Fragmentation Transitions in Adaptive Voter Models

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Fragmentation Transitions in Adaptive Networks for Opinion Formation

Many simple models for opinion formation [1–5] can reach two absorbing states:

(a) Global consensus state (all nodes in the same state)

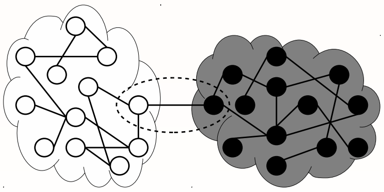
(b) Fragmented state (several disconnected components which are internally in consensus)

Changes in parameters can lead to a *fragmentation transition*, where the long term behavior changes from consensus to fragmentation. Here we present a simple analytical approach for the precise estimation of fragmentation thresholds [6].

General Approach

Our approach is based on linear stability analysis of the perturbed fragmented state (see fig.). The procedure for a general model is the following:

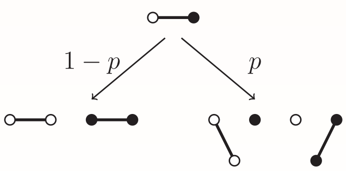
1. Consider a configuration close to the fragmentation threshold, where the network is almost fragmented, i.e. where the number of active links is much smaller than the number of inert links.
2. Determine the evolution equations for a set of appropriate *active motifs* in terms of the parameters of the model.
3. Construct the Jacobian which corresponds to the linear system of differential equations for the active motifs.
4. Solve the condition $\lambda = 0$ for the leading eigenvalue λ of the Jacobian.



Example: Adaptive Voter Model

In order to demonstrate our approach we consider the adaptive voter model. Starting with a random graph of nodes in two states (say, black and white), opinions evolve according to the following update rule:

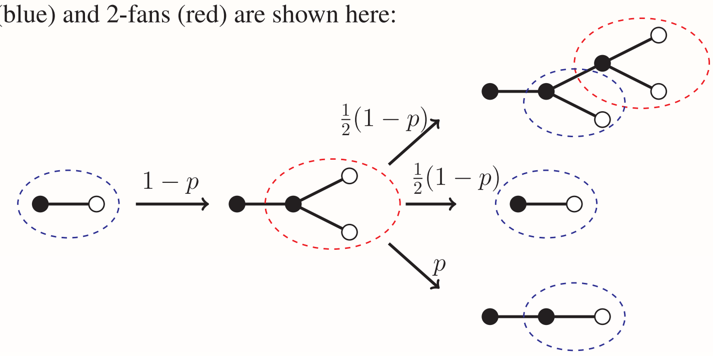
1. Choose a random active link.
2. With probability p: Cut the link and create a new one between one of the nodes and a randomly chosen node of the same state.
3. With probability $1-p$: Copy the state of one of the nodes to the other.



The goal is to estimate p_c using the described approach.

Procedure for fan-motifs and $k = 3$

For the simple case of $k = 3$ the updates occuring to single active links (blue) and 2-fans (red) are shown here:



From the above scheme we arrive at the evolution equations for the two *fan*-motifs in terms of the Jacobian

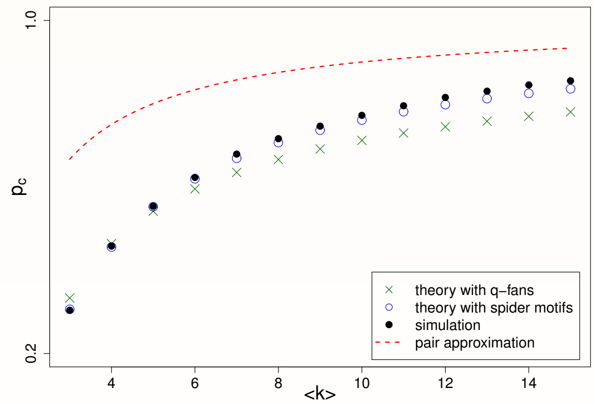
$$\mathbf{J} = \begin{pmatrix} -1 & 2 \\ 1-p & -1-p \end{pmatrix}.$$

The critical rewiring rate for this simple example yields $p_c = 1/3$. Analogously, the evolution equations for arbitrary degree k can be derived. A refined variant of this fan-motif set, a basis of *spider*-motifs, accounts for inert links and heterogeneous degree distributions.

Conclusion

The proposed motif-expansion allows for a simple and precise estimation of fragmentation transitions in contact processes.

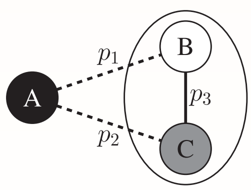
Results

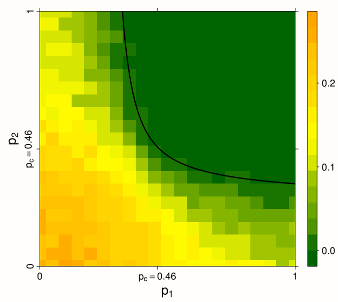


Comparison of the results from the procedure described above with simulation results shows that the simple approach already allows for a much better estimation of the critical point than the pair-approximation.

Extension: 3-state model

In voter models with many states our approach can be used to predict whether the system reaches consensus, partial fragmentation or full fragmentation [7].





For example in the three-state model, a cluster of A-nodes separates from the two remaining states in the green region.

References

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QR code provides link to article [6]: