phys. Rev. E 83 (2011) 1

Contact networks: SocioPatterns

What's in a crowd? Analysis of face-to-face behavioral networks.

L. Isella et al.

J. Theor. Bio. 271 (2011) 166

Recent review

- Temporal networks, P. Holme and J. Saramaki, arxiv:1108.1780

Topology affects emergent collective properties SYNCHRONIZATION

- One of the paradigmatic examples of emergent behavior
- **Engineering**: consensus, unmanned vehicle motion
- **Nature**: flashing fireflies, brain
- **Society**: people clapping, Millenium bridge

Synchronization in complex nets

- **Review**

Interplay between topology and dynamics

A. Arenas, A.D.-G., J. Kurths, Y. Moreno, C. Zhou, Phys. Rep. 469, 93 (2008)

- **Spectral properties of Laplacian matrix**
    - **Synchronizability = eigenratio** λ_n/λ_2

      Master Stability Function:

      M. Barahona and L. M. Pecora, Phys. Rev. Lett. 89, 054101 (2002)
      N. Fujiwara, and J. Kurths, Eur. Phys. J. B 69, 45 (2009)

    - **Time to synchronize =** $1/\lambda_2$

      J. Almendral, A.D-G, New J. Phys. 9, 187 (2007)

- **Network topology is fixed**

Synchronization in complex nets

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- **Network topology is fixed**

What happens if topology changes in time?
Is spectral approach possible?

Fast switching (mean field) approximation

- Approximation when the **time scale of the agents' motion** is much shorter than that of the oscillator dynamics
  M. Frasca, A. Buscarino, A. Rizzo, L. Fortuna, and S. Boccaletti, Phys. Rev. Lett. 100, 044102 (2008)

- Replace the time-dependent **Laplacian** matrix L(t) with its **time average** <L>, whose matrix element is the probability that two agents are connected

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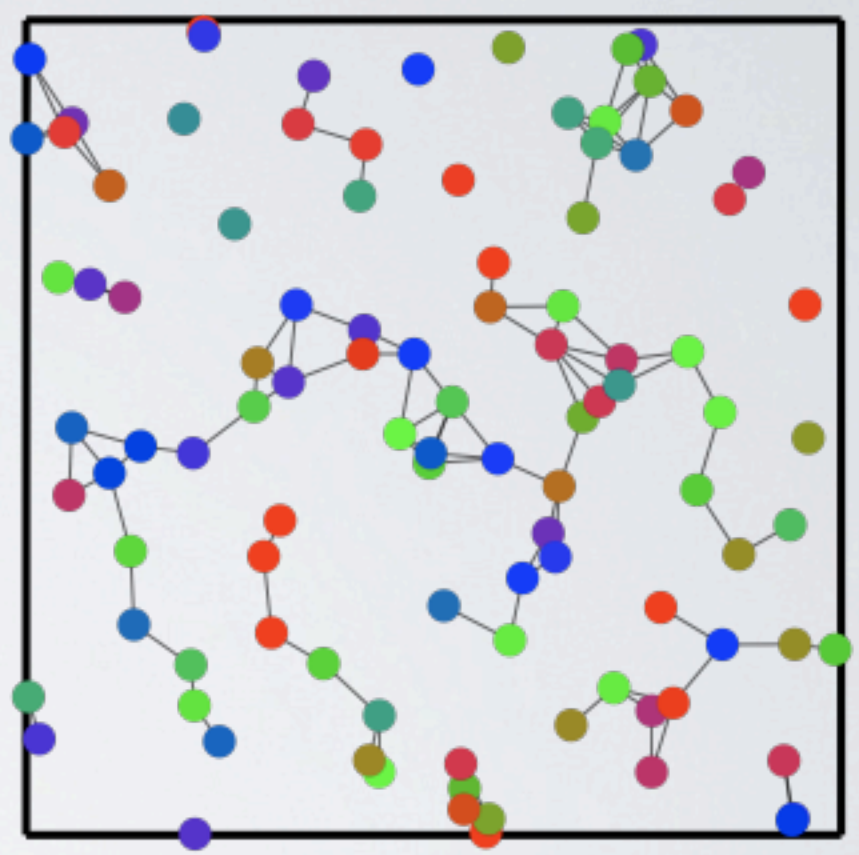
When synchronization is much faster than the motion of agents, we get local synchronization of spatial clusters

Model

N. Fujiwara, J. Kurths, A.D-G, PRE (2011)

- **Network topology**: N, L, d

  Instantaneous topology:
  continuum percolation
  (random geometric graph)



- **Agent dynamics**: v, τ_M

$$
\begin{aligned}
x_i(t_i^k + \Delta t) &= x_i(t_i^k) + v \cos \xi_i(t_i^k) \Delta t \mod L \\
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\end{aligned}
$$

$$t_i^{k+1} - t_i^k = \tau_M$$

- **Oscillator dynamics**: σ, τ_P

$$\varphi_i(t + \tau_P) = \varphi_i(t) + \sum_{j=1}^{N} \sigma(d_{ij}) \sin(\varphi_j(t) - \varphi_i(t))$$

$$\sigma(d_{ij}) = \begin{cases} \sigma & (d_{ij} < d) \\ 0 & (d_{ij} > d) \end{cases}$$

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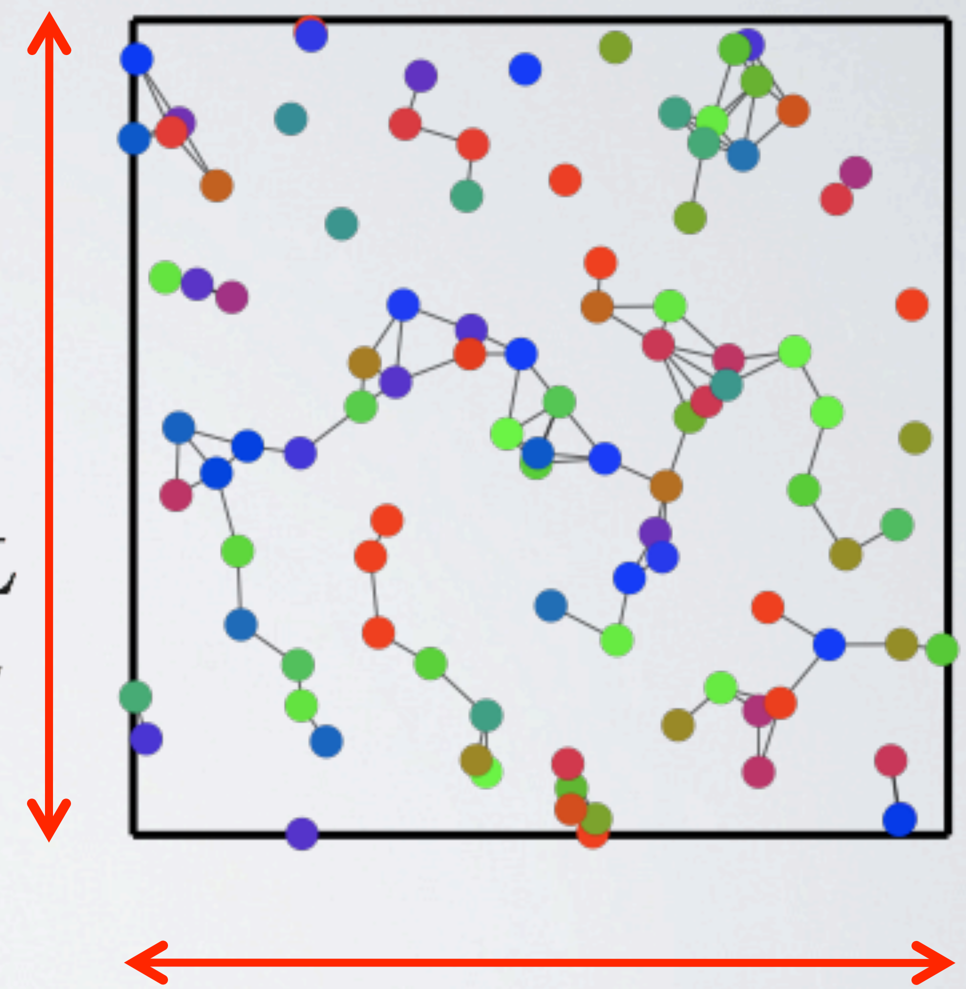

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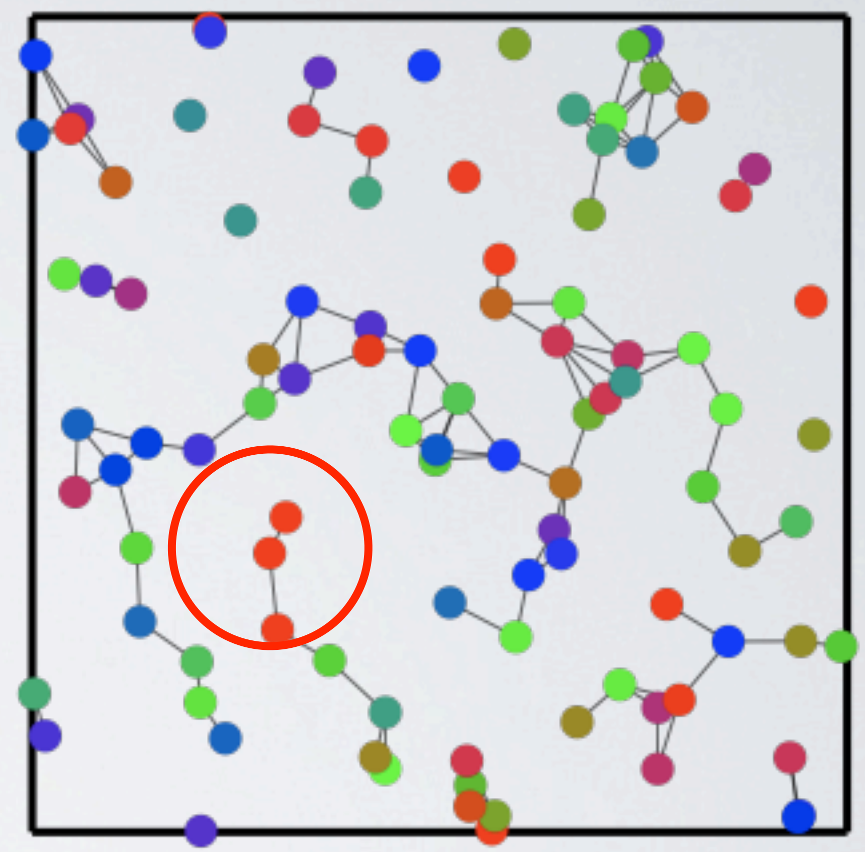

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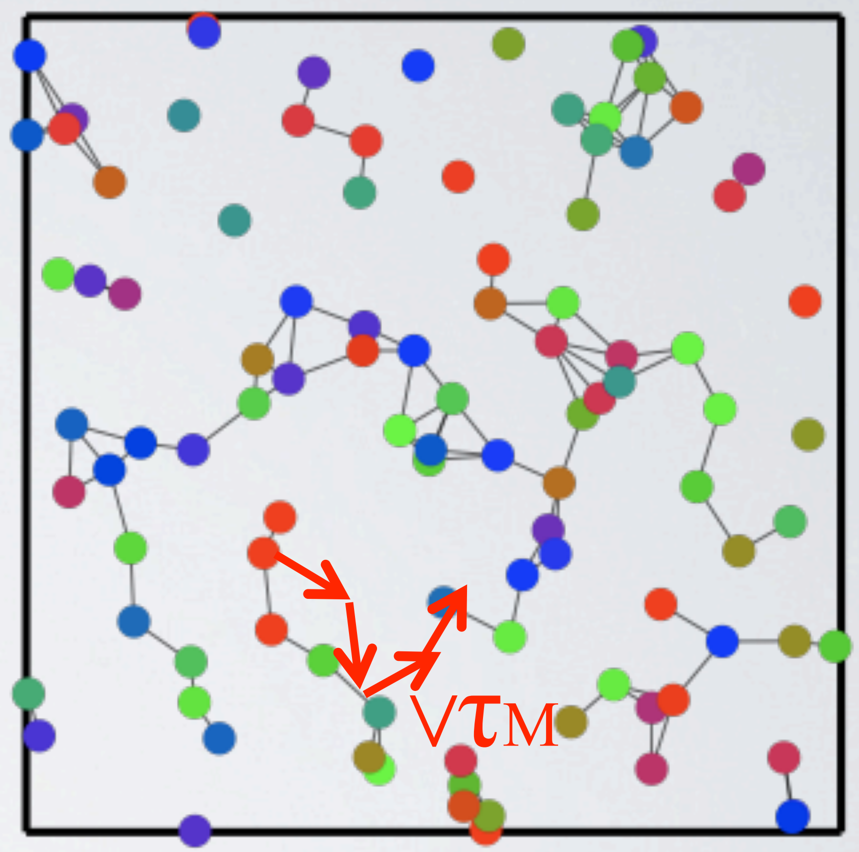

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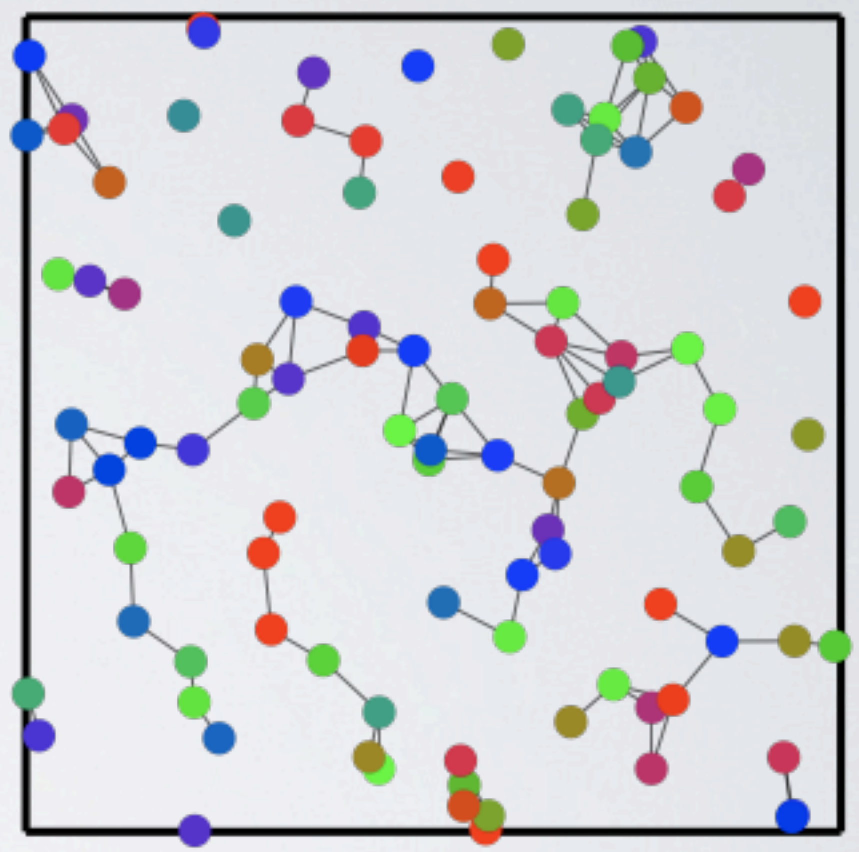

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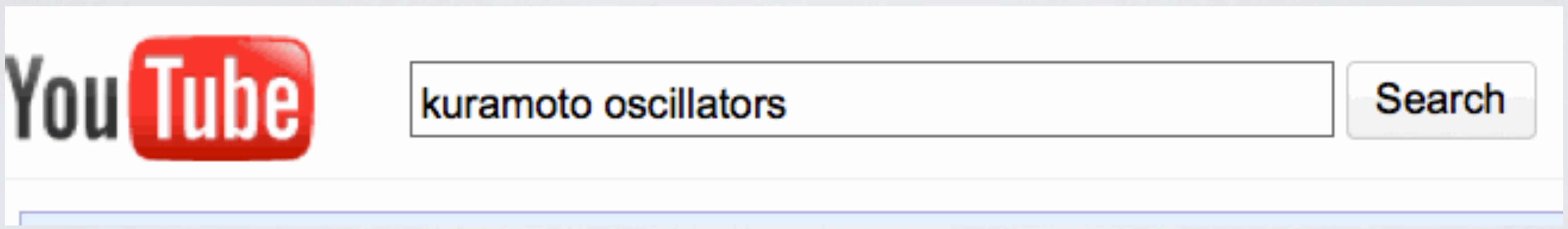
Applet

- Java applet simulation

  http://complex.ffn.ub.es/~albert/mobile/Kuramoto.html

Movies

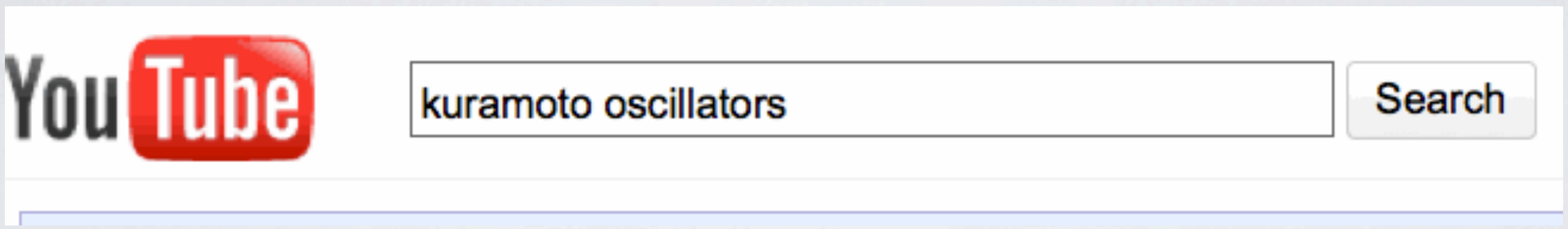


global multiple cluster

local multiple cluster

single cluster

Movies



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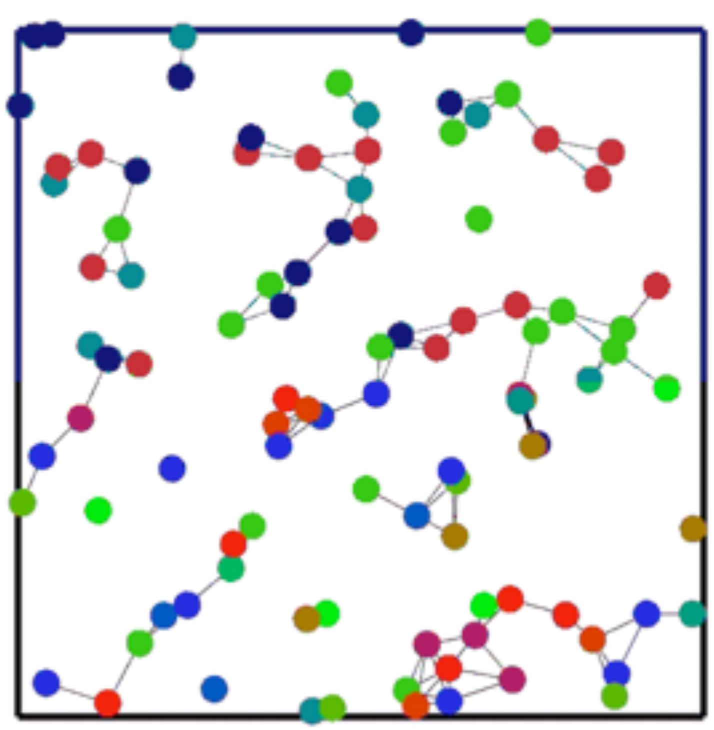
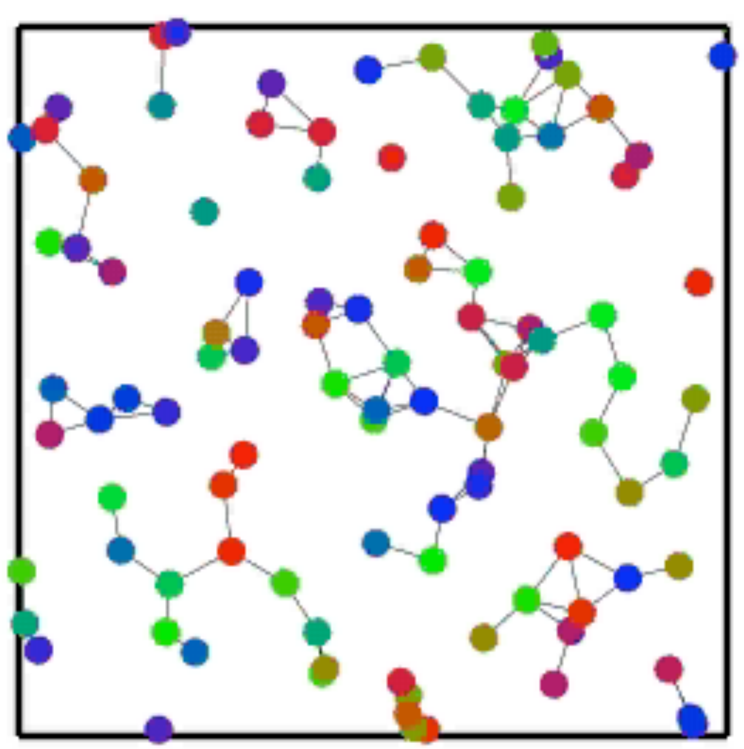


local multiple cluster



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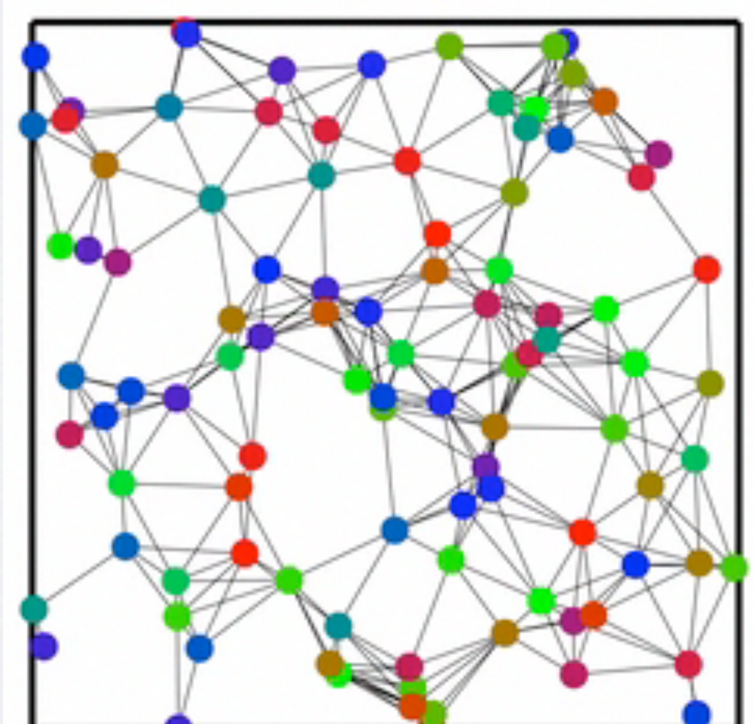
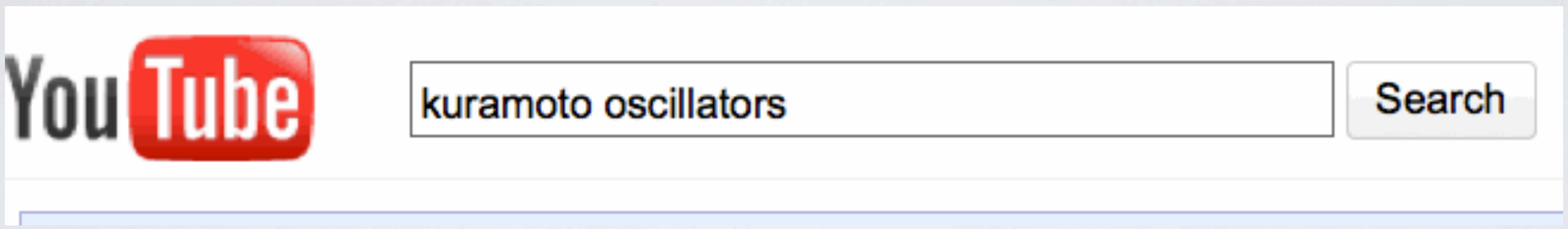


Movies



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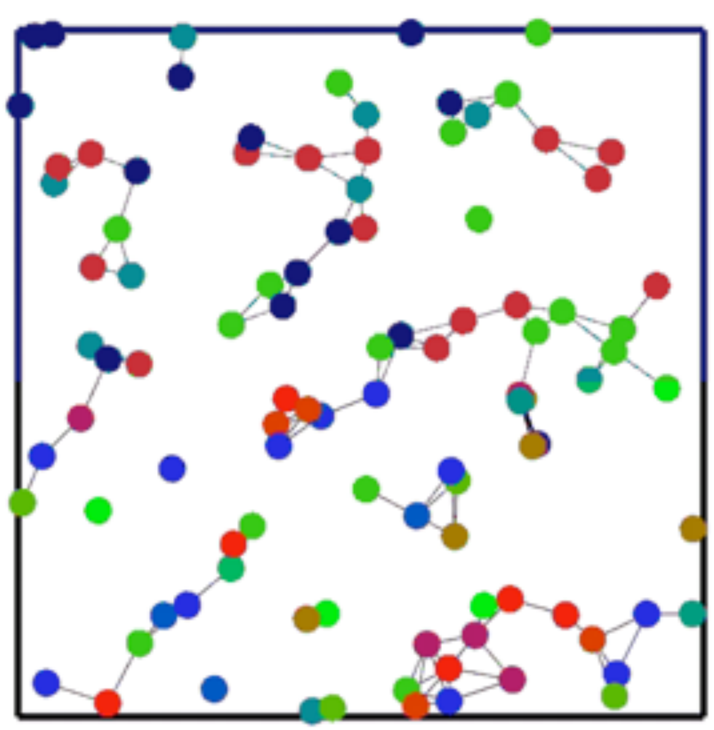

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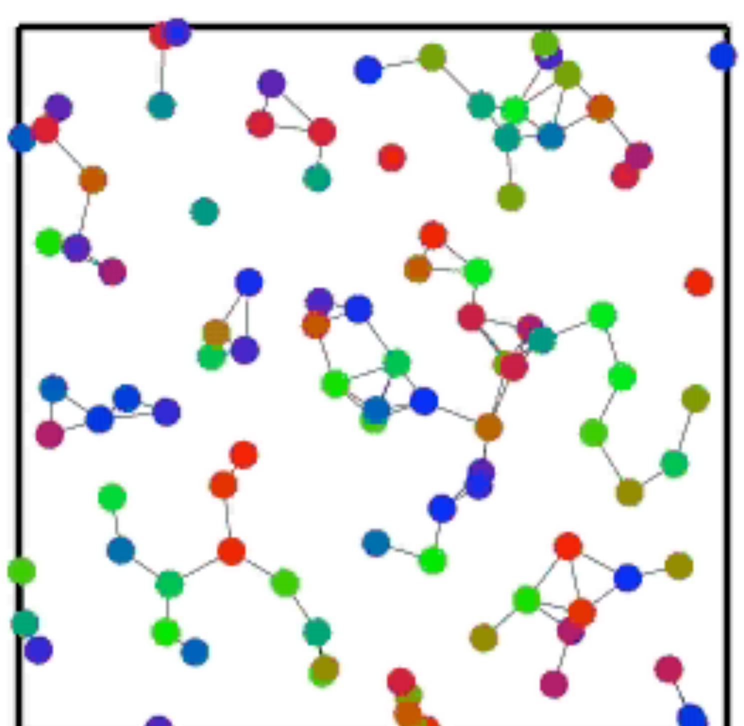

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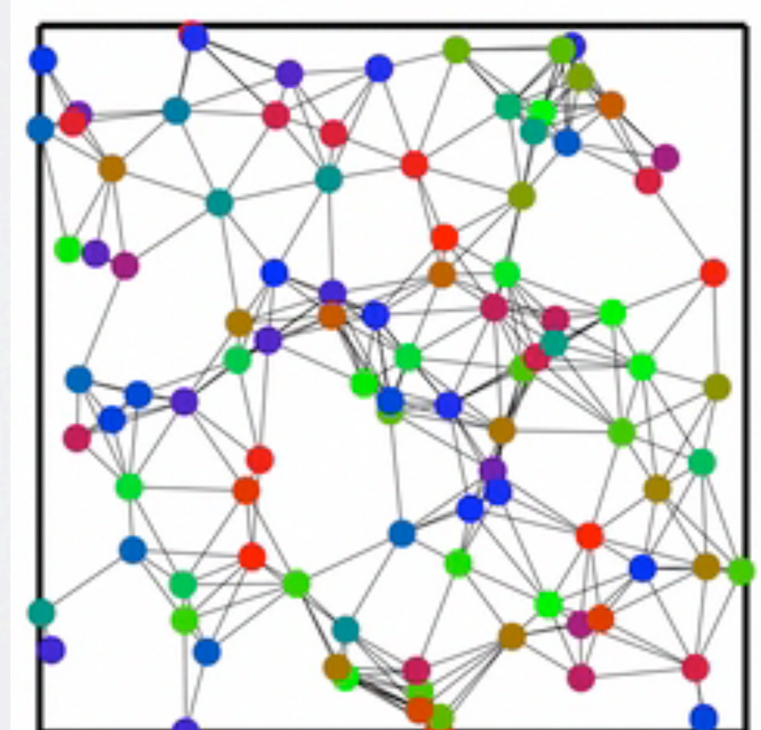
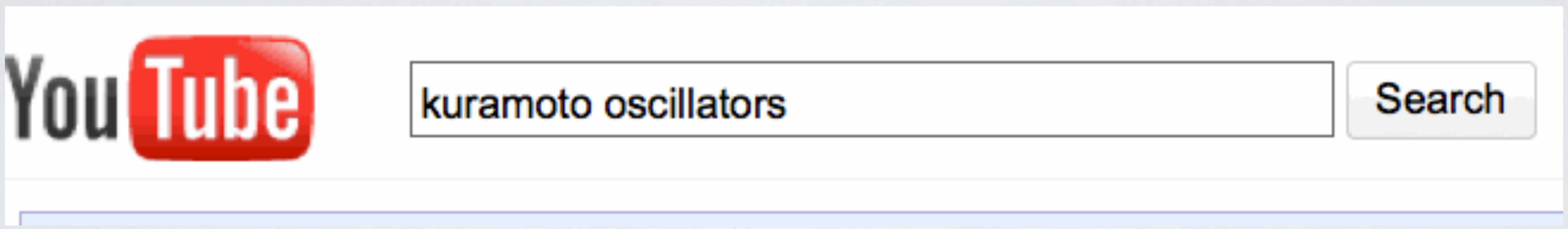


Movies



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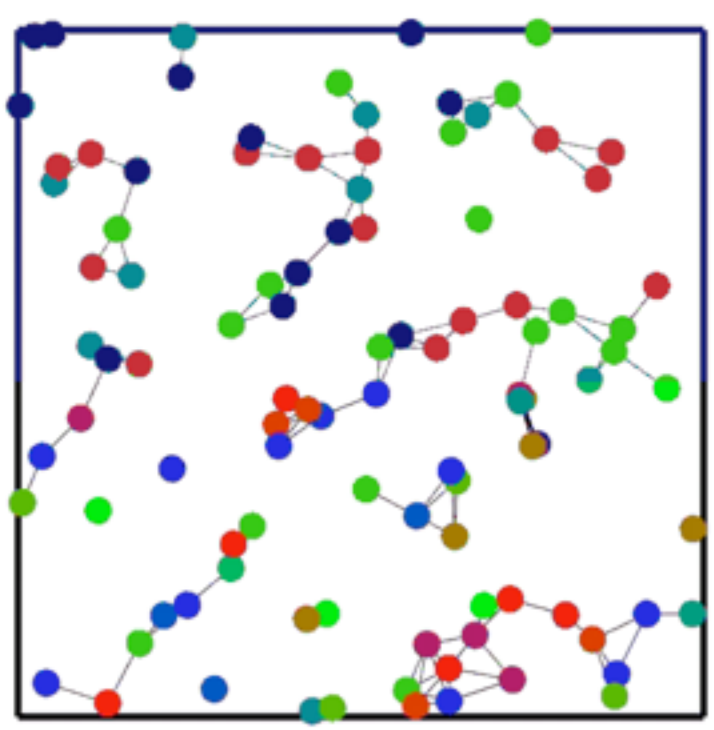

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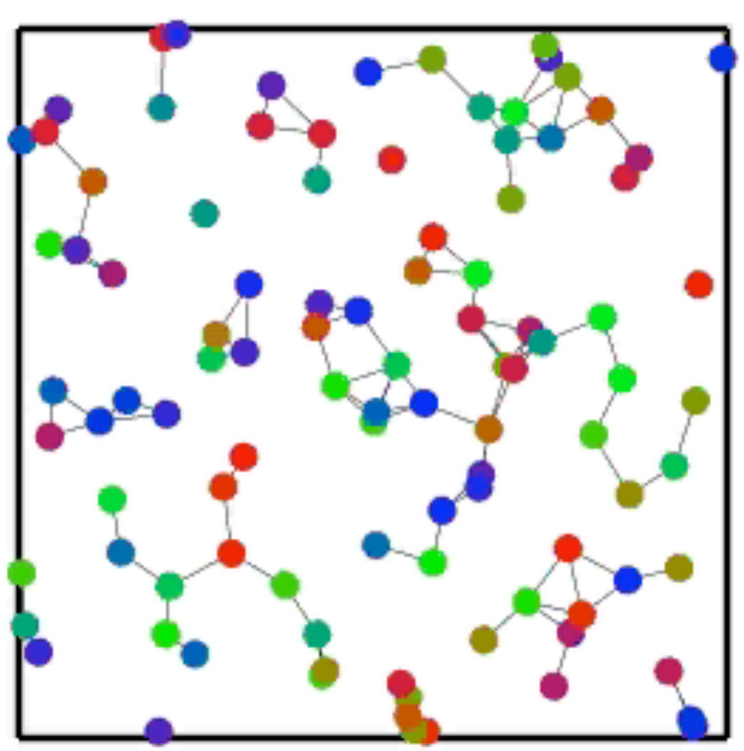

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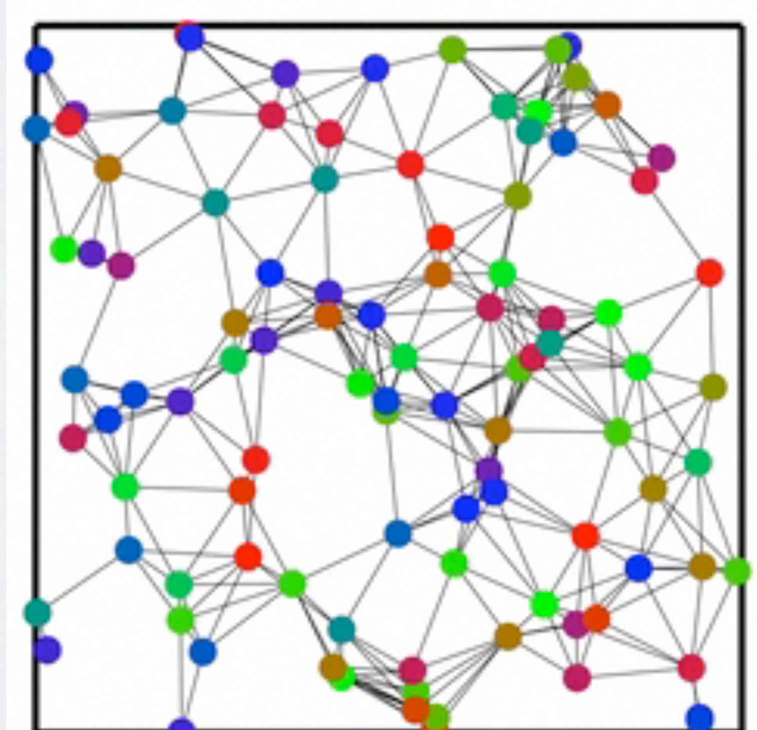
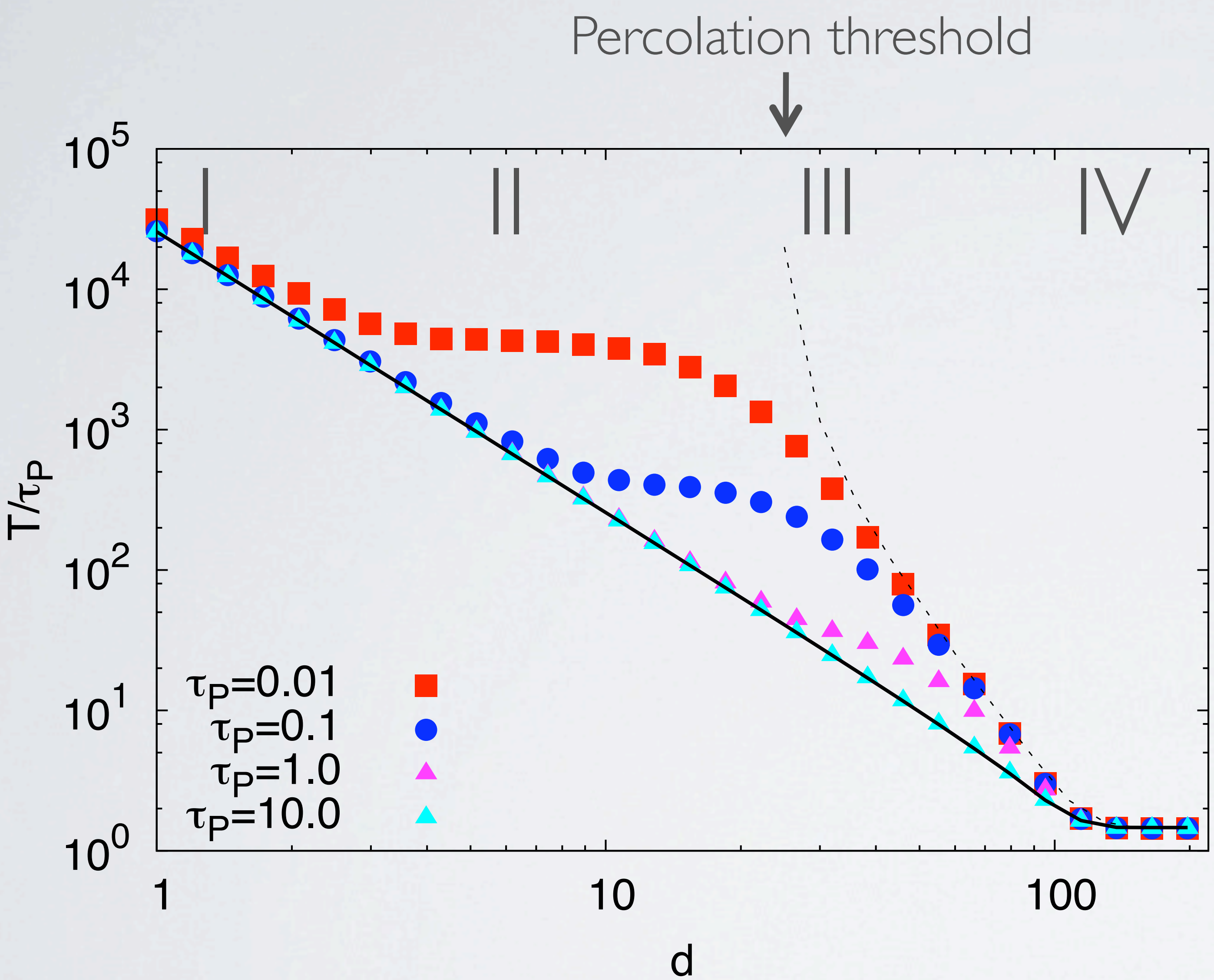


d (interaction range) dependence



$N = 100, \; L = 200, \; v = 10, \; \tau_M = 1.0, \; \sigma = 0.005$

- I: fast switching
- II: multi cluster
  - local synchronization
  - slow topology change
- III: single cluster
  - local synchronization
- IV: complete graph

Dynamic transition: local to global synchronization

- Number of steps for a cluster to internally synchronize

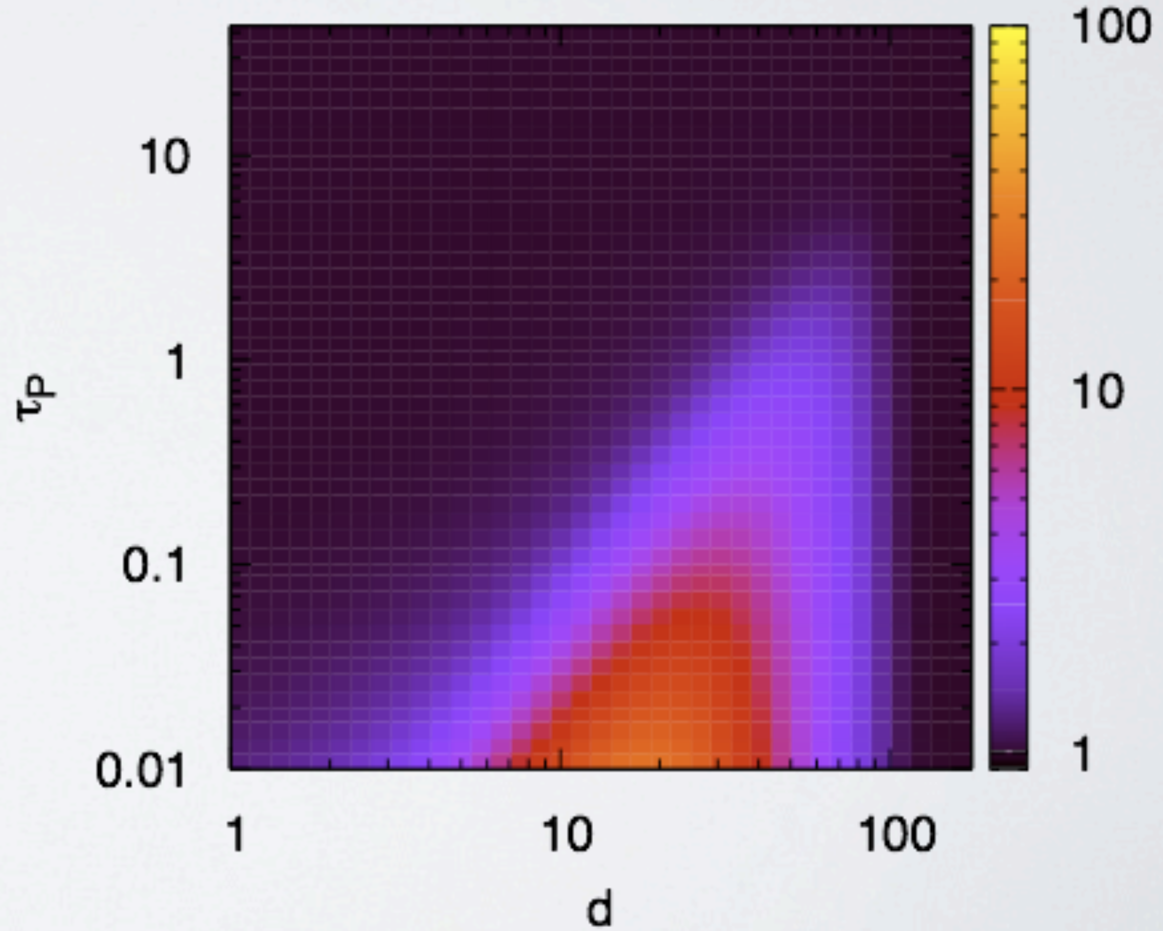
$$n_s = \frac{1}{\sigma \lambda_2^c(d)},$$

- Number of steps for an agent to leave a cluster

$$n_m = \frac{\xi^2(d)}{v^2 \tau_M \tau_P}.$$

Transition

$$\eta = \frac{n_m}{n_s} = \frac{\sigma f(d)}{v^2 \tau_M \tau_P}.$$



Matrix product for linearized equation

- When the phase difference is small, the linearized equation describes the synchronization dynamics

$$\varphi_i(t+\tau_P) = \varphi_i(t) - \sigma \sum_{j=1}^{N} L_{ij}(t)\varphi_j(t),$$

In our case **Laplacian matrix depends on time**

- consider the transformation of the normal modes (eigenmode of L)

$$\varphi_j(t) = \sum_{k=1}^{N} U_{jk}(t)\theta_k(t), \quad \sum_{k=1}^{N} L_{jk}(t)\theta_k(t) = \lambda_j(t)\theta_j(t)$$

- we get the time evolution of the normal modes as

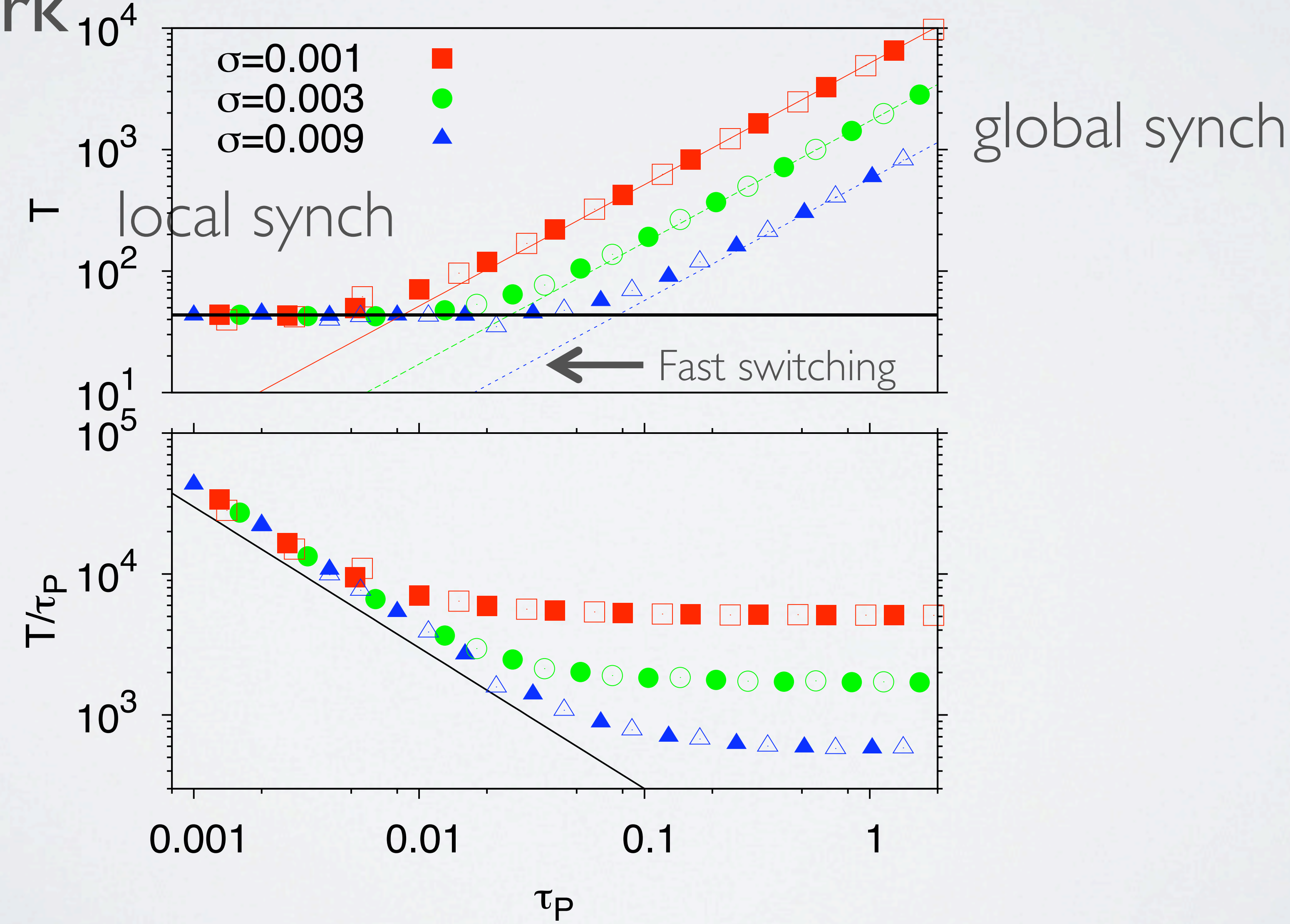
$$\theta_l(t+\tau_P) = \sum_{i,k} U^T_{li}(t+\tau_P)U_{ik}(t)\left[1-\sigma\lambda_k(t)\right]\theta_k(t)$$

$$\equiv \sum_k \underbrace{O_{lk}(t)}_{\text{agent mobility}}\underbrace{[1-\sigma\lambda_k(t)]}_{\text{oscillator dynamics}}\theta_k(t)$$

agent mobility oscillator dynamics

Matrix product for linearized equation

- Finally, we get $\theta_{k_n}(t+n\tau_P) = \prod_{q=0}^{n-1}\left[\sum_{k_q=1}^{N} O_{k_{q+1}k_q}(1-\sigma\lambda_{k_q})\right]\theta_{k_0}(t)$
- Compare empirical T with second smallest eigenvalue of the product of matrices (independent way), and they coincide for any value of the parameters even when fast switching approximation does not work



Derivation of fast switching approximation

$$\theta_{k_n}(t+n\tau_P) = \prod_{q=0}^{n-1}\left[\sum_{k_q=1}^{N} O_{k_{q+1}k_q}(1-\sigma\lambda_{k_q})\right]\theta_{k_0}(t)$$

- If the time scale of the oscillator is much longer than that of agent, eigenvalues for each time step are independent. Therefore we can replace product of oscillator dynamics part as

$$\prod_{q=1}^{n}(1-\sigma\lambda_{l_q}) \approx e^{n\langle\log(1-\sigma\lambda)\rangle} \qquad T = -\tau_P/\langle\log(1-\sigma\lambda)\rangle$$

- Up to the lowest order, characteristic time is approximated as

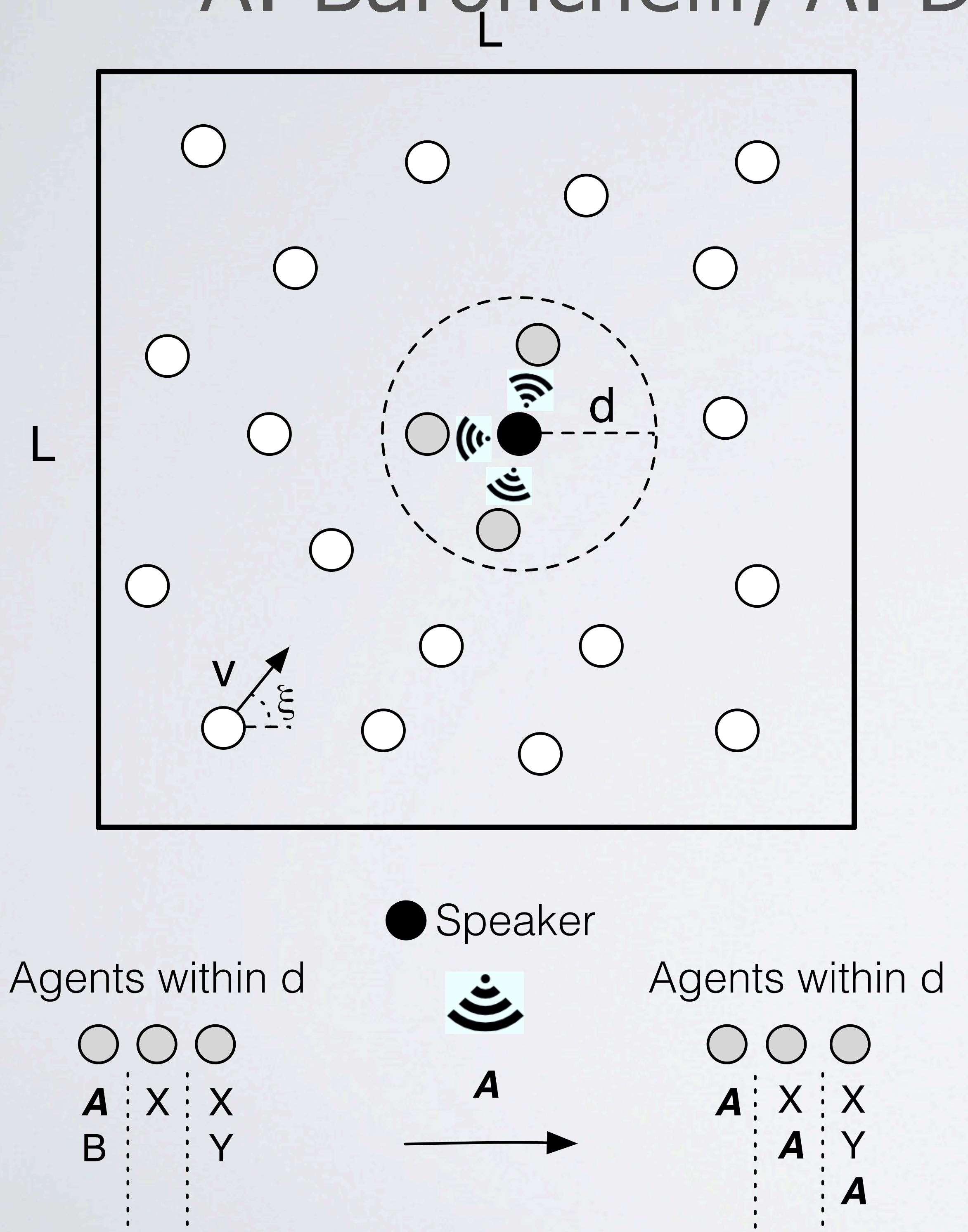
$$\frac{\tau_P}{T} = \sigma\langle\lambda\rangle$$

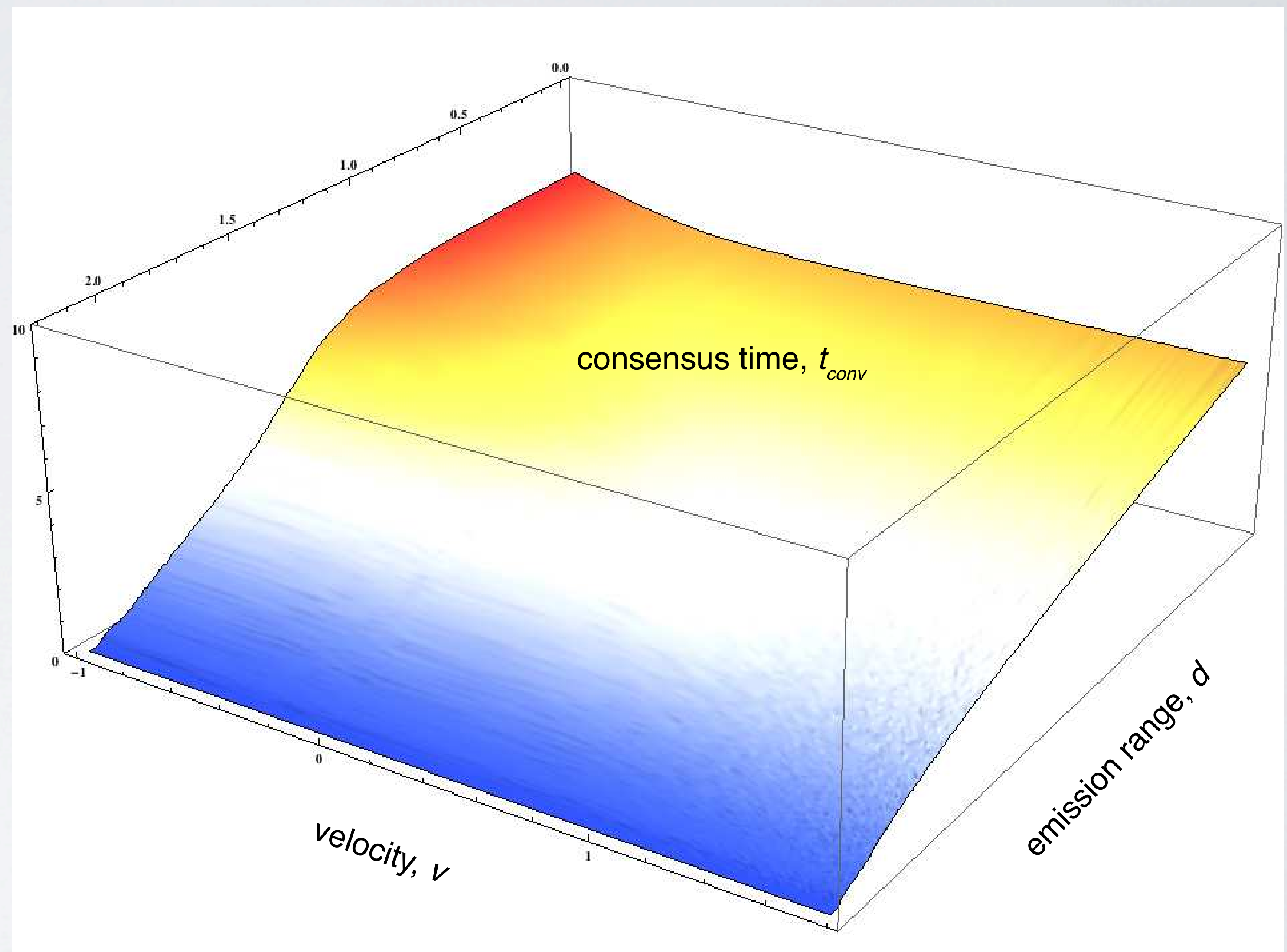
- Since average eigenvalue of the Laplacian matrix is average degree, we get

$$\frac{\tau_P}{T} = (N-1)\sigma\rho \qquad \rho = \begin{cases} \pi d^2/L^2 & d < \frac{L}{2} \\ L\sqrt{4d^2-L^2} + d^2[\pi - 4\cos^{-1}(\frac{L}{2d})] & \frac{L}{2} < d < \frac{L}{\sqrt{2}} \\ 1 & d > \frac{L}{\sqrt{2}} \end{cases}$$

Naming games

A. Baronchelli, A. D-G PRE 85 (2012) 016113



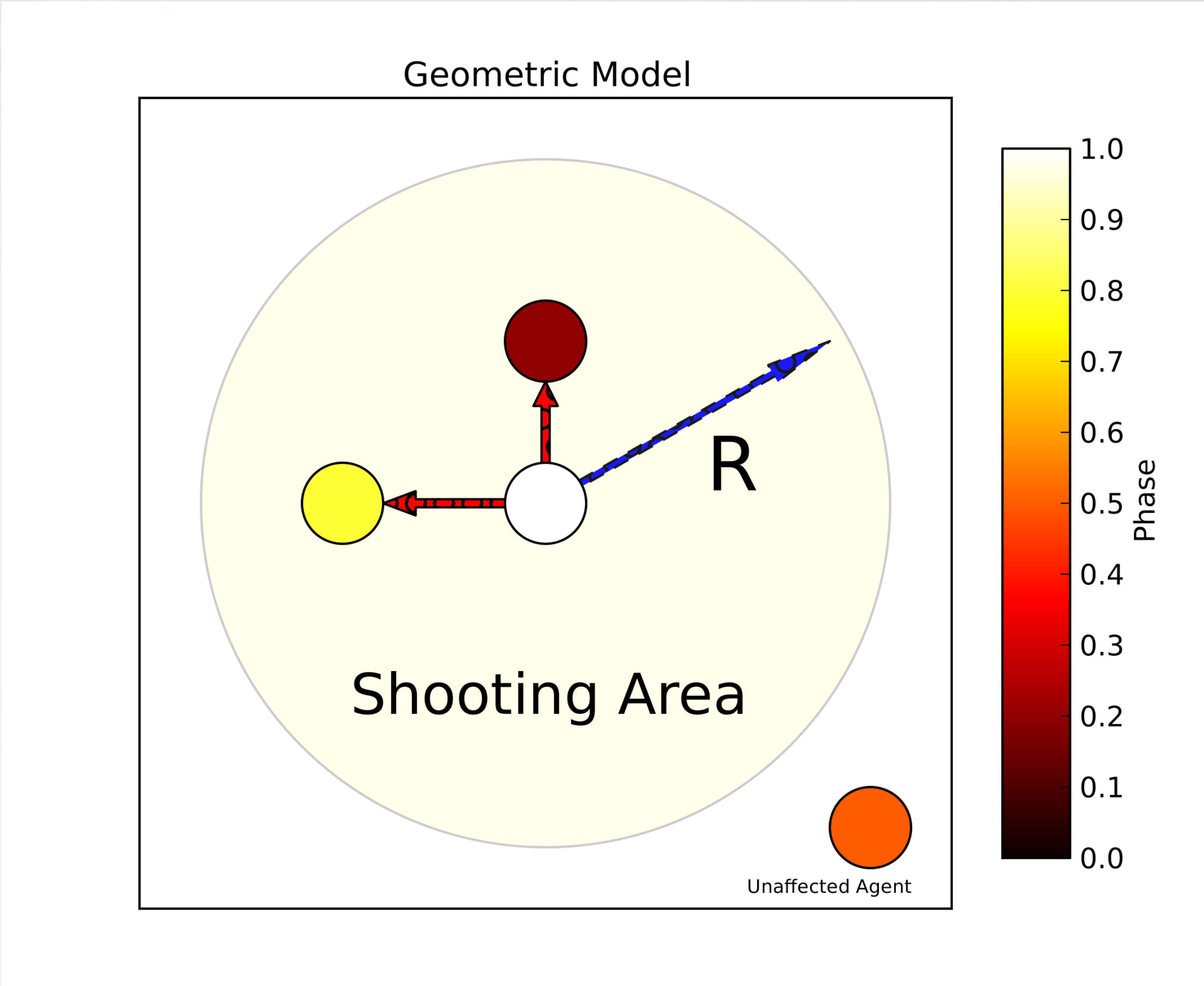


IFO's: instantaneous firings

L. Prignano, O. Sagarra, P.M. Gleiser, A.D-G

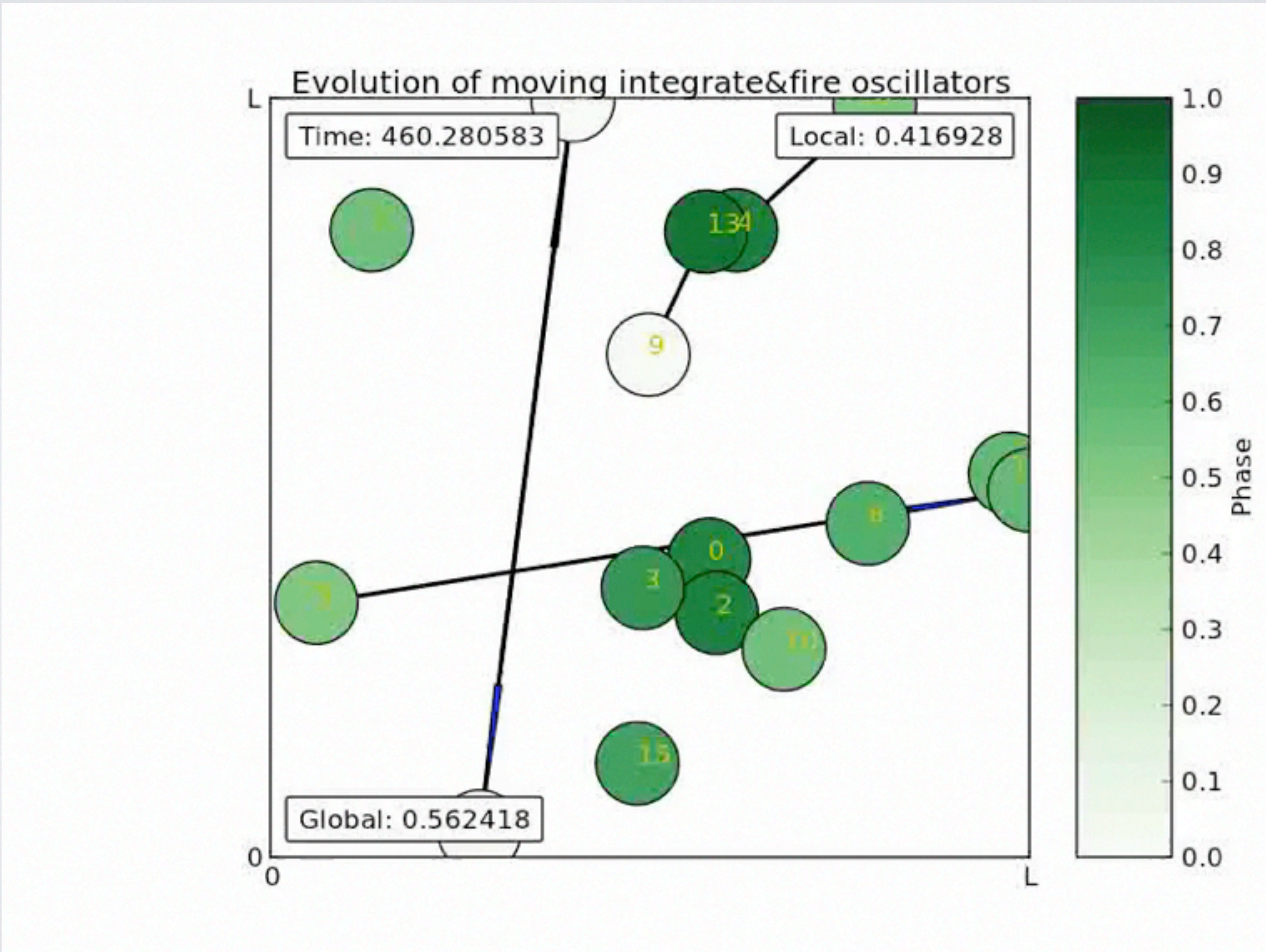
IJBC(in press)

$$\frac{d\phi_i}{dt} = \frac{1}{\tau}$$



$$\phi_i(t^-) = 1 \Rightarrow \begin{cases} \phi_i(t^+) = 0 \\ \phi_{\rm nn}(t^+) = (1+\epsilon)\phi_{\rm nn}(t^-) \\ \theta_i(t^+) \in [0, 2\pi] \end{cases} .$$

IFO's: instantaneous firings



Regimes

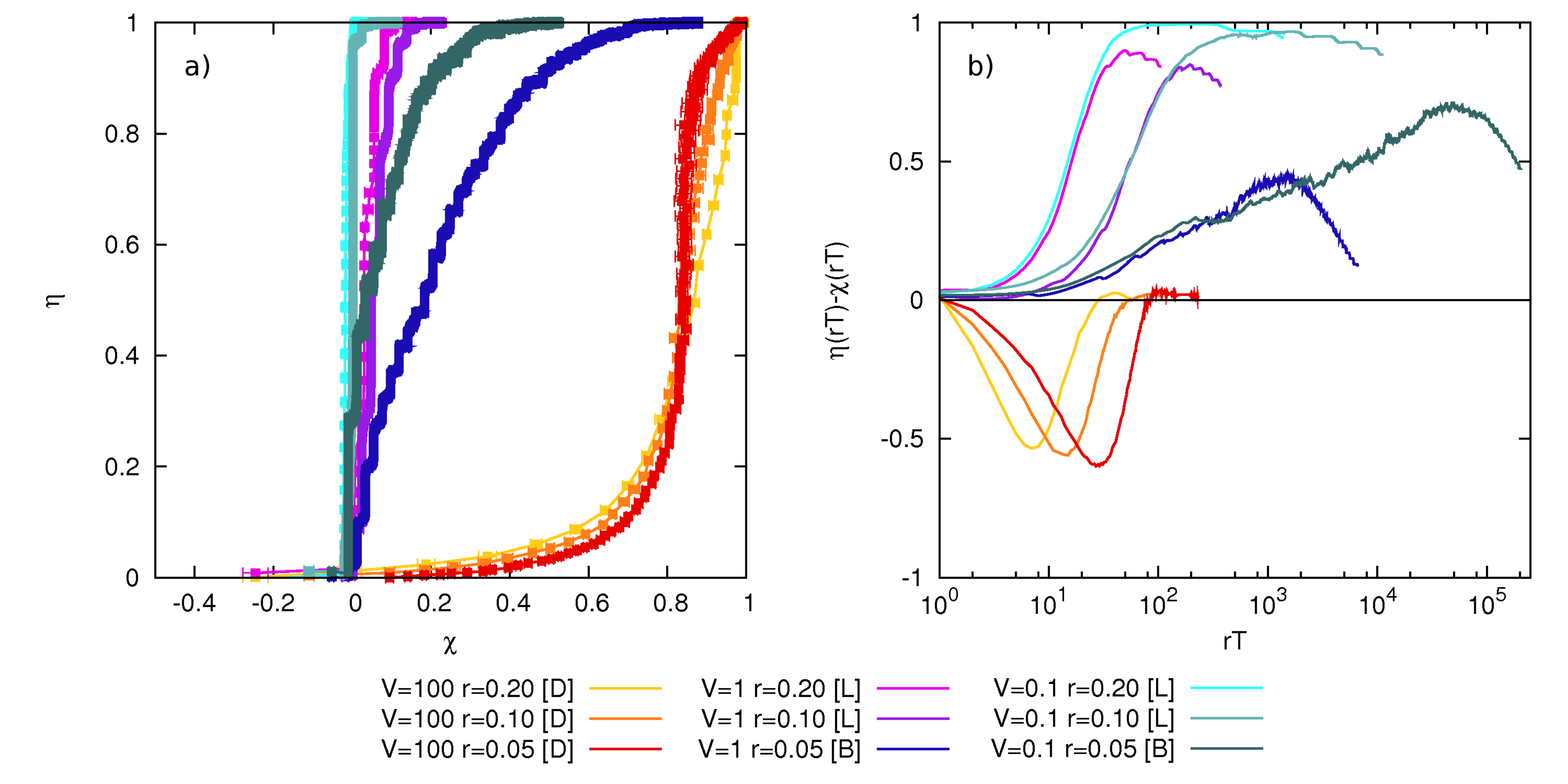
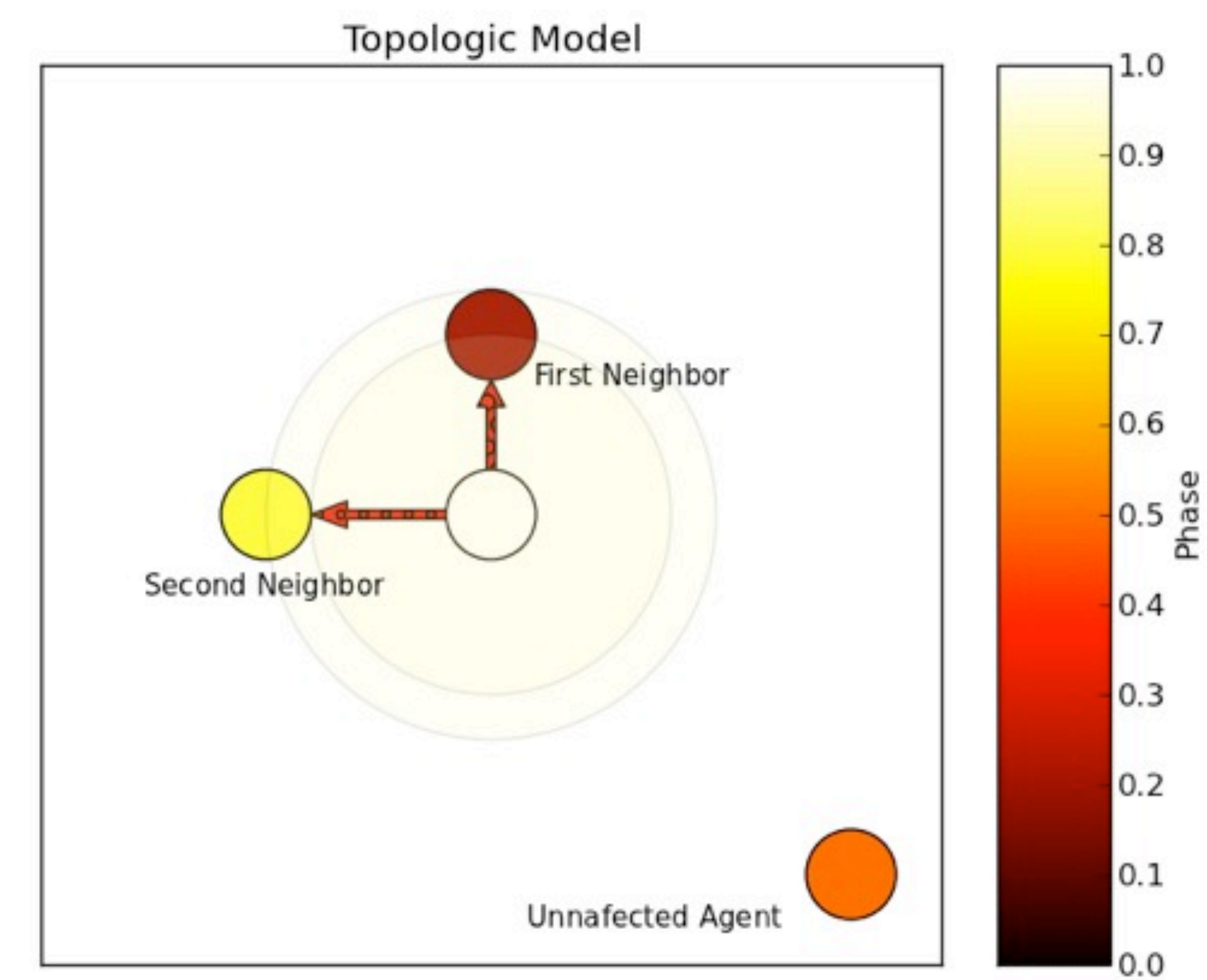


Fig. 3. (Colors online) Panel a): η against χ for several values of r and V. Panel b): the difference between the two contro parameters $(\eta - \chi)$ as a function of rT. Letters [D], [L] and [B] stand respectively for "diffusive", "local" and "bounded regimes. The values of η and χ at each time instant have been calculated averaging over 1000 realizations.

Minimal model:poster
L. Prignano & O. Sagarra

$$\frac{d\phi_i}{dt} = \frac{1}{\tau}$$

- Fire to the closest neighbor and change direction

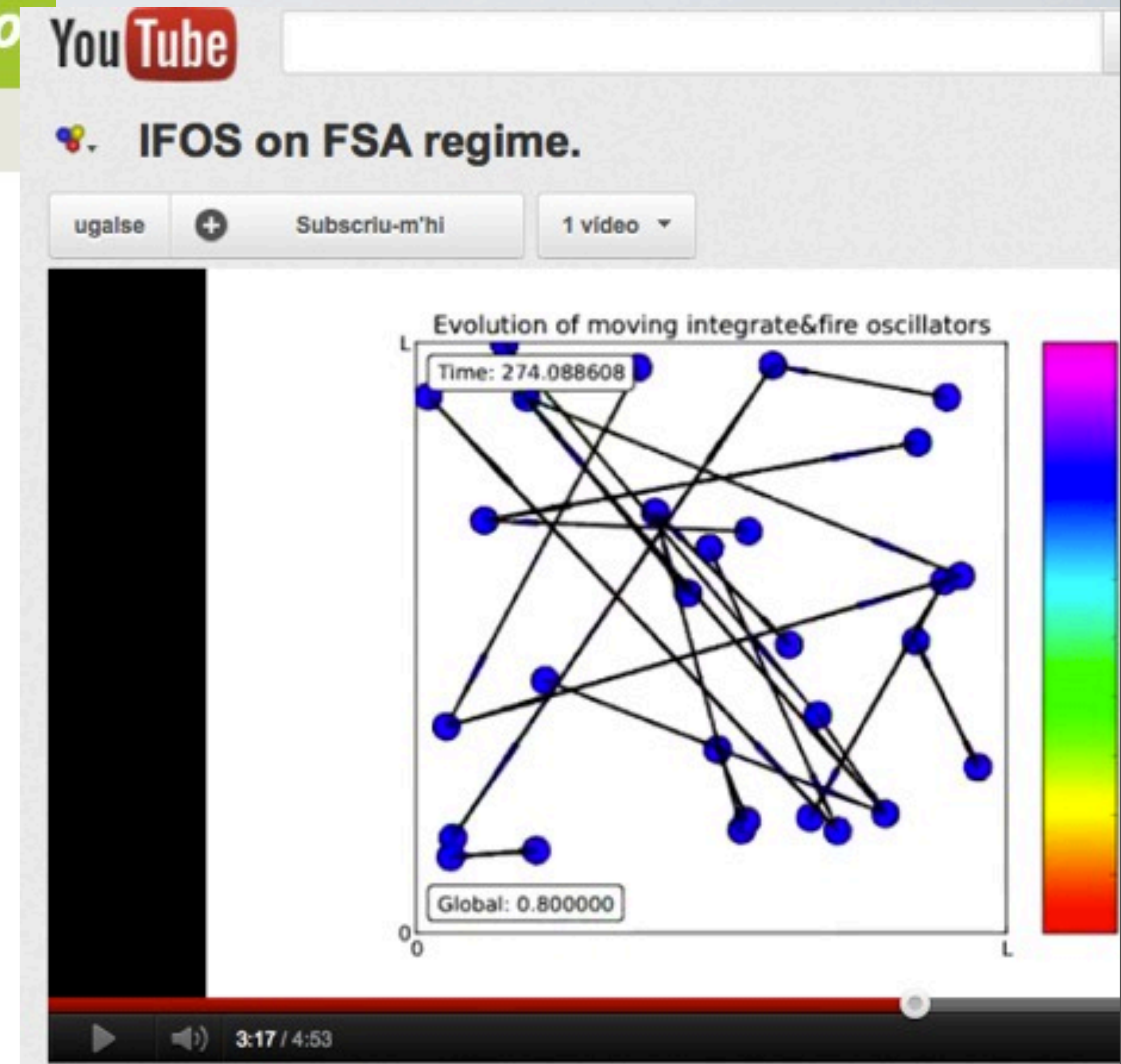


Minimal model

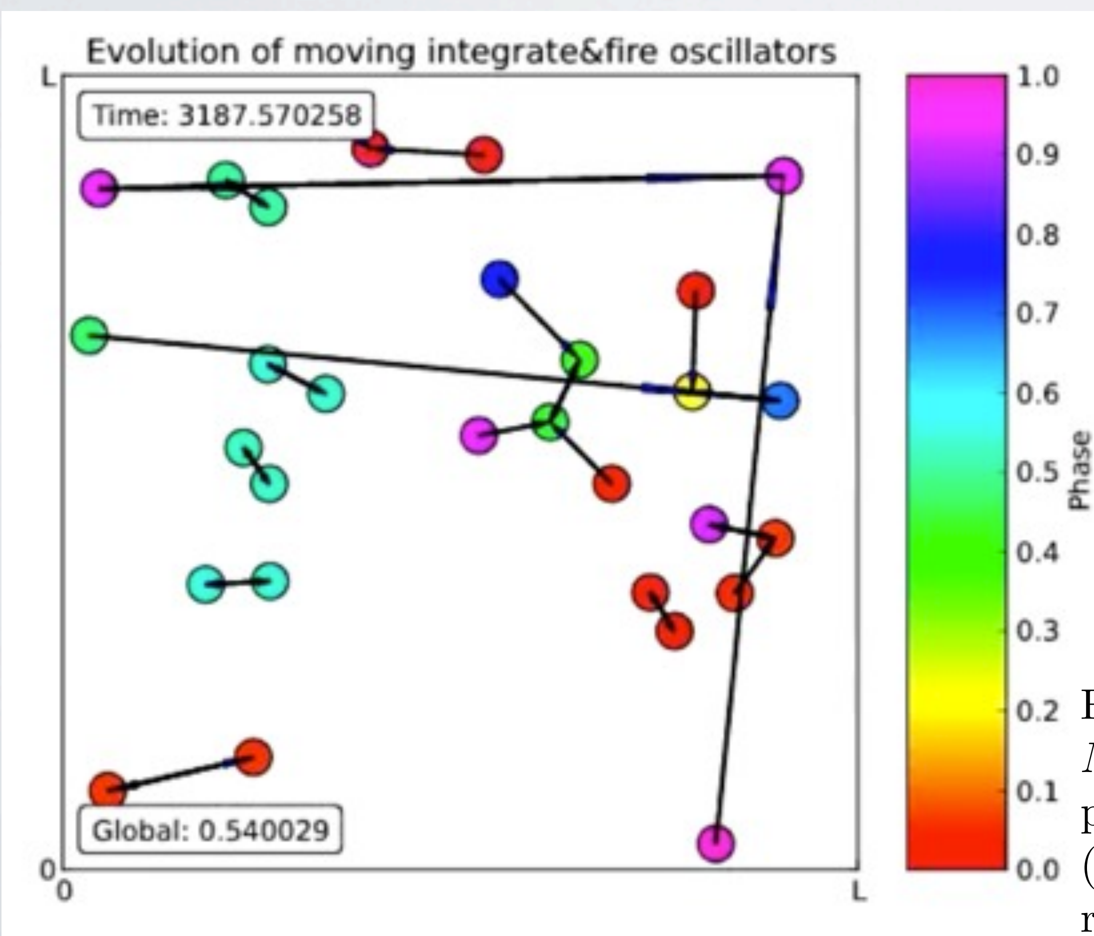
- Outdegree: 1 for all
- Indegree: 1 on average

Movies


vimeo
Join vimeo
Search videos for ifos synchronization
We found 2 videos. See all videos tagged with "ifos synchronization".
IFOS where movement prevents synchronization
IFOS where slow movement allows synchronization


YouTube
IFOS on FSA regime.
ugalse
Subscriu-m'hi
1 vídeo
Evolution of moving integrate&fire oscillators
Time: 274.088608
Global: 0.800000
3:17 / 4:53

Regimes



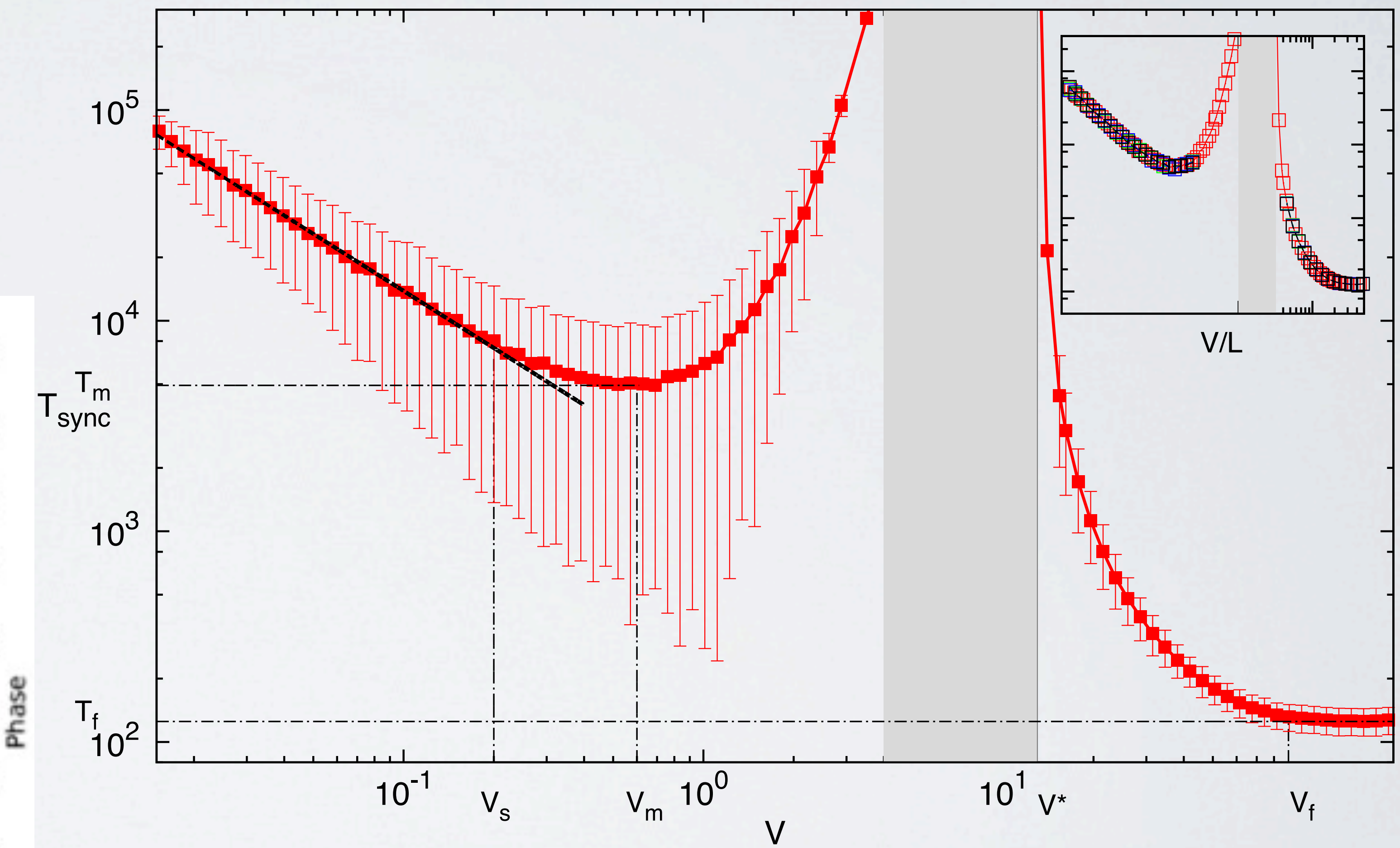


Figure 4.7: The average synchronization time T_{sync} as a function of V, for $L = 4$
$N = 20$, $\epsilon = 0.1$. In the following, when not otherwise states, the values of t
parameters are those used in this figure. In the inset: T_{sync} against V/L, for $L = 12$
(black), 800 (blue), 400 (red) and 200 (green). Averages are performed over 20
realizations.

Contact networks

- Instantaneously: single links
- Reciprocal?
- Very sparse

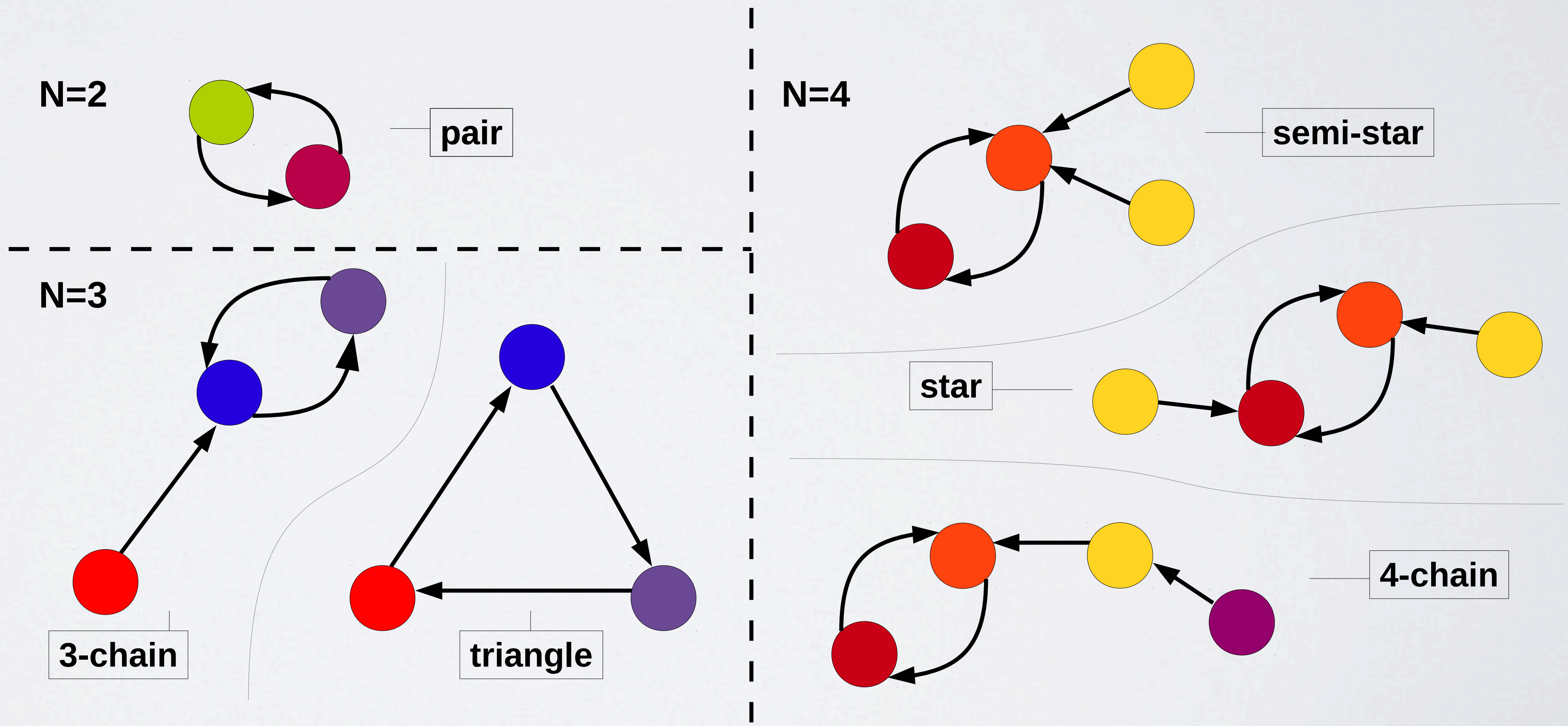


Figure B.3: Connected cluster of size $N = 2, 3, 4$.

Cumulative individual interaction network

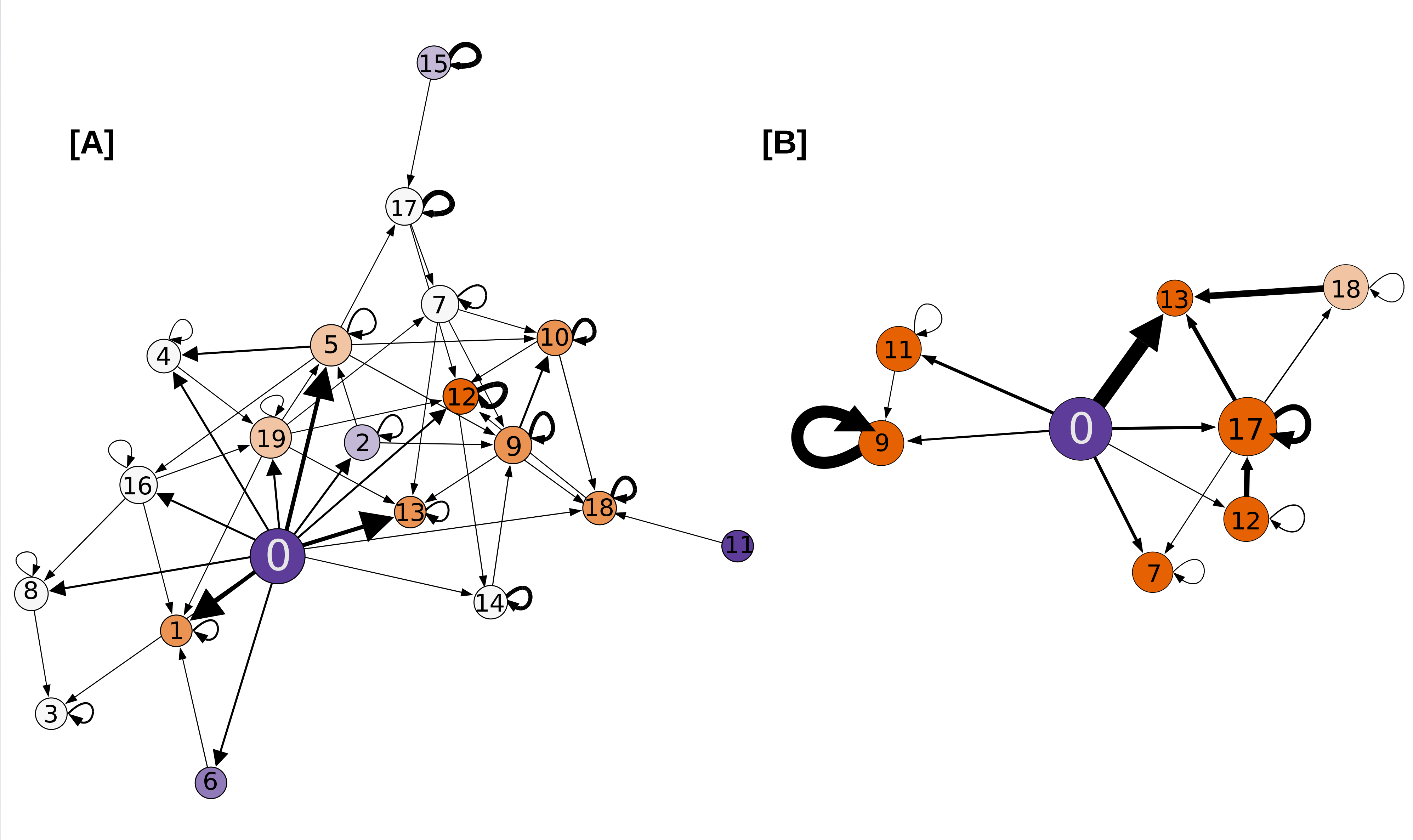


Figure 4.8: Final ($T = T_{sync}$) network of the interactions mediated by a single oscillator (labeled "0"), respectively in the fast limit, at $V = 2V_f$ (panel A) and at $V = V_m$ (panel B). Node color changes from purple to orange increasing the in-degree. Size increases with increasing out-degree. The weights of the links are proportional to occurrence of the interactions.

Cumulative total interaction network

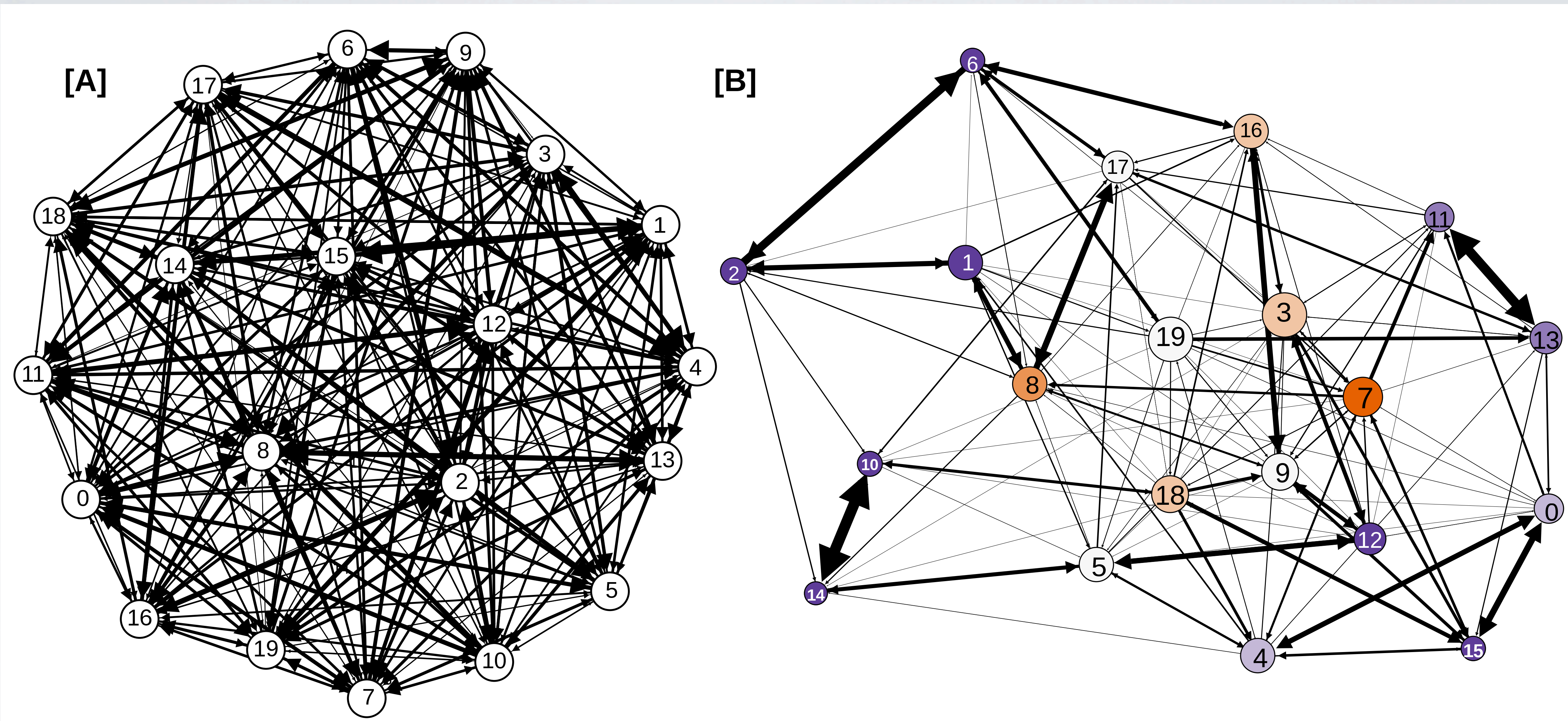


Figure 4.9: Final ($T = T_{sync}$) total interaction networks, respectively in the fast limit (panel A) and at $V = V_m$ (panel B). Node color changes from purple to orange increasing the in-degree. Size increases with increasing out-degree. The weights of the links are proportional to occurrence of the interactions.

Conclusions

- New features of complex networks:
  - networks of networks
  - networks are interconnected
  - time dependent
- Emergent properties depend also on the dynamics of the network
- There are feedback effects between topology and dynamics
- Non-universality: depend on rules of interaction, dynamics of the units, ...