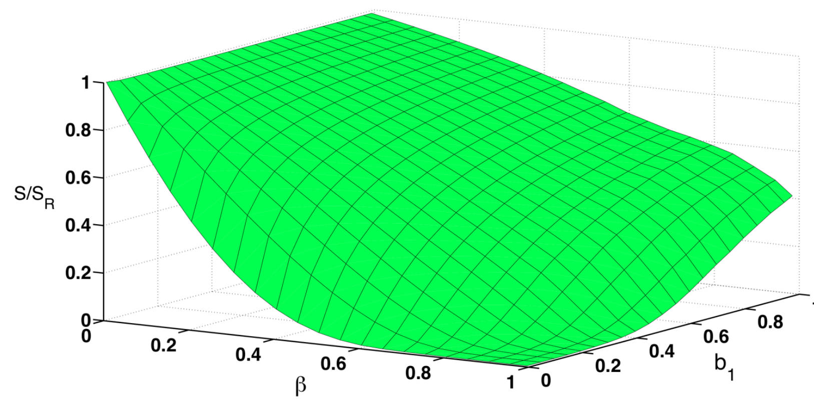
Dresden MAPCON12, May 14-18 2012

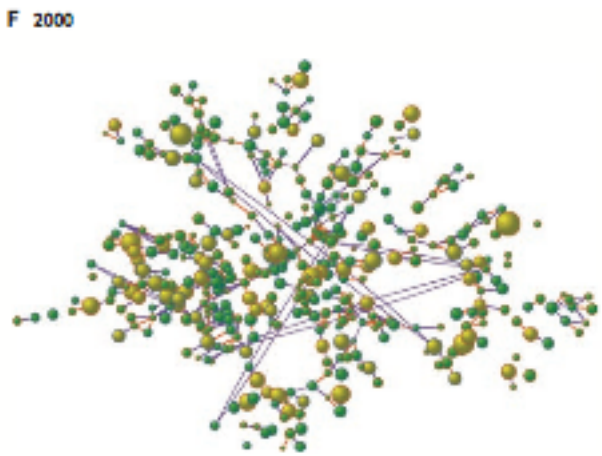
Entropy of Dynamical Social Networks

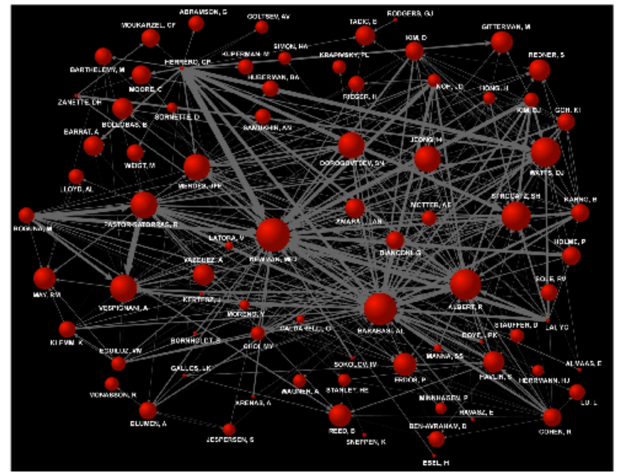


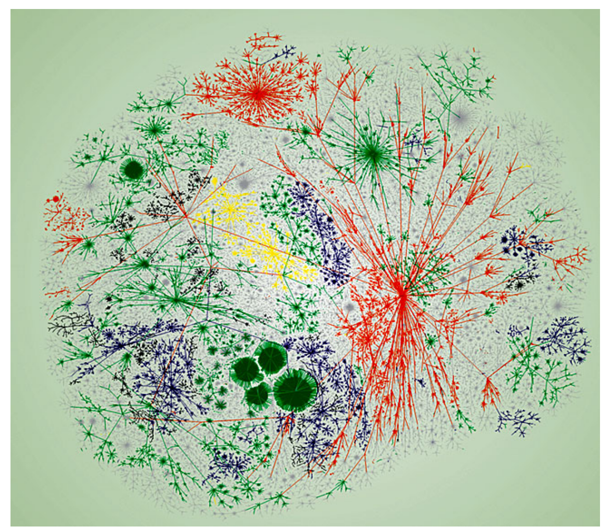
Ginestra Bianconi

Department of Physics, Northeastern University

The structure of human society is a set of interacting networks

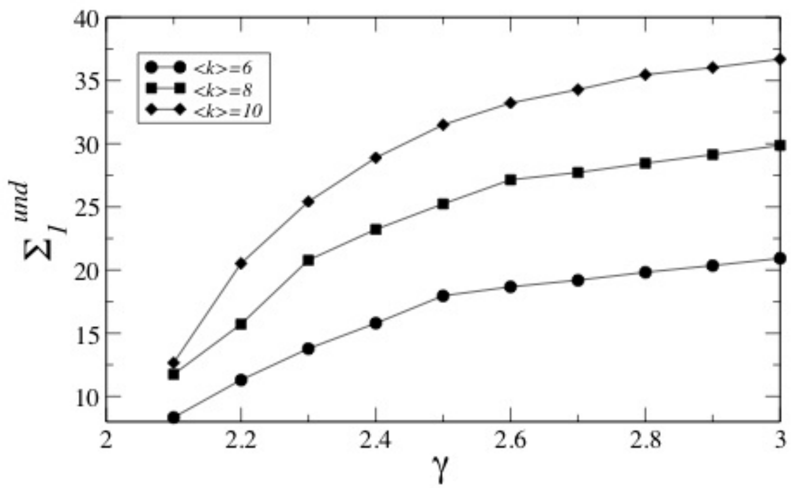




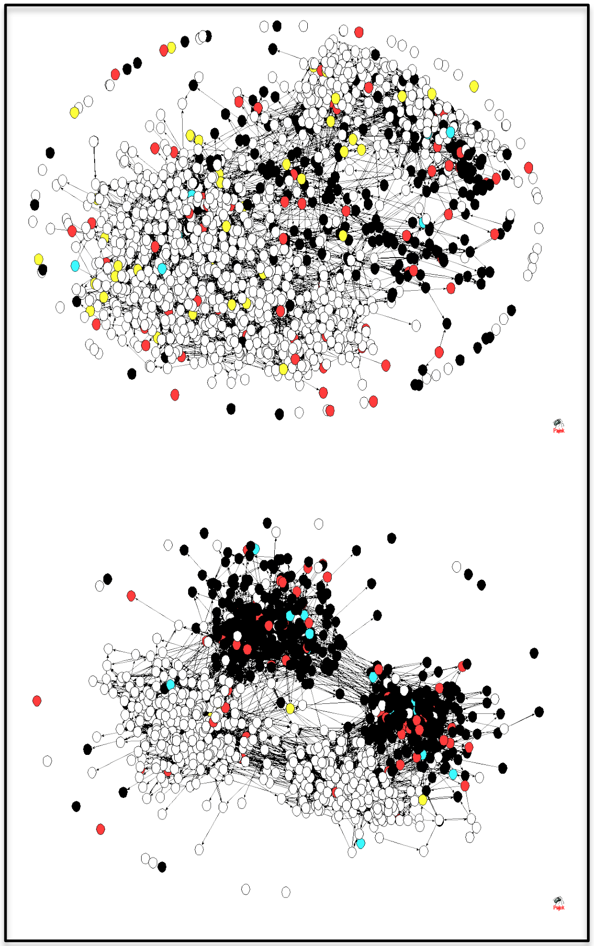


Complex networks encode for information

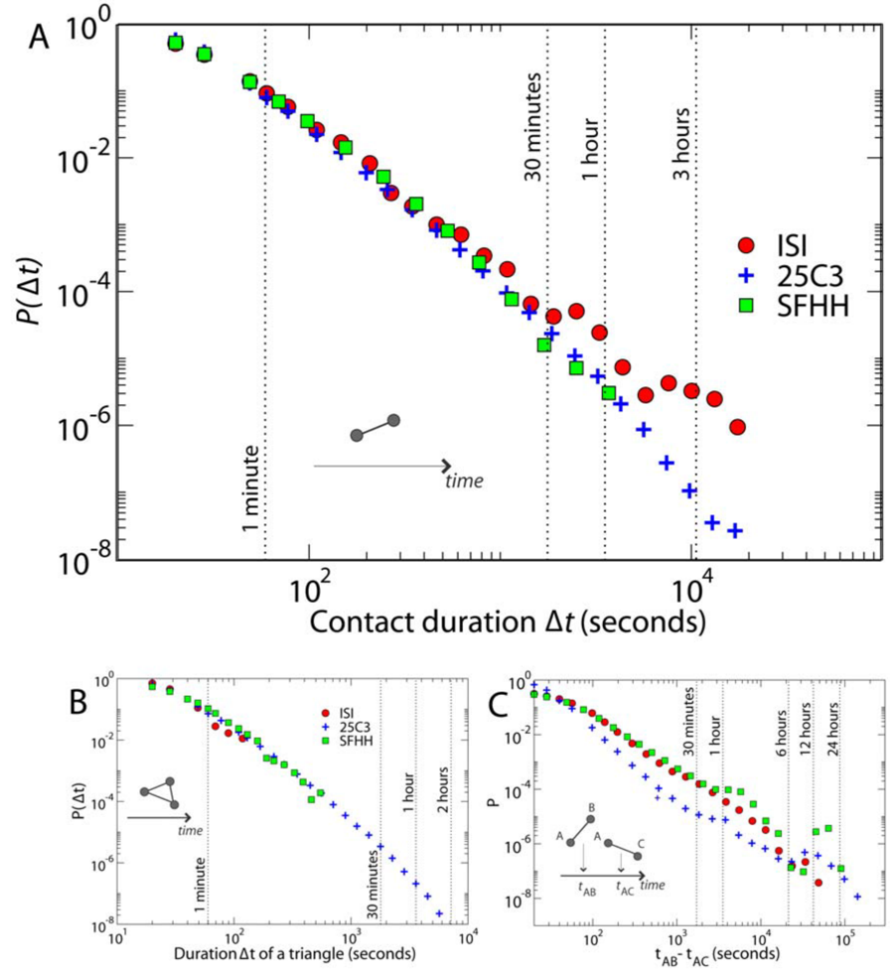
$$\Sigma = \frac{1}{N}\log(\aleph)$$



G. Bianconi et al. PNAS 2009, G. Bianconi PRE 2009,Europhys. Lett2008, Anand et al. PRE 2009, PRE 2010, PRE 2011



Duration of Face-to-face Interaction



A. Barrat, et al. PlosOne (2010)

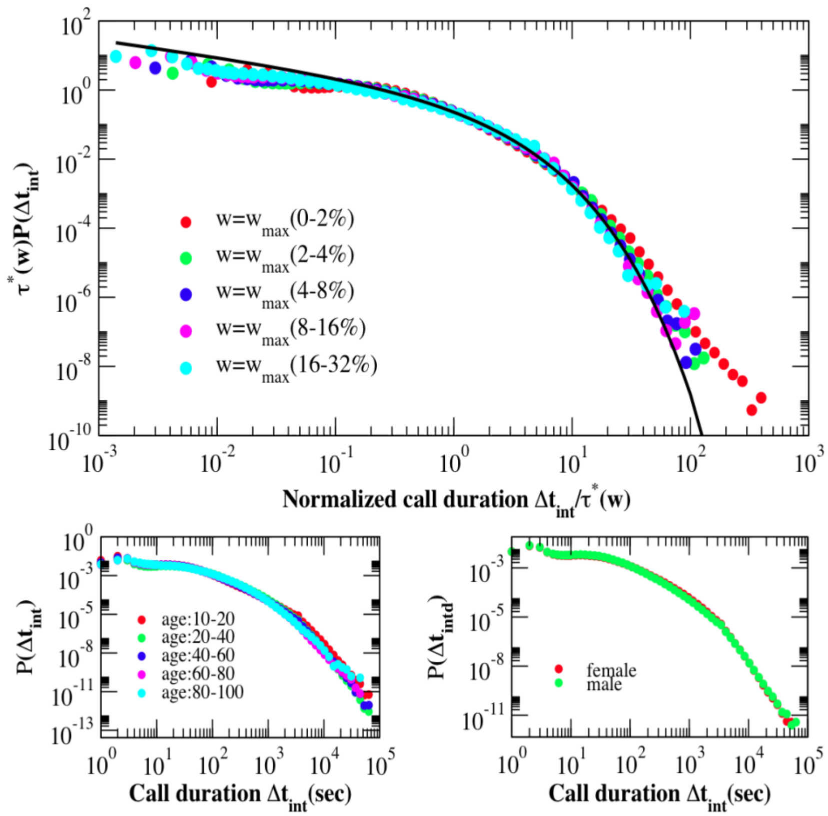
Cellphone Communication Dataset

6 millions users

3-months long observation period

Distribution of Call Duration

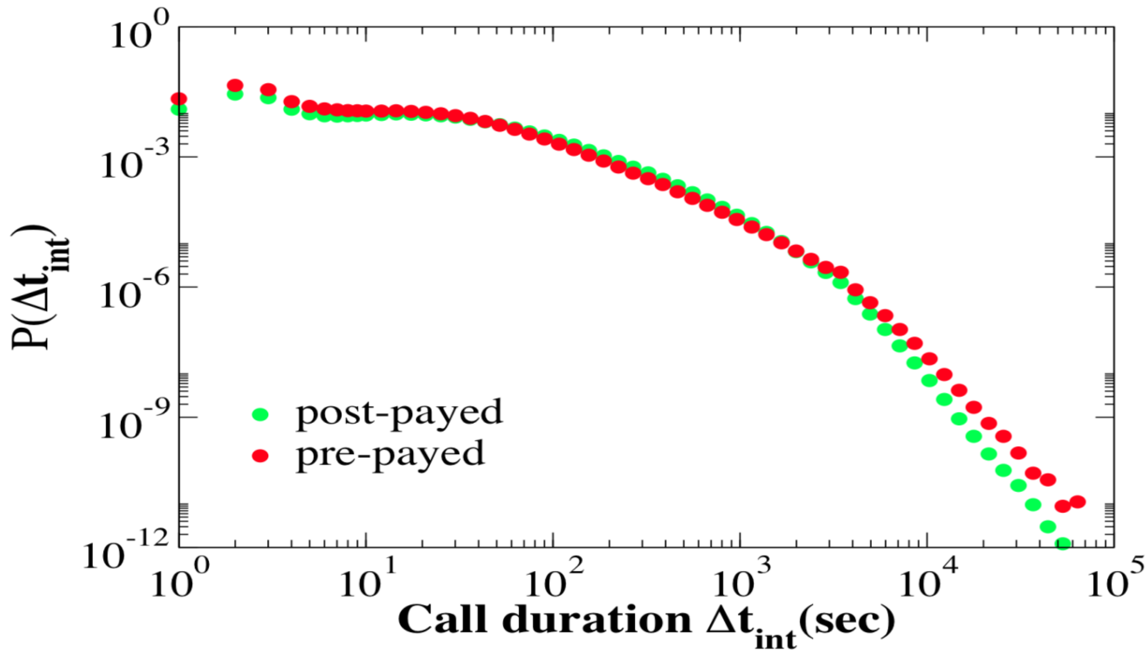
$$\tau^*(w)P(x=\Delta t_{\text{int}}/\tau^*(w)) \propto x^{-\beta}\exp[-x^{1-\beta}/(1-\beta)]$$



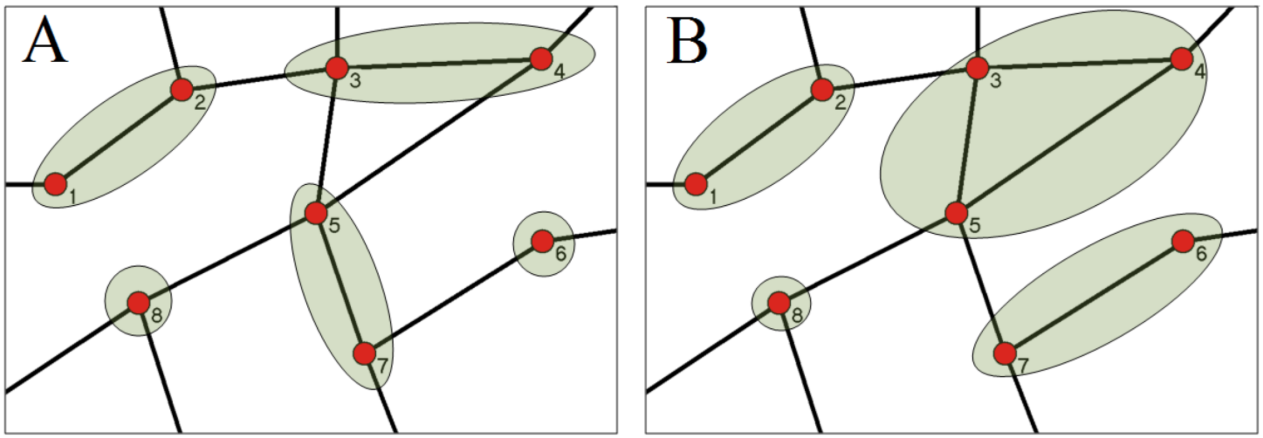
Weight of the link	Typical time $\tau^*(w)$ in seconds (s)
(0-2%) w_{max}	111.6
(2-4%) w_{max}	237.8
(4-8%) w_{max}	334.4
(8-16%) w_{max}	492.0
(16-32%) w_{max}	718.8

K. Zhao, M. Karsai and G. Bianconi PloSOne 2011

Call Duration in Different Contract



Dynamical Social Networks



At any given time t, static network G will be partitioned into interacting groups (indicated by green shaded area).

$$g_{i_1, i_2, \ldots, i_m}(t) = 1$$ If $i_1, i_2, \ldots i_m$ are interacting in a maximum group

$$g_{i_1, i_2, \ldots, i_m}(t) = 0$$ otherwise

Entropy of Dynamical Networks

Likelihood of the configuration:

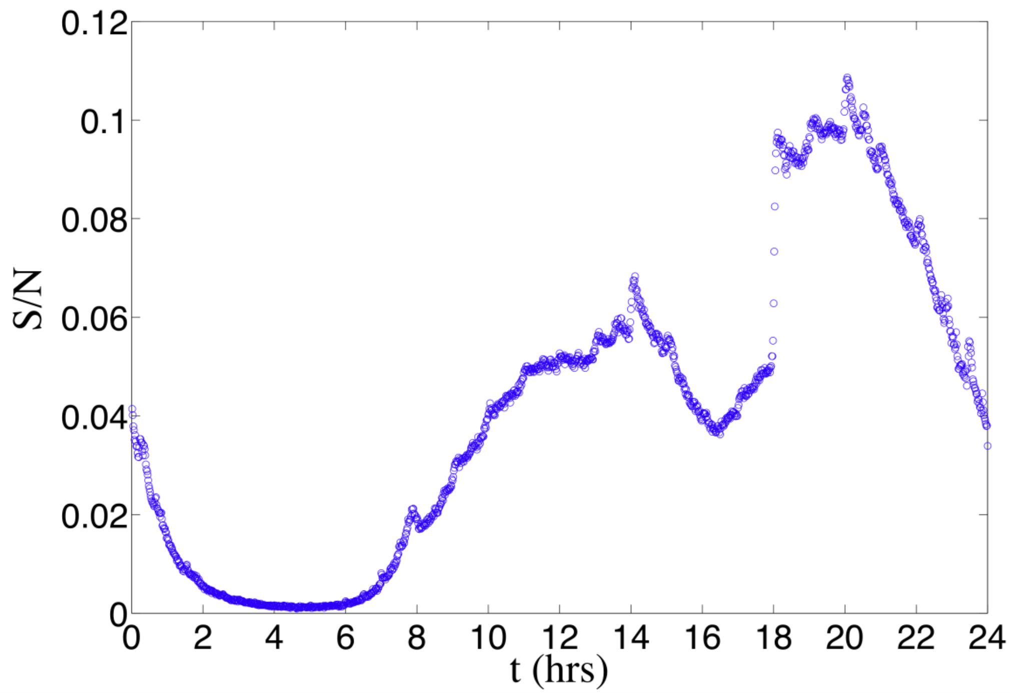
$$L = \prod_{(i_1, i_2 \ldots i_m) \in G} p(g_{i_1, i_2 \ldots i_m}(t) = 1 | h_t)^{g_{i_1, i_2 \ldots i_m}(t)}$$

Entropy of the network:

$$S = -\sum_{(i_1, i_2 \ldots i_m) \in G} p(g_{i_1, i_2 \ldots i_m}(t) = 1 | h_t) \log p(g_{i_1, i_2 \ldots i_m}(t) = 1 | h_t)$$

(h_t is the history of configurations at $t' < t$)

Entropy Analysis of The Cellphone Model



Entropy as a function of time in a typical week-day of cellphone data

Reinforcement Dynamics in Social Interactions

For the interacting individual

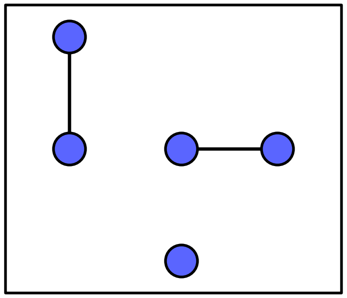
The longer an individual interacts with a group the less is likely to leave the group

For the isolated individual

The longer and individual is isolated the less is likely to interact with a group

The Dynamical Model

1. Randomly choose one agent i, n_i is the size of his group, t_i is the last time that the agent has changed his state

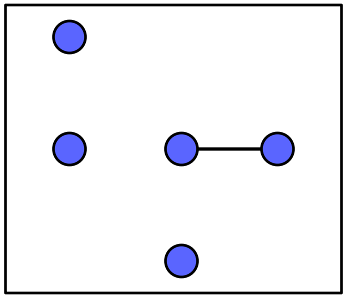


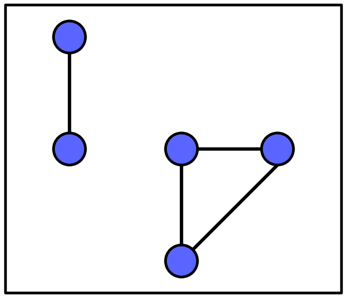
2. (a) If n_i=1, with probability $f_1(t_i,t)$ he will interact with another agent j chosen with probability proportional to $f_1(t_j,t)$

   (b) If n_i>1, with probability $f_n(t_i,t)$ he will change his state.

   (i)with probability λ he will leave the group

   (ii)with probability 1- λ he will introduce another agent j to the group





The Dynamical Model

Face-to-face Model:

$$f_1(t_i, t) = \frac{b_1}{(1+\tau)}$$

$$f_n(t_i, t) = \frac{b_2}{(1+\tau)}$$

$$\tau = (t - t_i) / N$$

Such choice indicates a reinforcement dynamics that the longer an agent stays in his current state the less possible he will change it.

The Dynamical Model

Rate equation for the face-to-face model

$$\frac{\partial N_1(t_0,t)}{\partial \tau} = -\left[2+(1-\lambda)c\right] f_1(t_0,t) N_1(t_0,t) + \pi_0(t_0)\delta_{t,t_0}$$

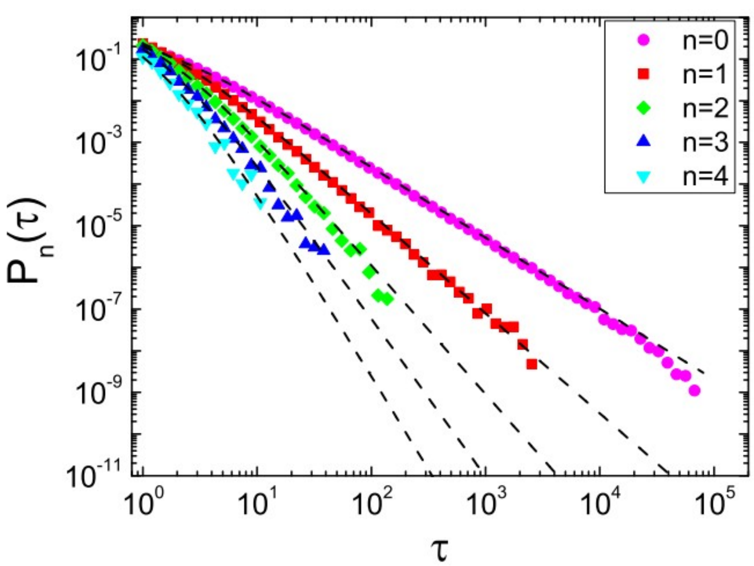
$$\frac{\partial N_n(t_0,t)}{\partial \tau} = -n f_n(t_0,t) N_n(t_0,t) + \pi_n(t_0)\delta_{t,t_0} \quad n \geq 2$$

with stationary solution

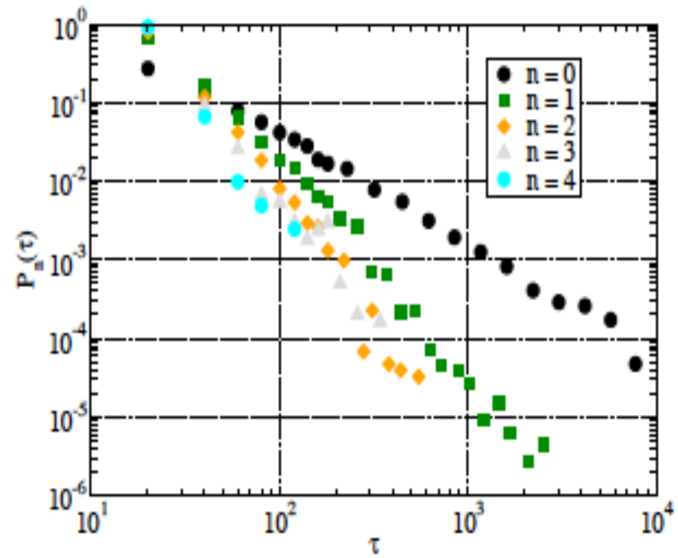
$$N_1(\tau) \propto (1+\tau)^{-[2+(1-\lambda)/(1-2\lambda)]b_1}$$

$$N_n(\tau) \propto (1+\tau)^{-nb_n}$$

The Dynamical Model



Dynamical model



Empirical data from ESRW conference

J. Stehle, A. Barrat, G. Bianconi PRE (2010).
Zhao K, J. Sthele, G. Bianconi, A. Barrat PRE (2011)

The Dynamical Model

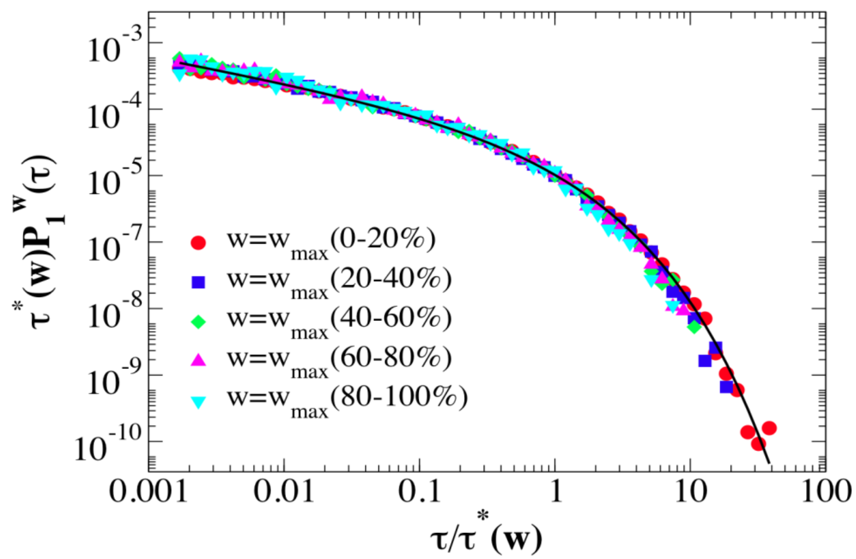
Cellphone Model:

$$f_1(t_i, t) = \frac{b_1}{(1+\tau)^{\beta}}$$

$$f_2(t_i, t \mid w_{ij}) = \frac{b_2 g(w)}{(1+\tau)^{\beta}}$$

β is a parameter to characterize the adaptability of human social interaction.

The Dynamical Model



Numerical result of interaction time of the dynamical cellphone model. The data are described by Weibull distribution

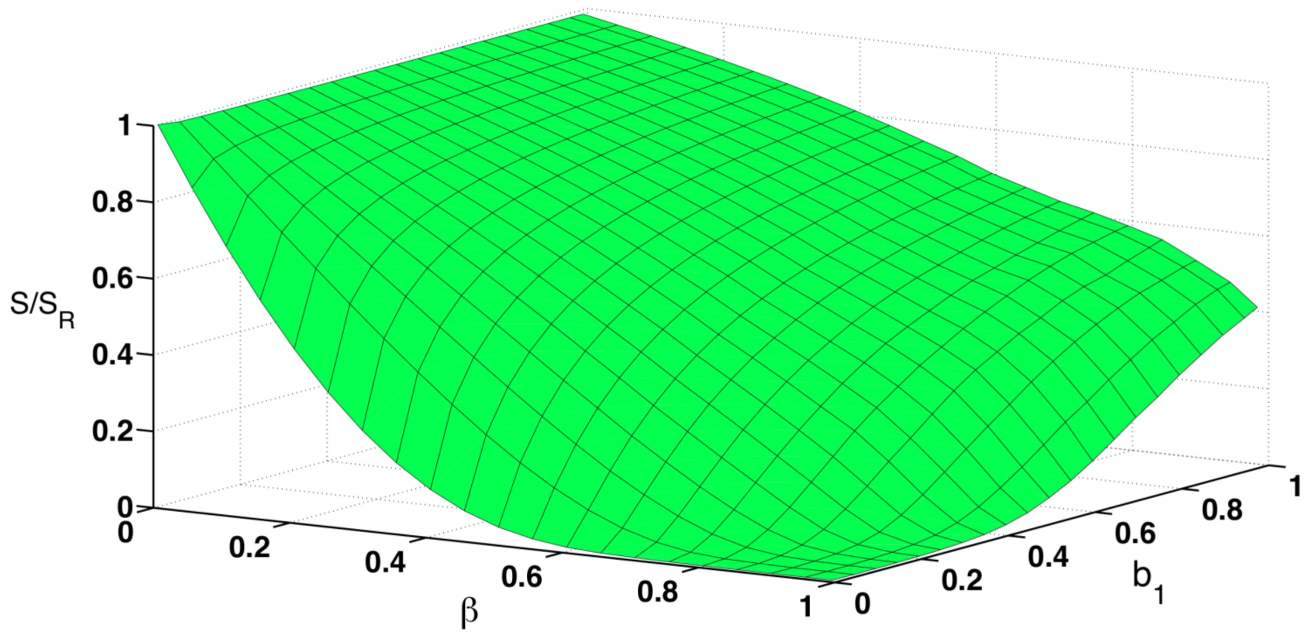
$$\tau^*(w)P(x=\Delta t_{int}/\tau^*(w)) \propto x^{-\beta}\exp[-x^{1-\beta}/(1-\beta)]$$

Human adaptability

Human adaptability to mobile phone technology can be seen as an effective modulation of the parameter β

from β=1 in face-to-face interaction to β=0.45.. in mobile phone communication

Entropy Analysis of The Cellphone Model



Conclusion

- Human social networks are highly dynamical and adaptive
- The entropy of dynamical social networks is able to characterize the information present in them
- Human social interaction on a fast timescales are characterized by a dynamics with reinforcement that is able to predict both power-law and Weibull distribution of durations of contacts
- The human dynamics is able to modulate the dynamical entropy of social interactions, during the day following circadian rhythms and when interfacing with a different technology as in mobile phone communication

Collaborators

Kun Zhao
Northeastern University, Boston, USA

Marton Karsai
Aalto University, Finland

Alain Barrat
University of Marseille, France

Juliette Stehle
University of Marseille, France