Controlling Networks

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Understand

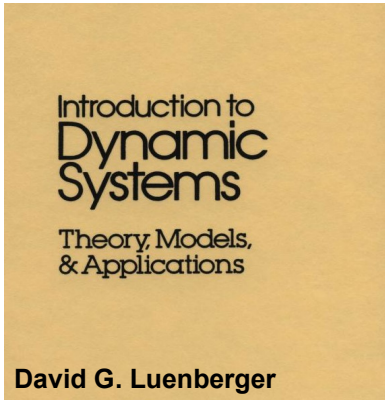
quantify

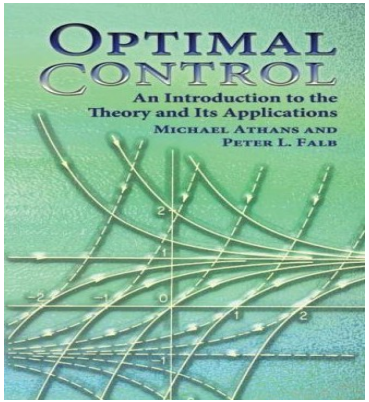
predict

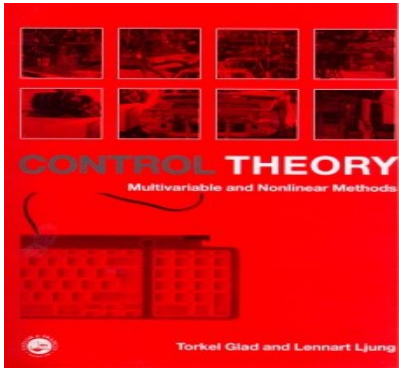
control

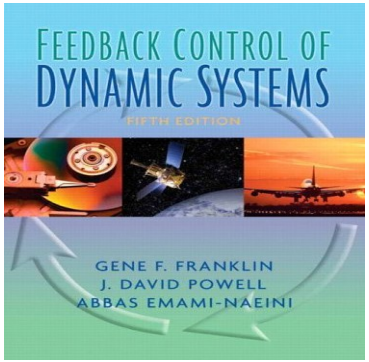
CONTROL YOUR EMOTIONS

Control Theory

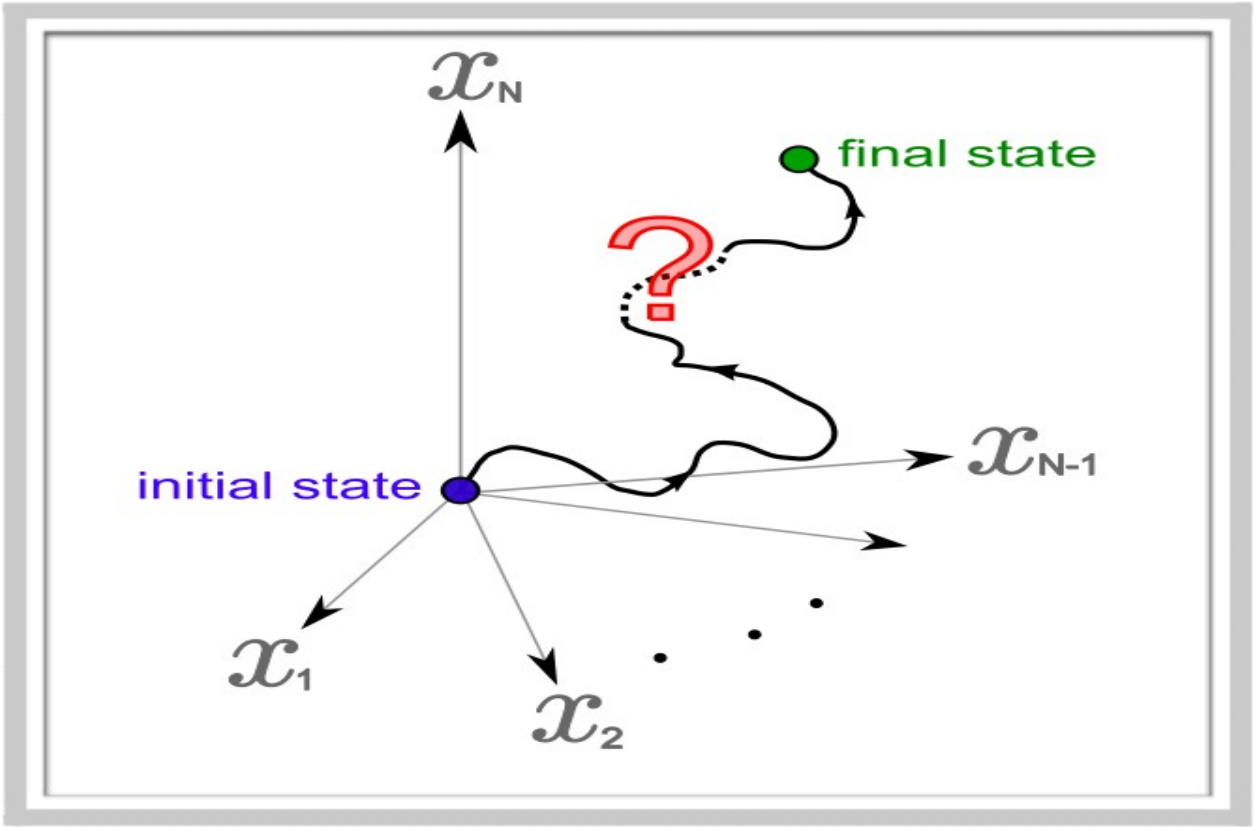




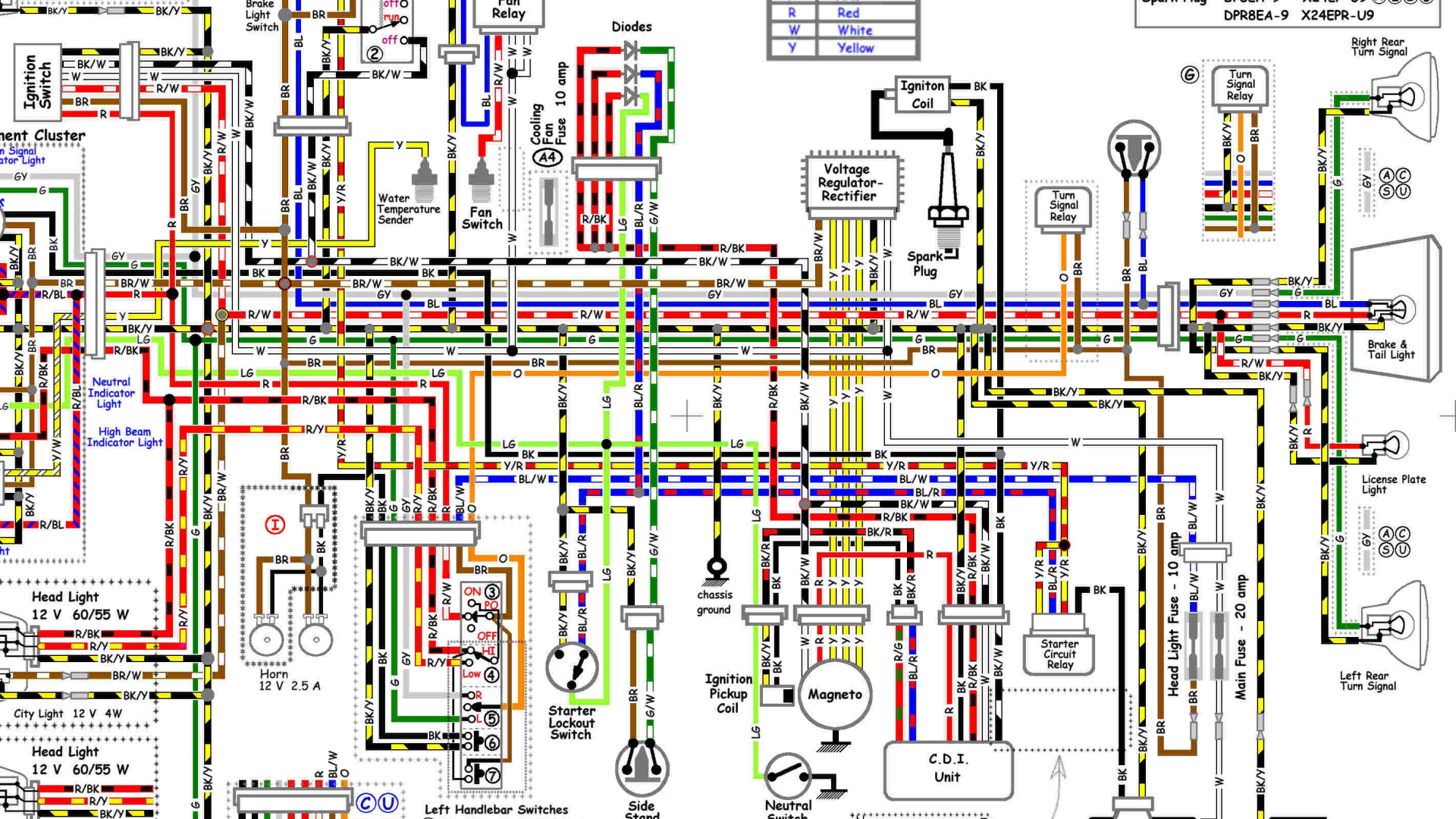


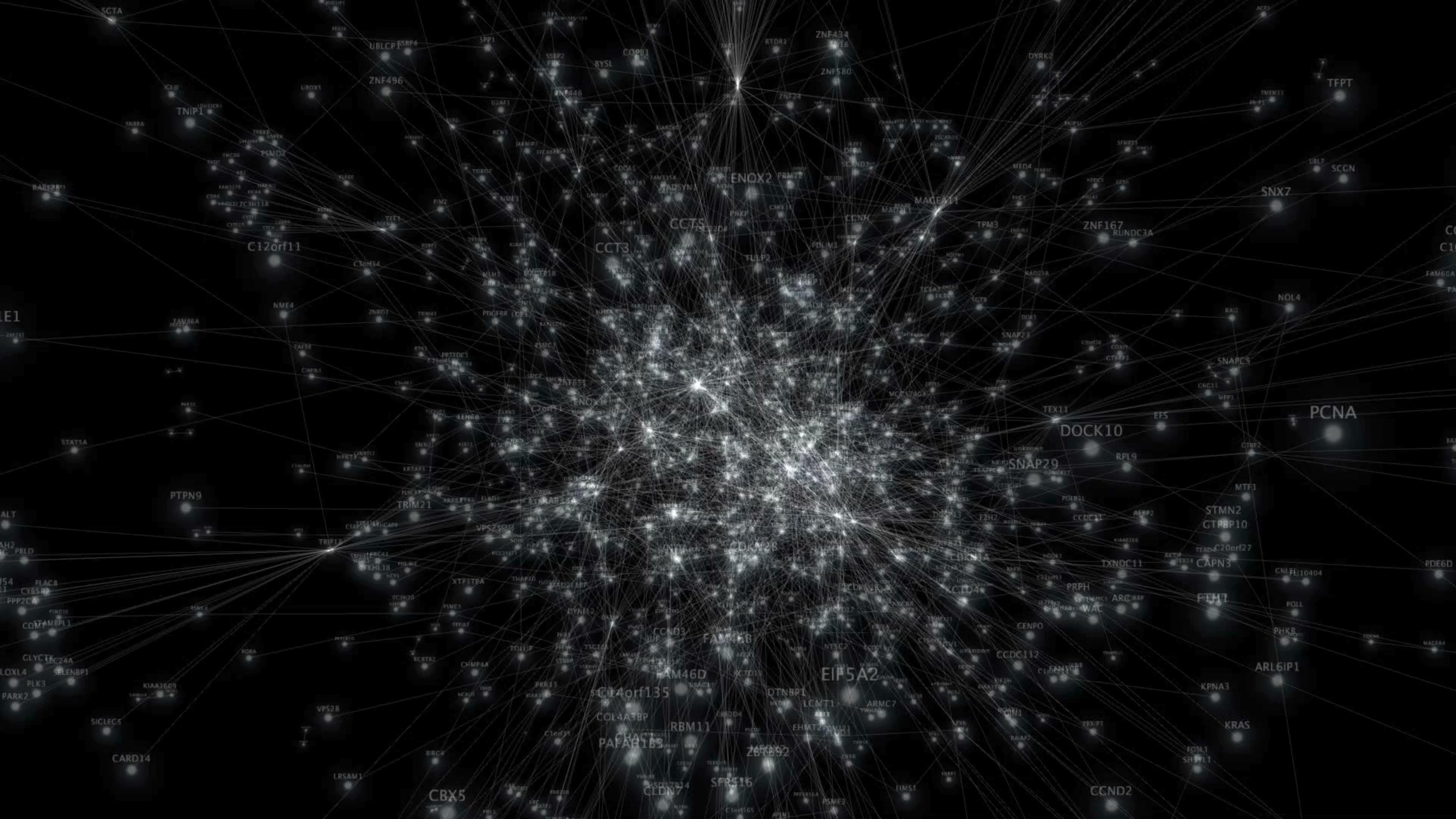


A system is controllable if it can be driven from any **initial state** to any desired **final state**.



A system is controllable if it can be driven from any initial state to any desired final state in finite time.


Ignition Switch
Brake Light Switch
Fan Relay
Diodes
Cooling Fan Fuse 10 amp
A4
Fan Switch
Water Temperature Sender
Igniton Coil
Voltage Regulator-Rectifier
Spark Plug
Turn Signal Relay
Right Rear Turn Signal
Left Rear Turn Signal
Brake & Tail Light
License Plate Light
Neutral Indicator Light
High Beam Indicator Light
Horn
12 V 2.5 A
Head Light
12 V 60/55 W
City Light 12 V 4W
Head Light
12 V 60/55 W
chassis ground
Starter Lockout Switch
Ignition Pickup Coil
Magneto
Starter Circuit Relay
C.D.I. Unit
Neutral Switch
Side Stand
Left Handlebar Switches
Head Light Fuse - 10 amp
Main Fuse - 20 amp
ON
RO
OFF
HI
Low
R
L
off
run
off
R Red
W White
Y Yellow
DPR8EA-9 X24EPR-U9


SGTA
UBLCP1
ZNF496
TNIP1
PSMD2
C12orf11
ZNF434
ZNF580
ENOX2
CCT5
CCT3
MAGEA11
TULP2
ZNF167
RUNDC3A
SNX7
SCGN
TFPT
NOL4
NME4
PCNA
DOCK10
TEX11
SNAP29
RPL9
STAT5A
PTPN9
TRIM21
MTF1
STMN2
GTPBP10
C20orf27
CAPN3
TXNDC11
PDE6D
PRPH
FTH1
CENPO
CCDC112
ARL6IP1
KPNA3
KRAS
FAM46B
FAM46D
EIF5A2
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ARMC7
COL4A3BP
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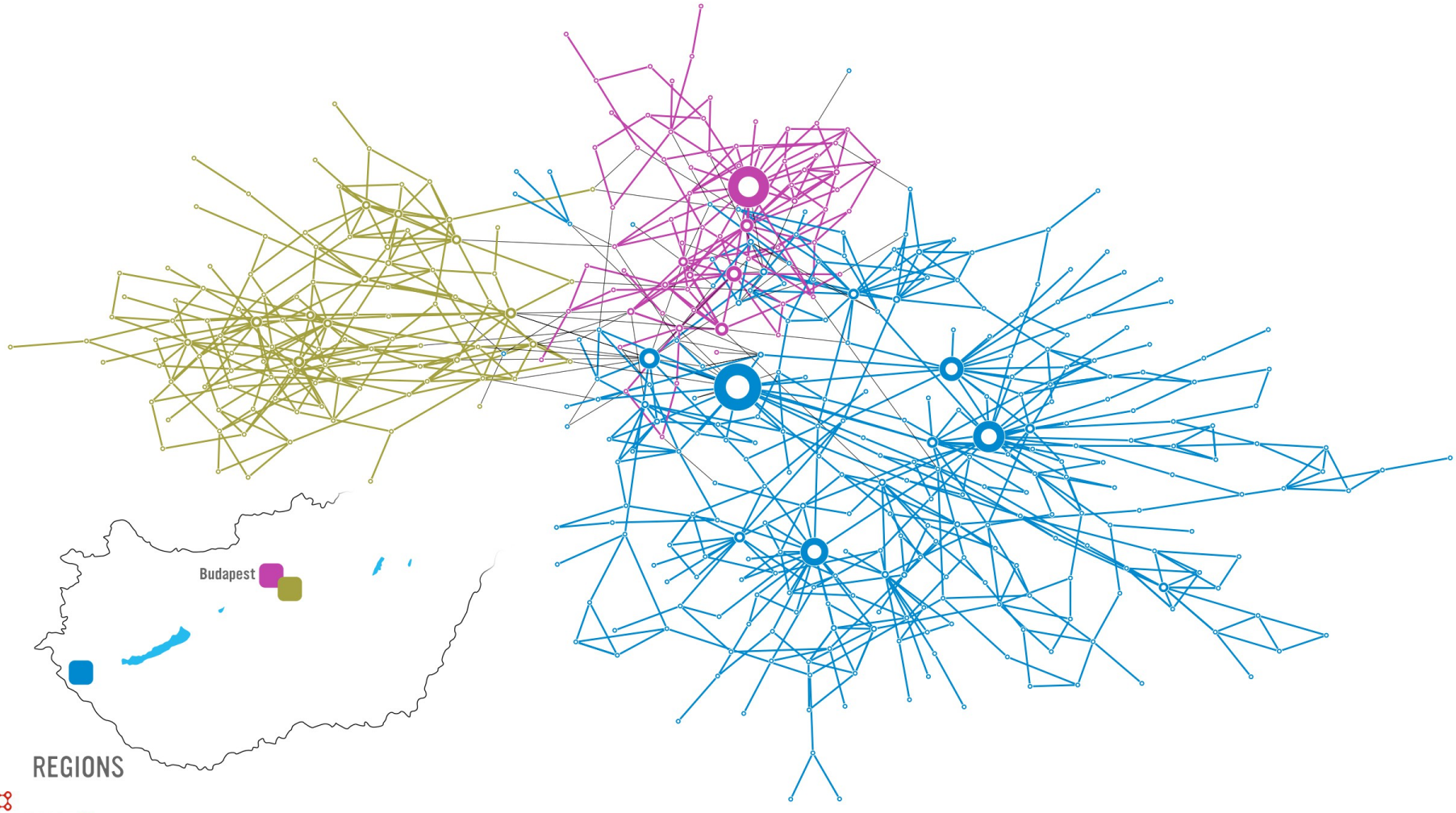
SIGLEC5
VPS28
CARD14
LRSAM1
CBX5
CCND2



ARTICLE

doi:10.1038/nature10011

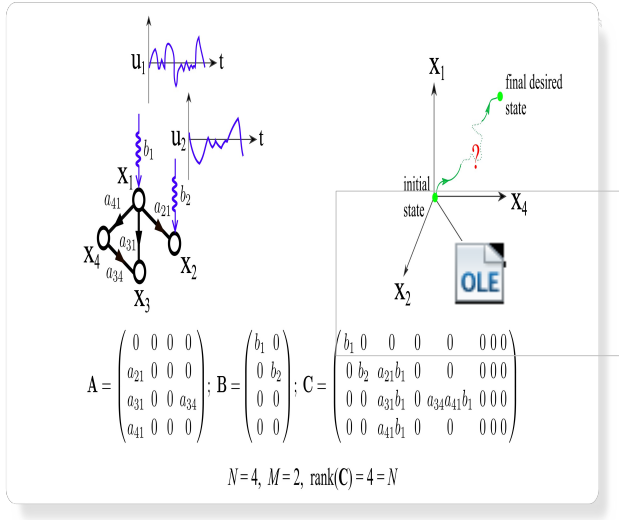
Controllability of complex networks

Yang-Yu Liu[1,2], Jean-Jacques Slotine[3,4] & Albert-László Barabási[1,2,5]

The ultimate proof of our understanding of natural or technological systems is reflected in our ability to control them. Although control theory offers mathematical tools for steering engineered and natural systems towards a desired state, a framework to control complex self-organized systems is lacking. Here we develop analytical tools to study the controllability of an arbitrary complex directed network, identifying the set of driver nodes with time-dependent control that can guide the system's entire dynamics. We apply these tools to several real networks, finding that the number of driver nodes is determined mainly by the network's degree distribution. We show that sparse inhomogeneous networks, which emerge in many real complex systems, are the most difficult to control, but that dense and homogeneous networks can be controlled using a few driver nodes. Counterintuitively, we find that in both model and real systems the driver nodes tend to avoid the high-degree nodes.

Linear System

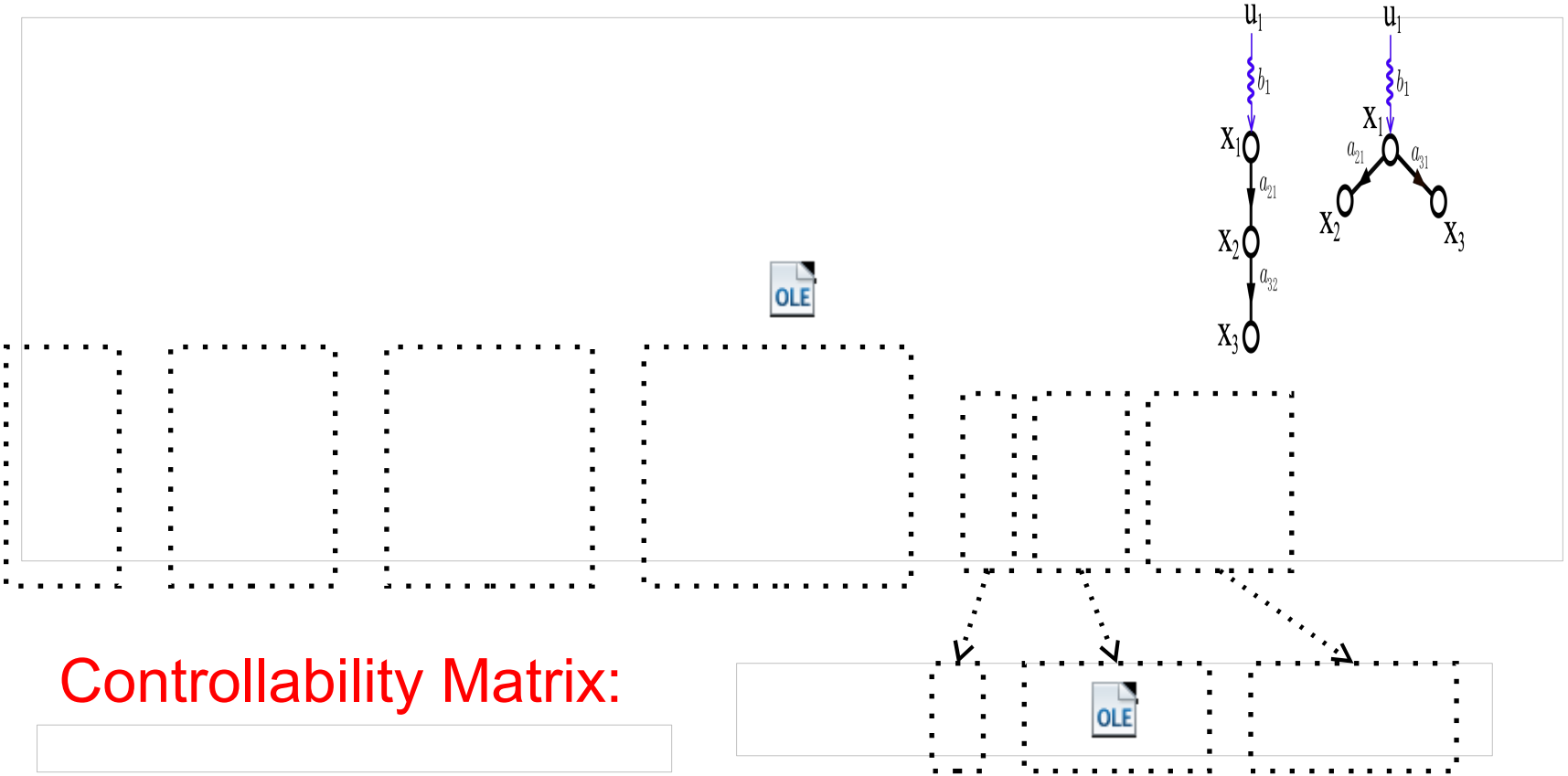
Linear Time-Invariant Dynamics



Kalman's Rank Condition: A system is controllable if its controllability matrix has full rank.

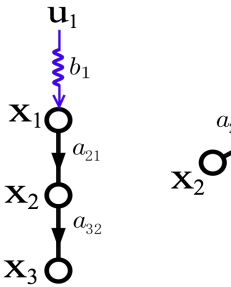
R. E. Kalman, *J.S.I.A.M. Control* (1963)

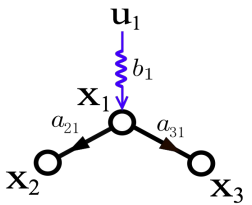
Example 1: Controllable

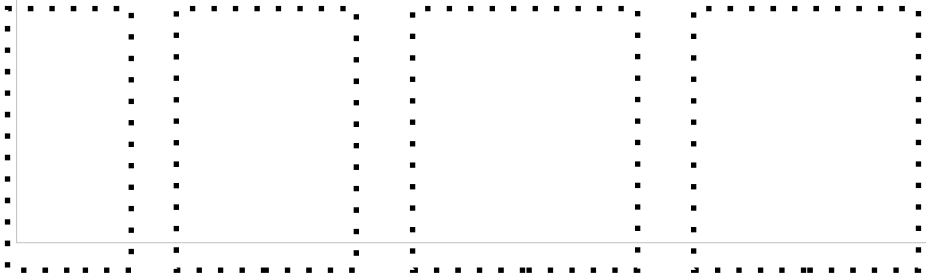


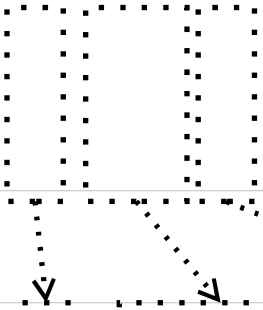
Controllability Matrix:

Example 2: Uncontrollable



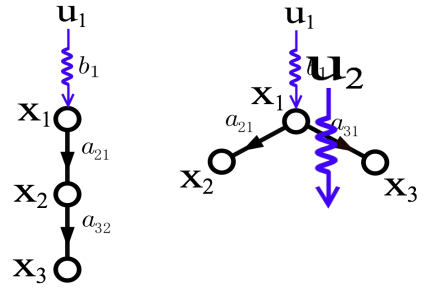


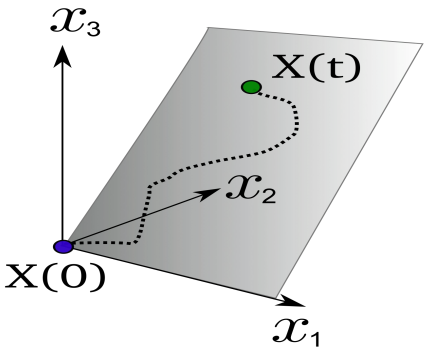




Controllability Matrix:

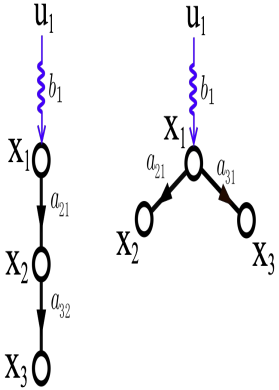
Example 2: Uncontrollable



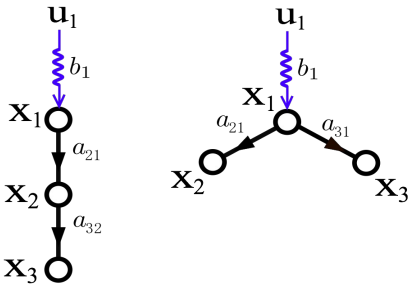


The system is stuck in a plane in the state space.

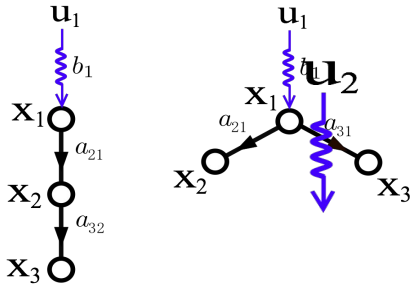
EXAMPLES: Controllable or not controllable?



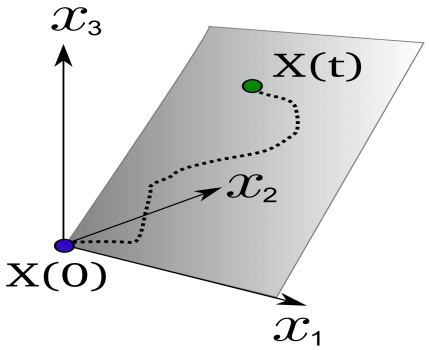
Yes



No



Yes



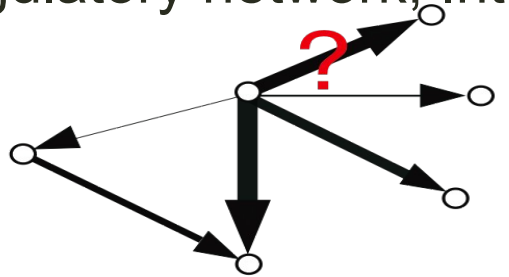
What's the minimum number of driver nodes (*N*D)?

How to efficiently identify them?

Which network characteristics determine *N*D?

Difficulties

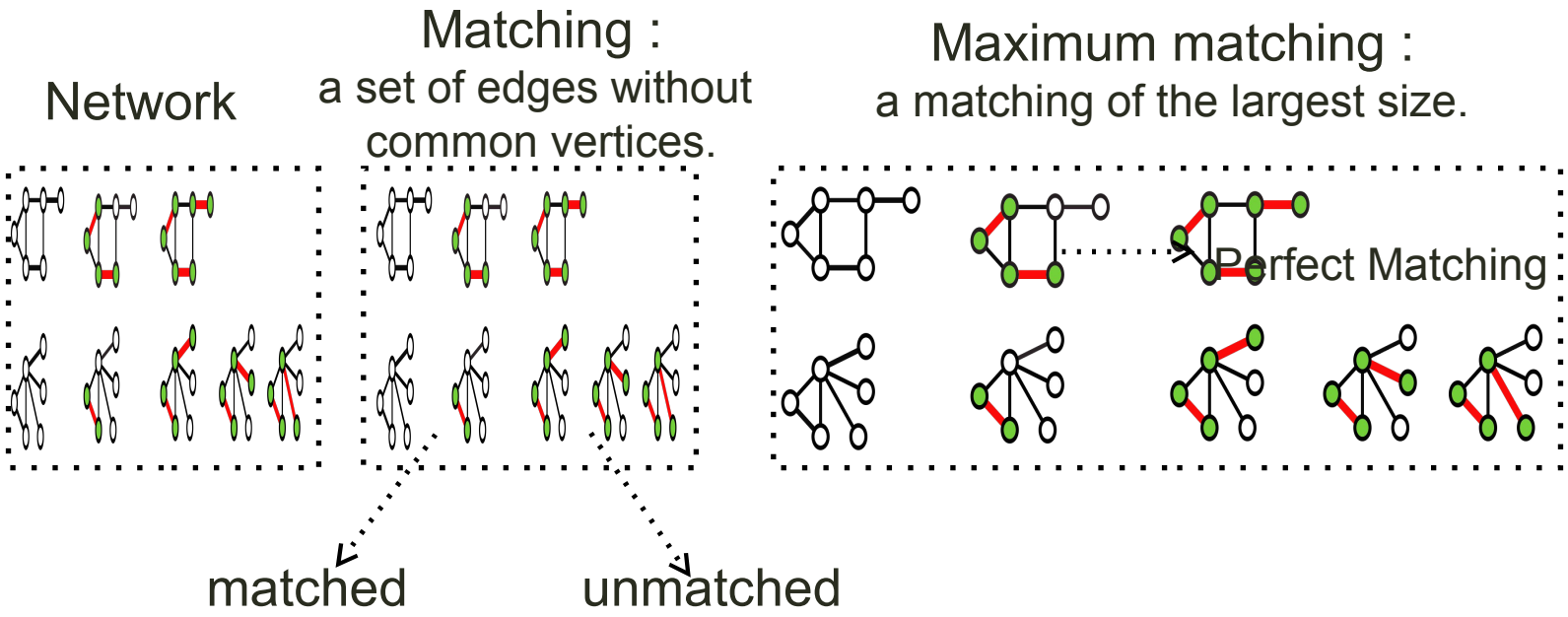
1. Parameters (link weights): usually unknown.
   e.g. gene regulatory network, Internet, etc.



2. If brute-force search: $(2N-1)$ combinations.

3. Kalman's rank condition is hard to check for large system.

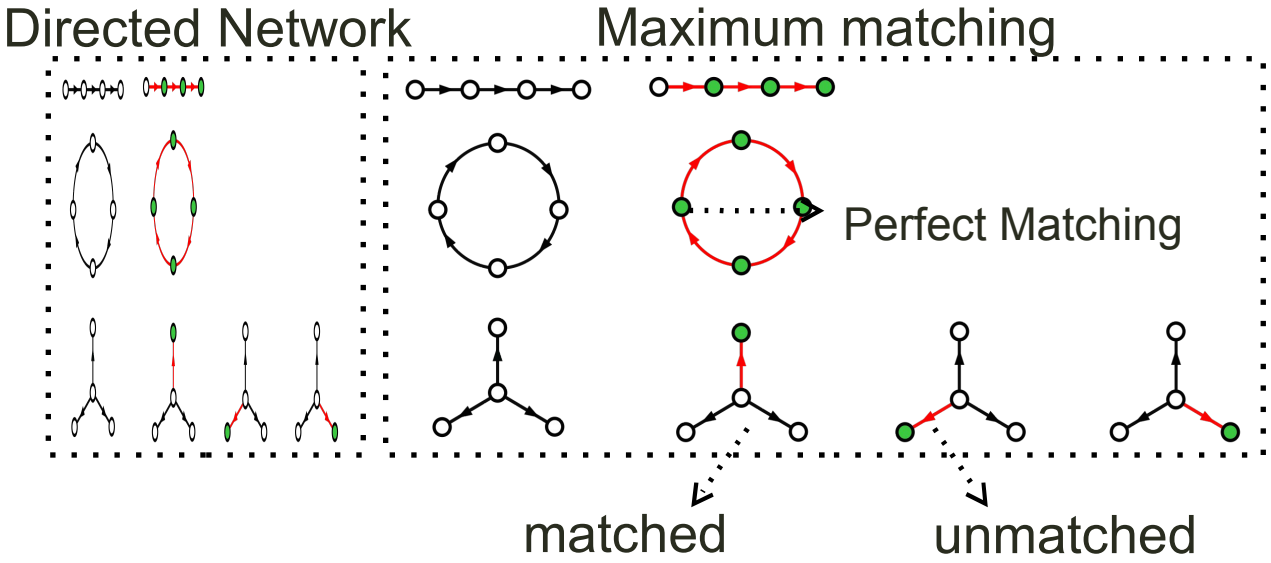
Matching



Lovász, L. & Plummer, M.D., *Matching Theory*

Matching in Directed Network

Matching : a set of edges without common heads or tails.

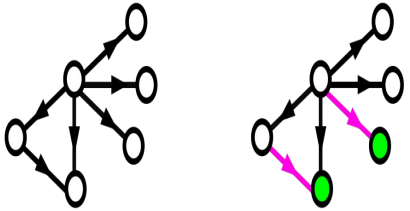


Minimum Input Theorem:
Driver nodes = Unmatched nodes

Y.-Y. Liu, J.-J. Slotine, A.-L. Barabasi, *Nature* (2011)

Example

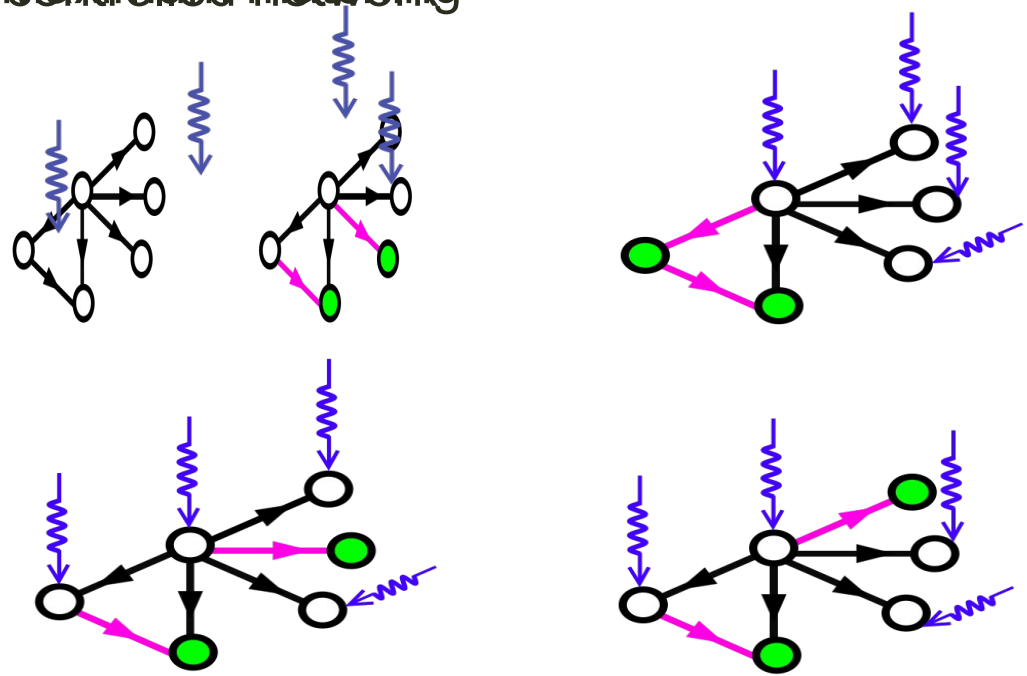
network



Brute-force search
$O(2N)$
~1030 for N=100.
Hopeless!

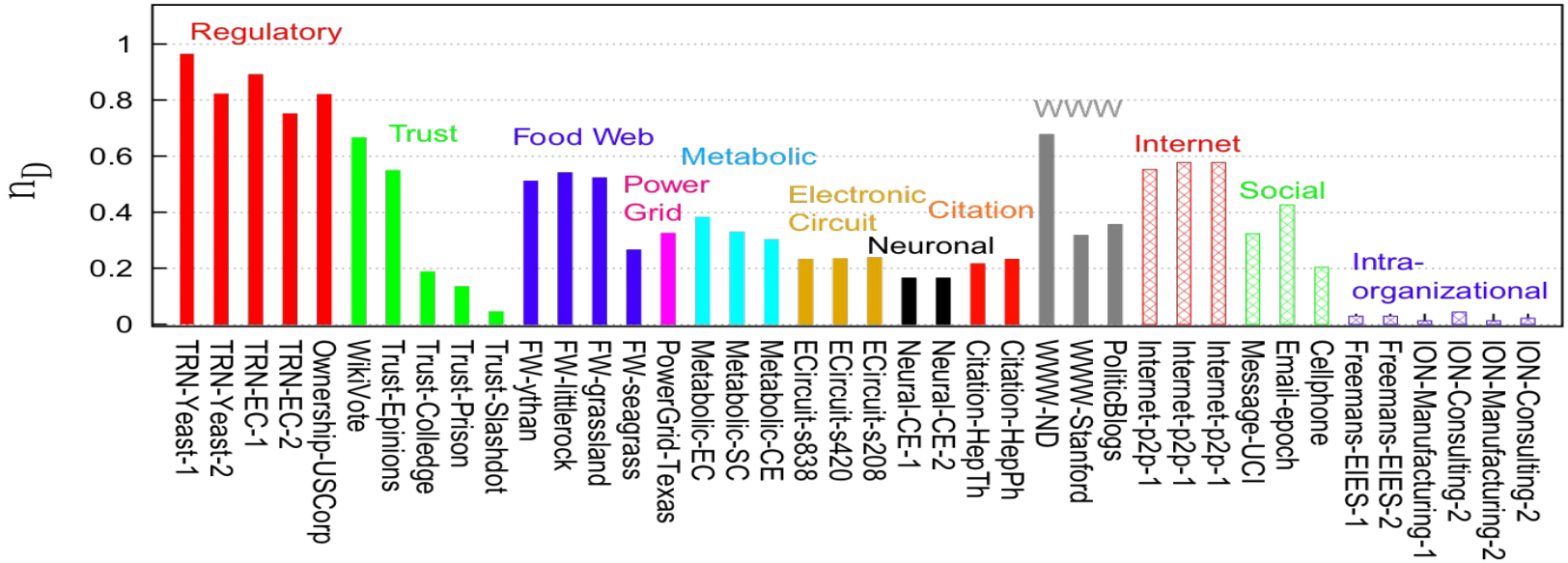
Hopcroft-Karp Algorithm
$O(N1/2L)$
Polynomial!
Fast even for N~106.

Maximum matching



Y.-Y. Liu, J.-J. Slotine, A.-L. Barabasi, *Nature* (2011)

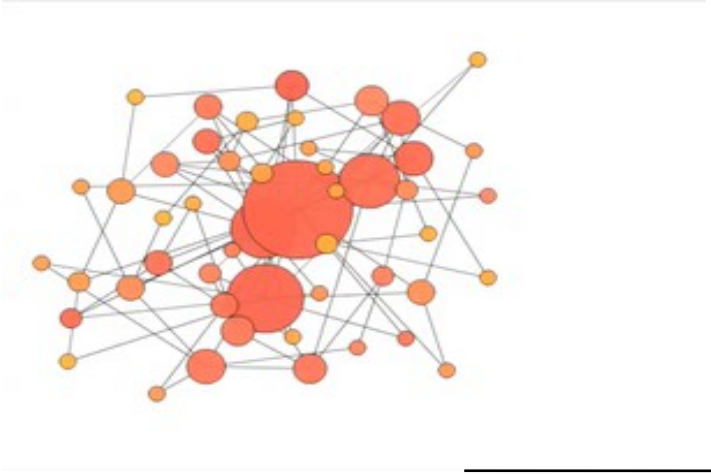
*N*D of real networks



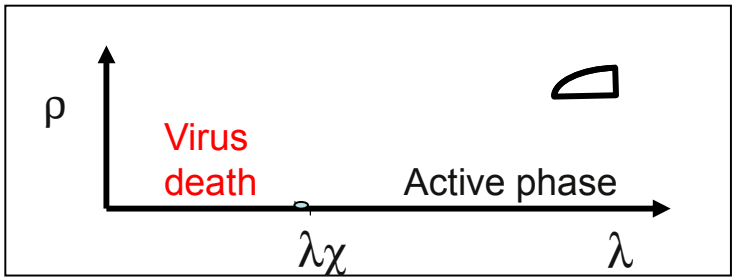
1. Overall we see no obvious trend in *n*D (or *N*D) across these networks.
2. As a group, regulatory networks display very high *n*D ≈ 0.8.
3. A few social networks display the smallest observed *n*D values.

Hubs matter!

Network Robustness

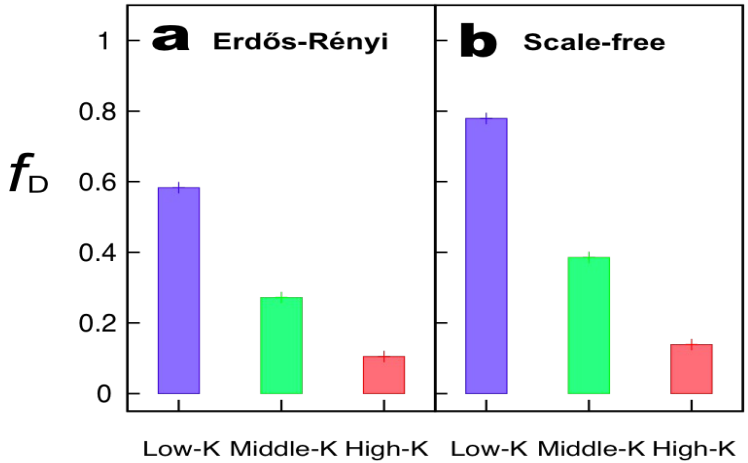


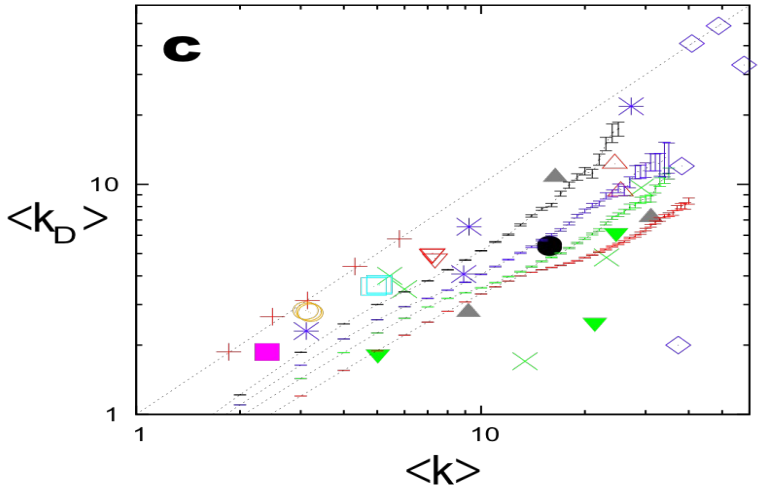
Spreading Phenomena



Cohen/Havlin, *PRL* (2001); Pastor Satorras & Vespignani, *PRL* (2001)

Role of hubs





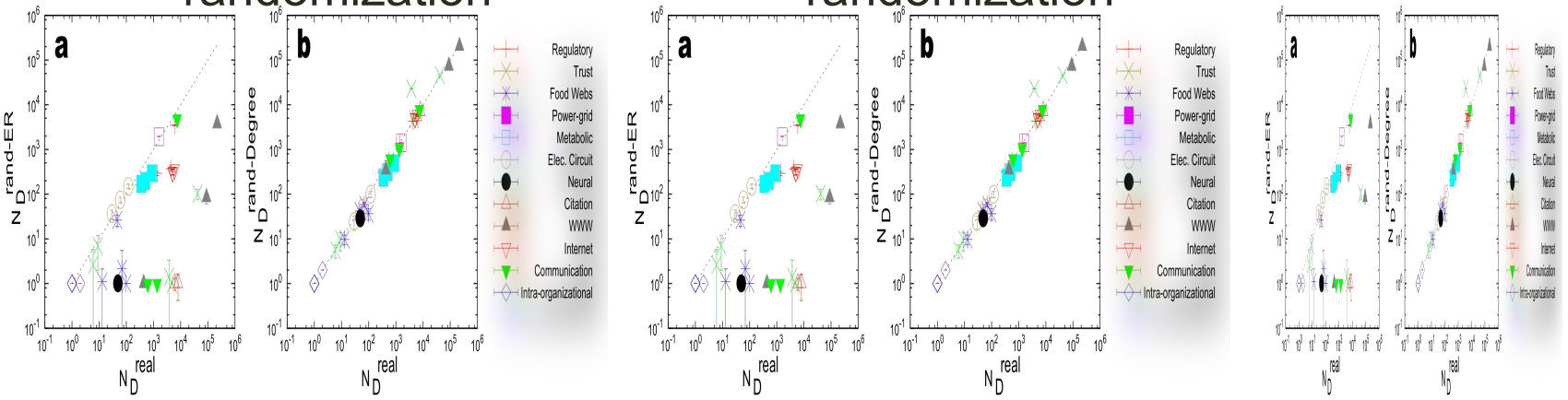
1. The fraction of driver nodes is significantly higher among low degree nodes than among the hubs.
2. Mean degree of driver nodes <*k*D> is either significantly smaller or comparable to <*k*>.

Driver nodes tend to avoid the hubs.

*N*Dreal vs. *N*Drand

Complete randomization

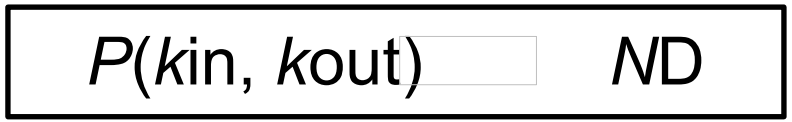
Degree-preserving randomization



*N*D is mainly determined by degree distribution.

Key Result

*N*D is primarily determined by degree distribution.

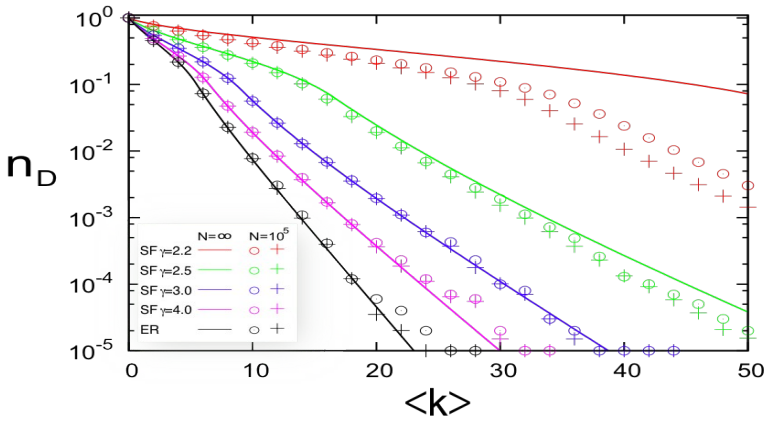


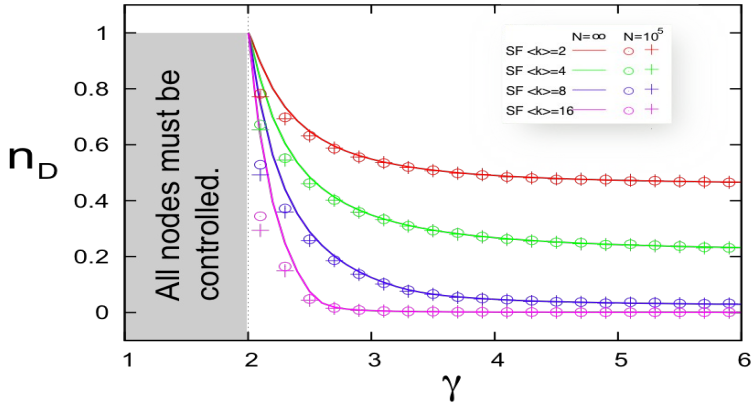
1. The number of driver nodes does not depend on the precise wiring diagram, but only on the degree distribution.

2. The wiring diagram is needed only if we want to know which are the driver nodes.

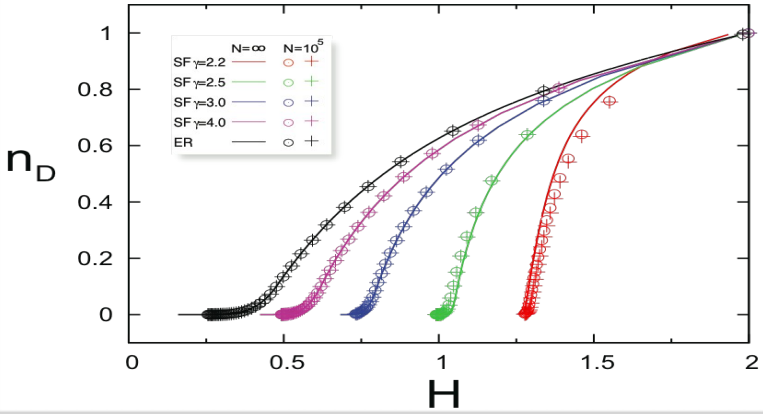
3. Allows us to analytically calculate the average *N*D over all network realizations compatible with *P*(*k*in, *k*out), using the ***cavity method***.

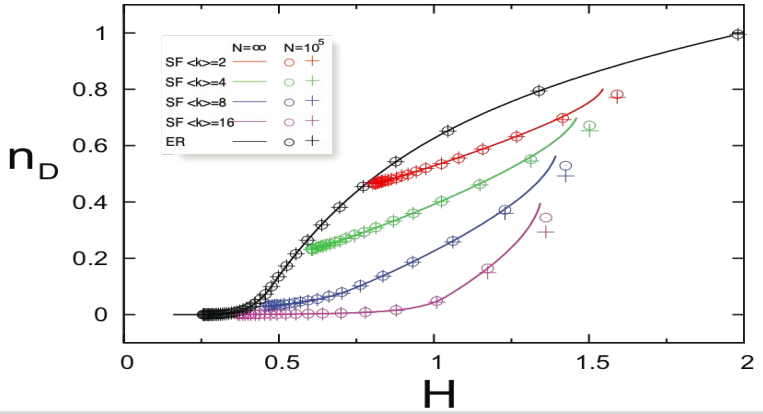
Degree Dependence





Degree Heterogeneity



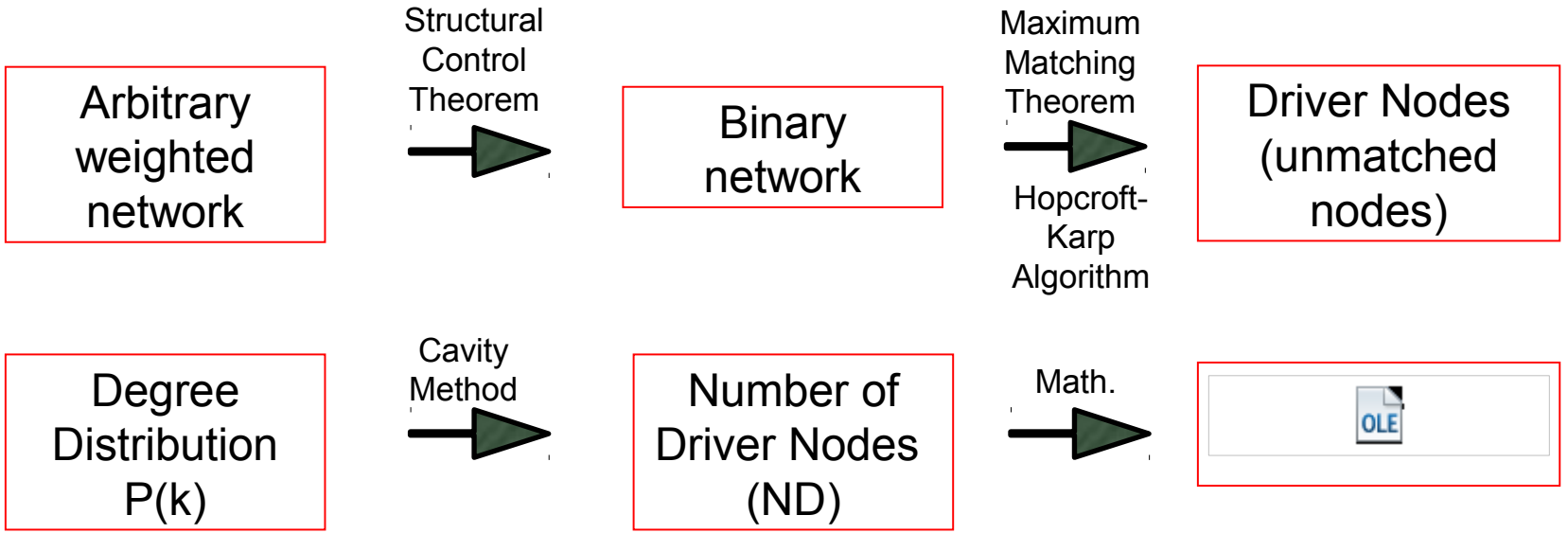


Degree heterogeneity $H = 2 \times$ Gini coefficient

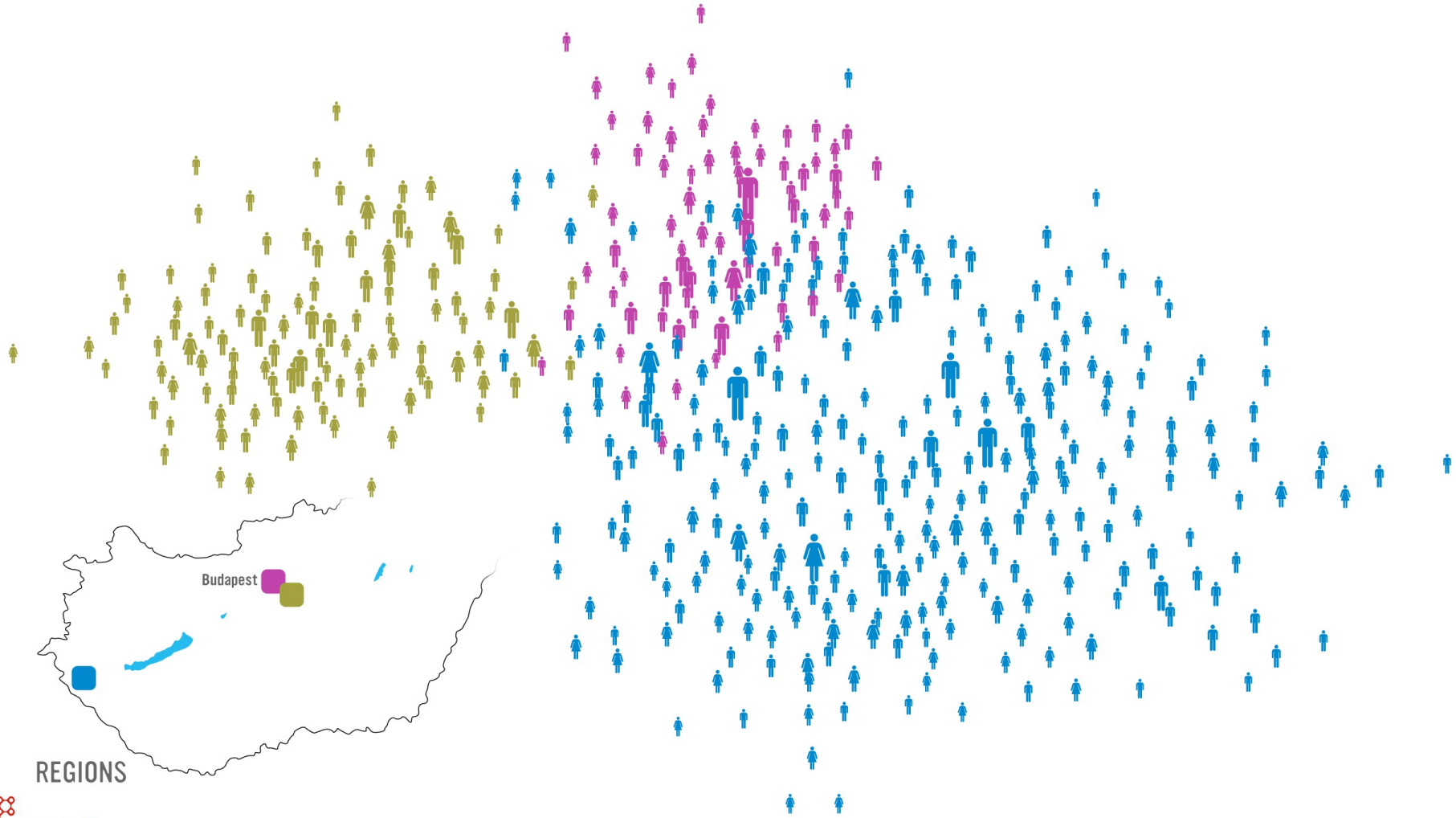
Results

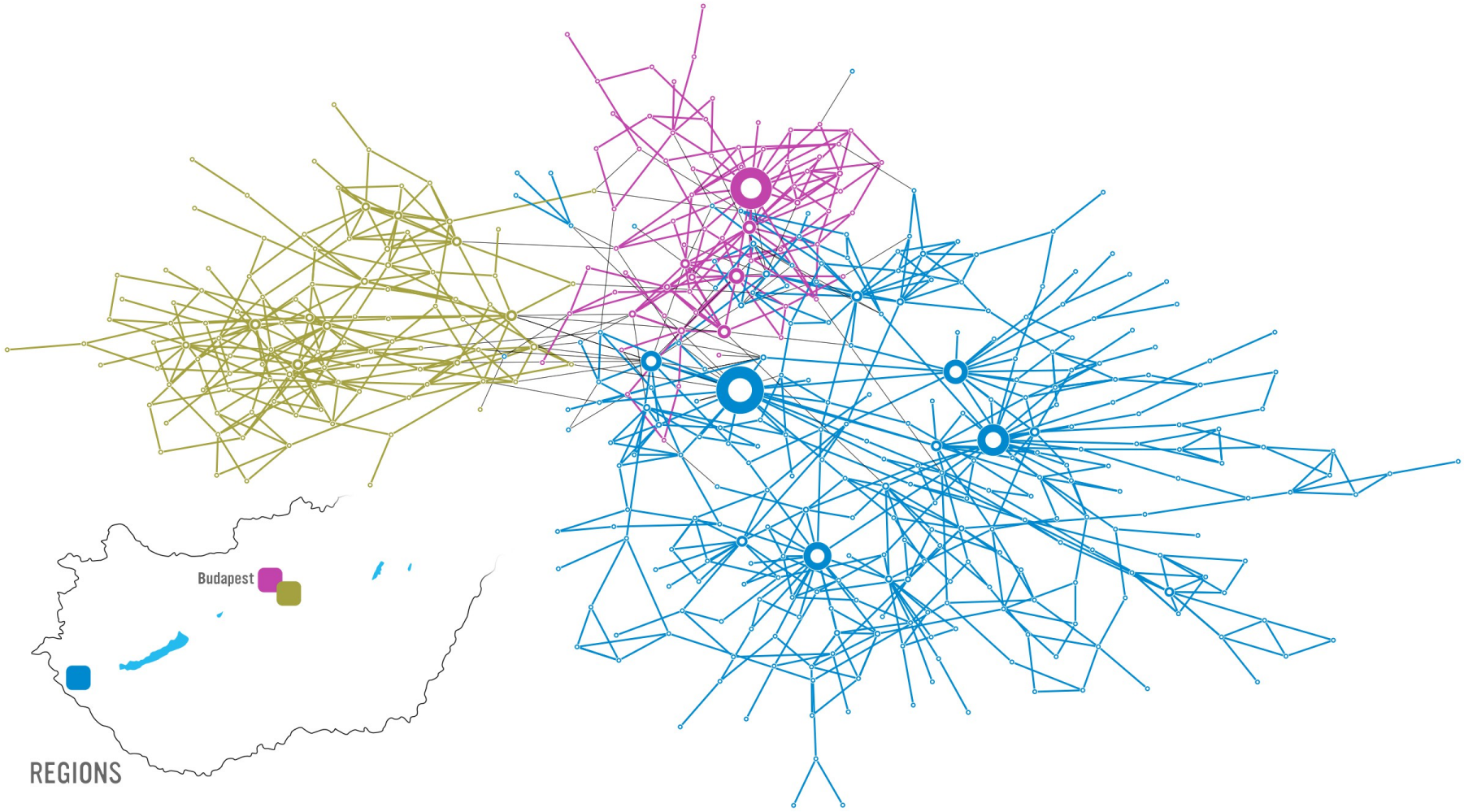
Mean degree <*k*> and **degree heterogeneity** *H* are the two main factors that determine *N*D.
Sparse and **heterogeneous** networks are harder to control than **dense** and **homogeneous** networks.

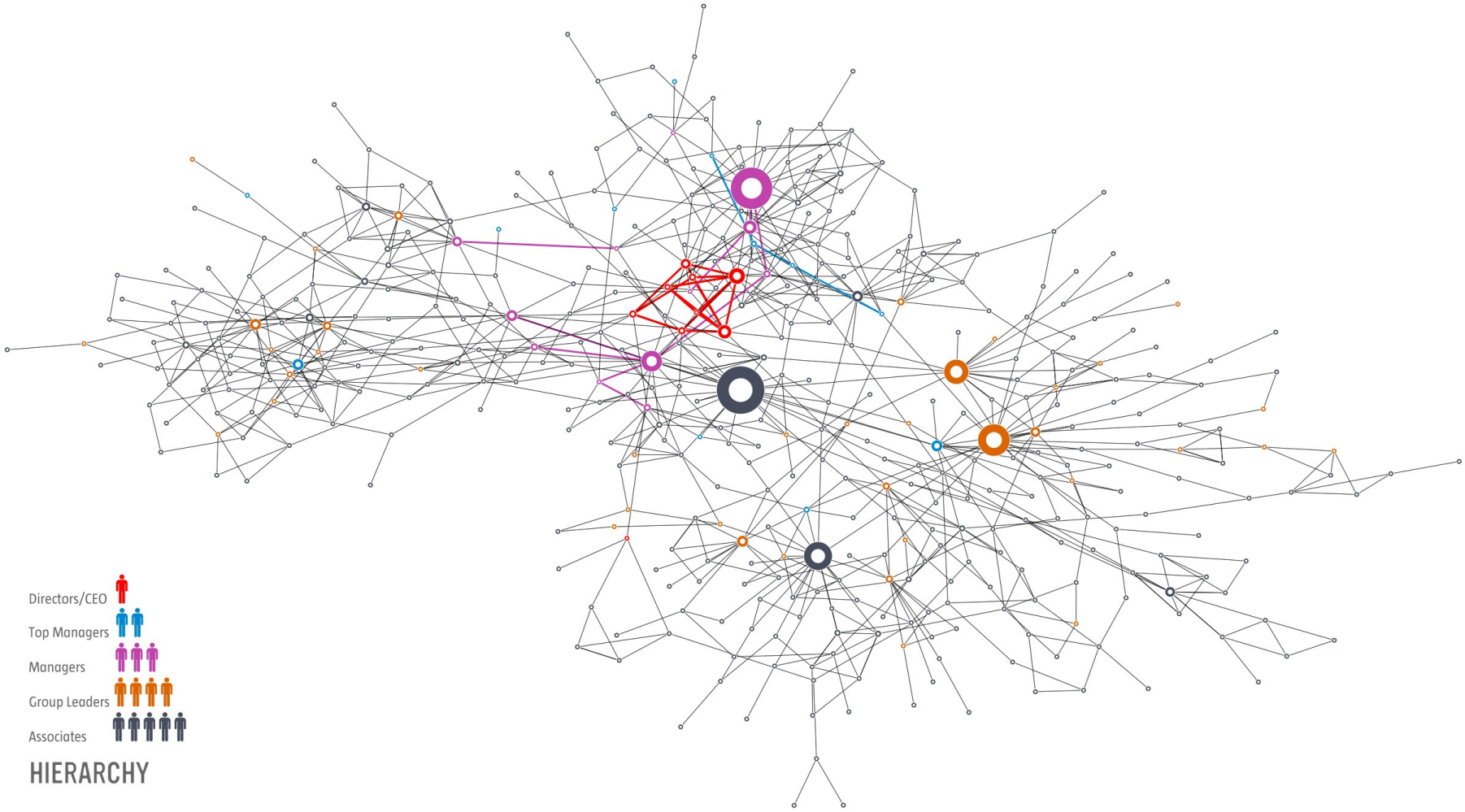
Summary/Outline



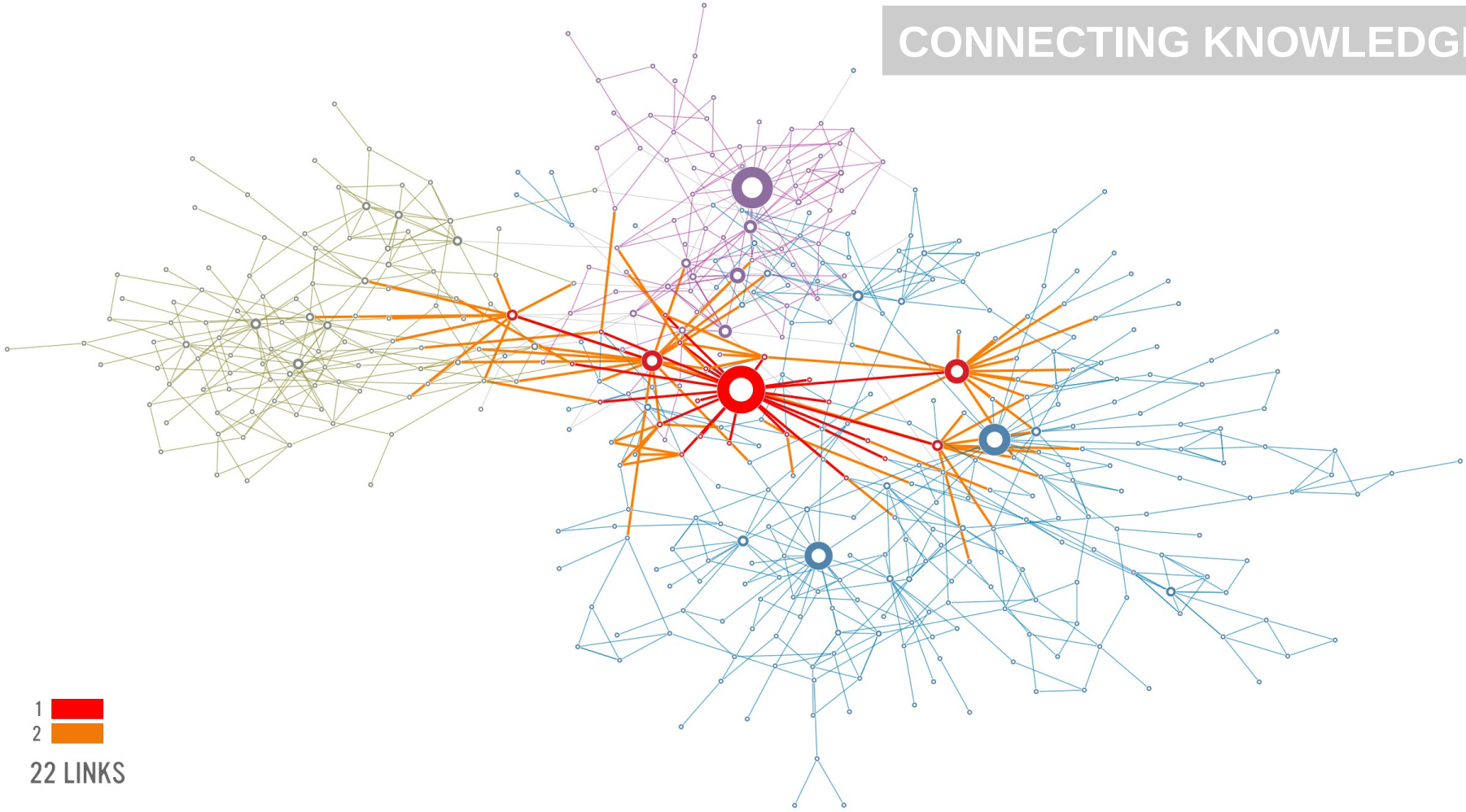
Y.-Y. Liu, J.-J. Slotine, A.-L. Barabasi, *Nature* (2011)

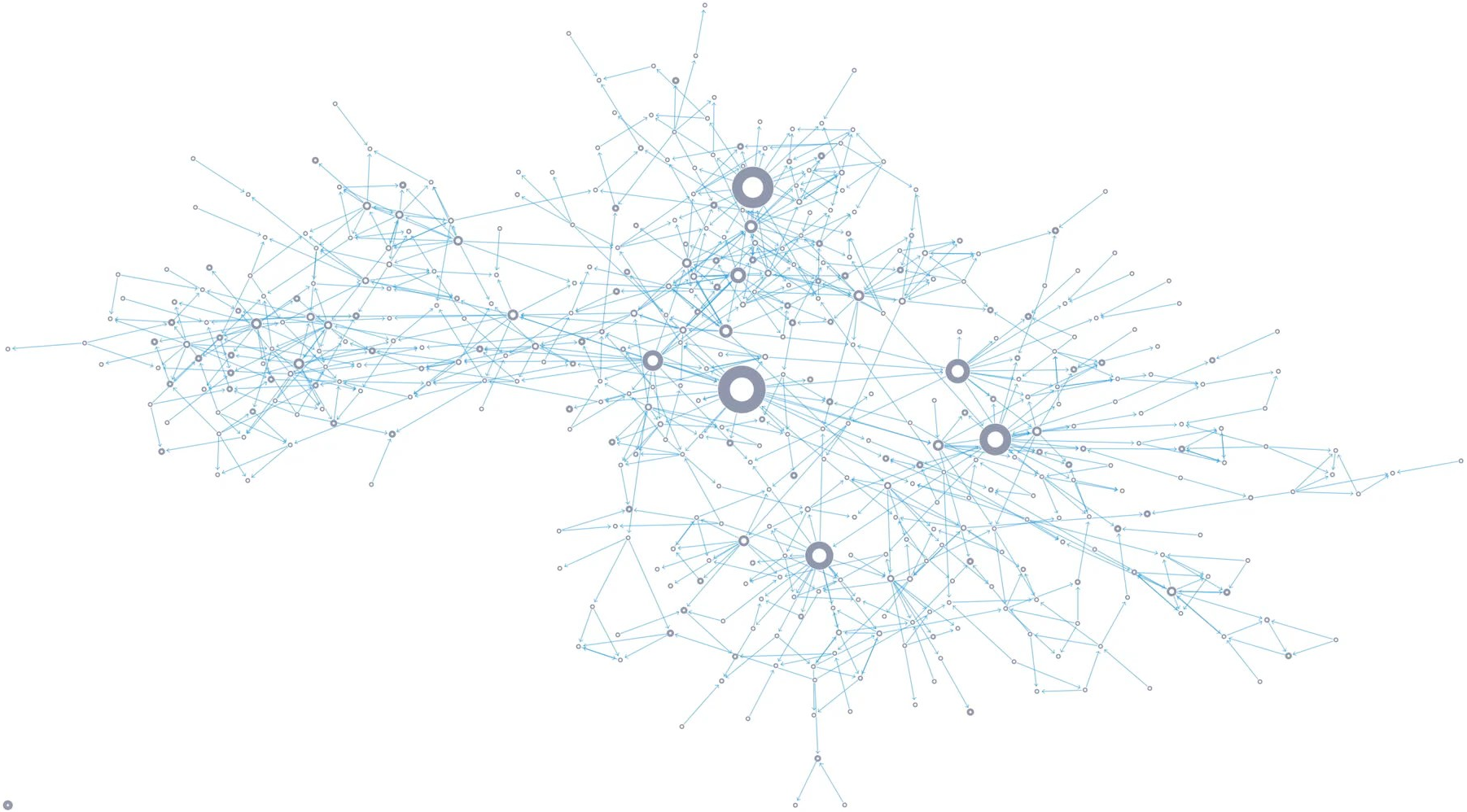



Budapest
REGIONS

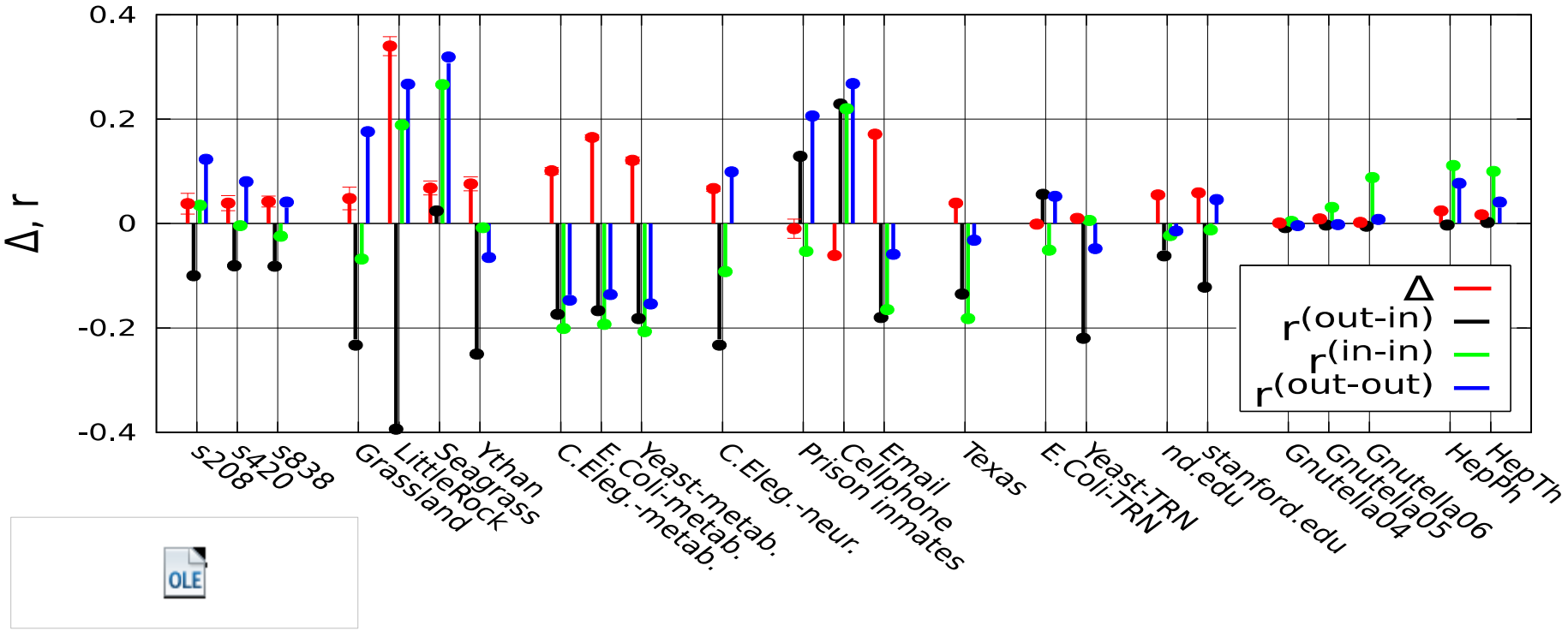

Directors/CEO
Top Managers
Managers
Group Leaders
Associates
HIERARCHY

CONNECTING KNOWLEDGE





Nepusz and Vicsek, arXiv:1112.5945



Clustering Coefficient

Communities

Degree correlations

DEGREE CORRELATIONS IN NETWORKS

Assortative:

hubs show a tendency to link to each other.

Neutral:

nodes connect to each other with the expected random probabilities.

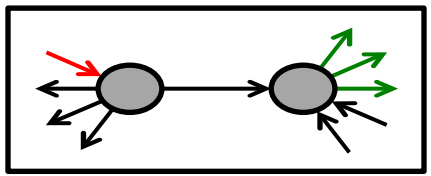
Disassortative:

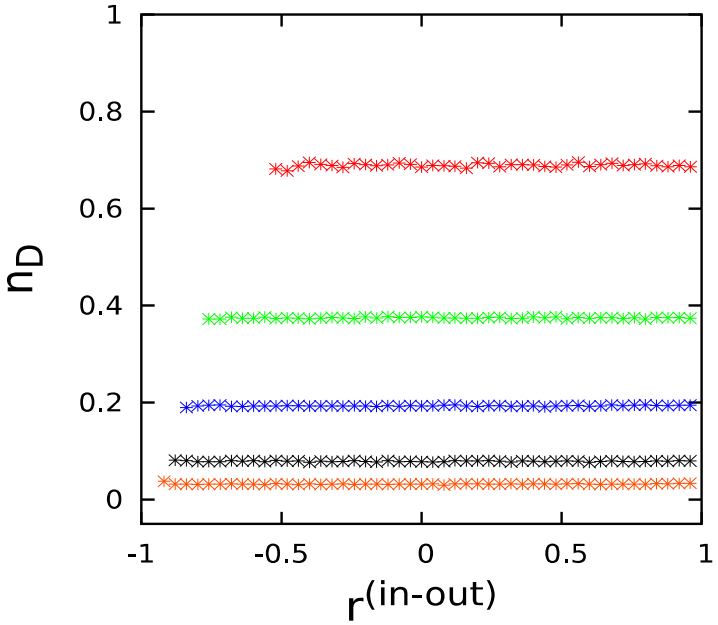
Hubs tend to avoid linking to each other.

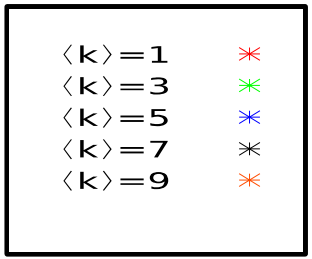
Quantifying degree correlations:

- full statistical description (Maslov and Sneppen, Science 2001)
- degree correlation function (Pastor Satorras and Vespignani, PRL 2001)
- correlation coefficient (Newman, PRL 2002)

In-Out Degree Correlations: No Dependence





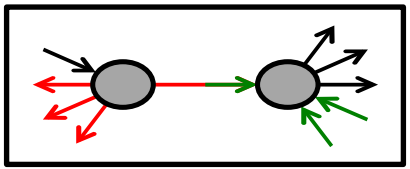


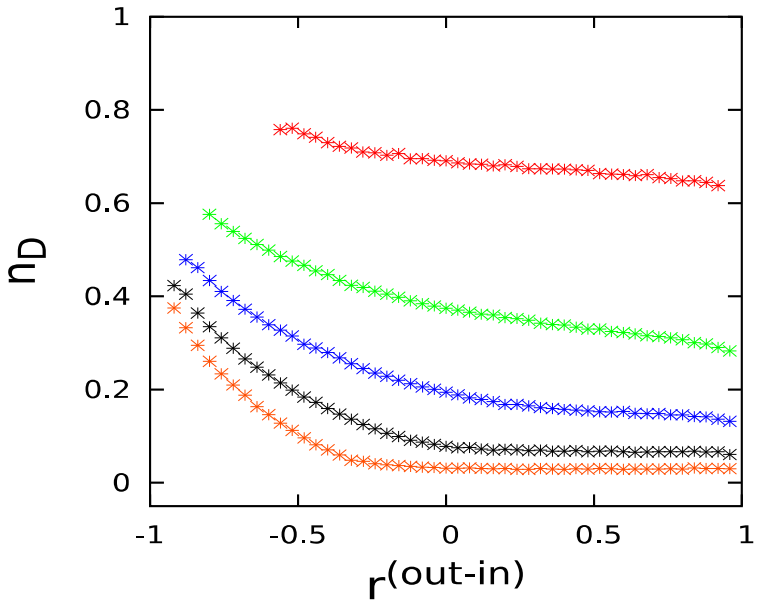
nD does *not* depend on in-out correlations

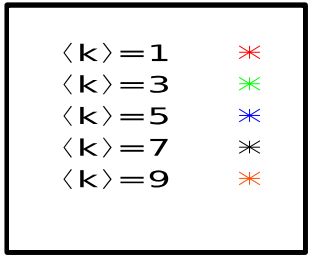
Scale-free model with structural cut-off (γ=2.5, N=1000).

Posfai, Liu, Slotine, Barabasi

Out-In Degree Correlations: Linear





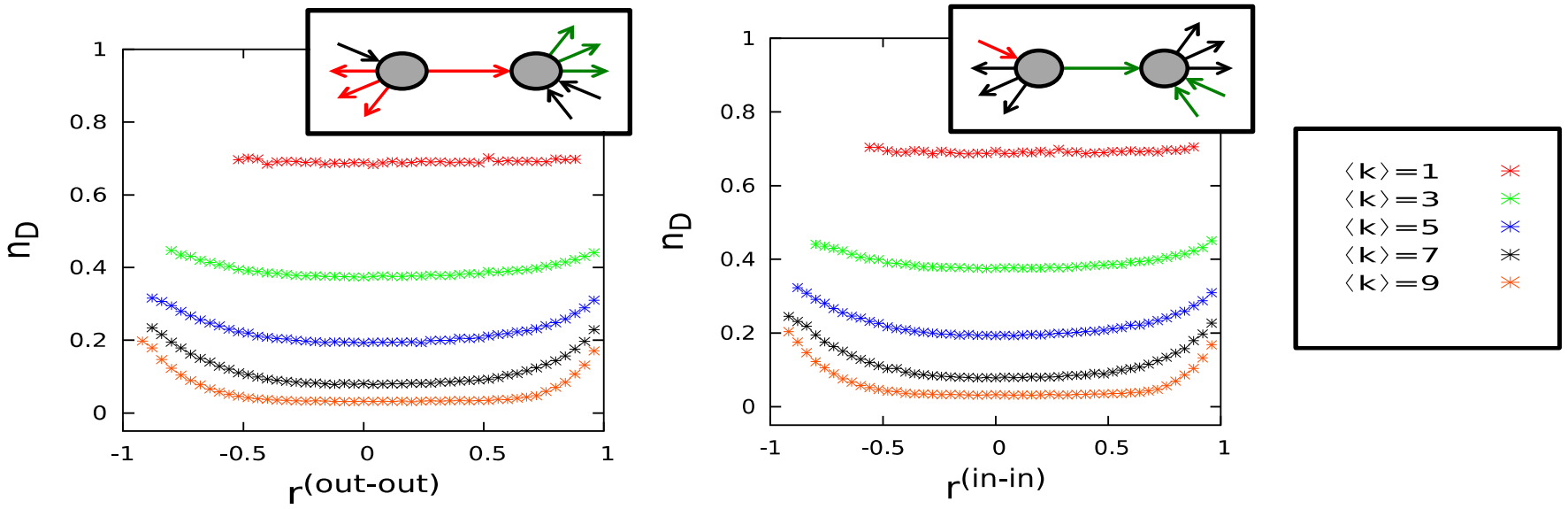


nD depends *linearly* on the out-on correlations coefficient

Scale-free model with structural cut-off (γ=2.5, N=1000).

Posfai, Liu, Slotine, Barabasi

Out-Our and In-In Degree Correlations: Quadratic



nD depends *quadratically* on *out-out* and *in-in* correlations

Scale-free model with structural cut-off (γ=2.5, N=1000).

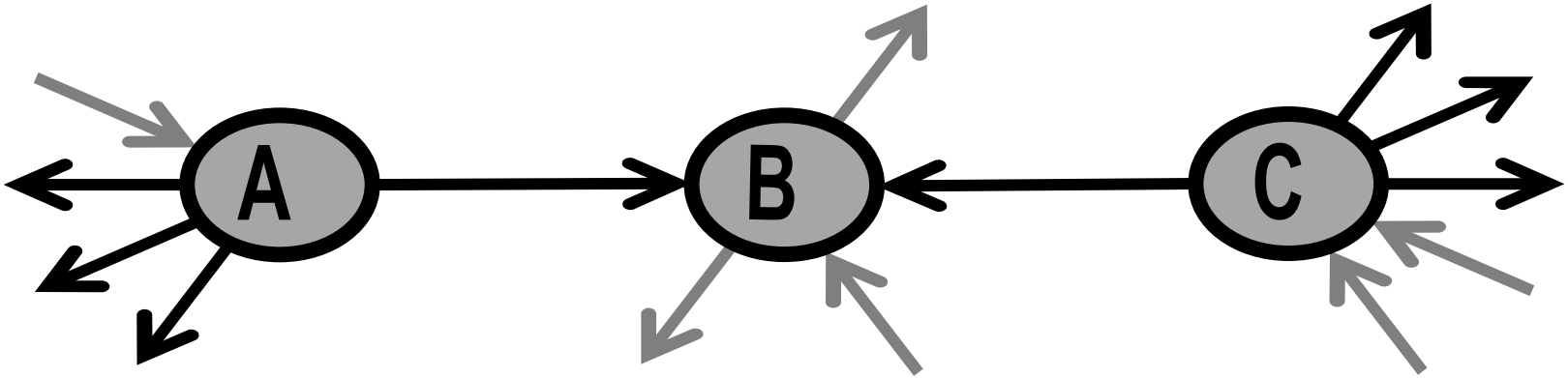
Posfai, Liu, Slotine, Barabasi

Yang-Yu Liu
CCNR/NEU

Jean-Jacques Slotine
MIT

Marton Posfai
CCNR/Budapest

BarabasiLab.com


A
B
C


OLE


OLE


OLE

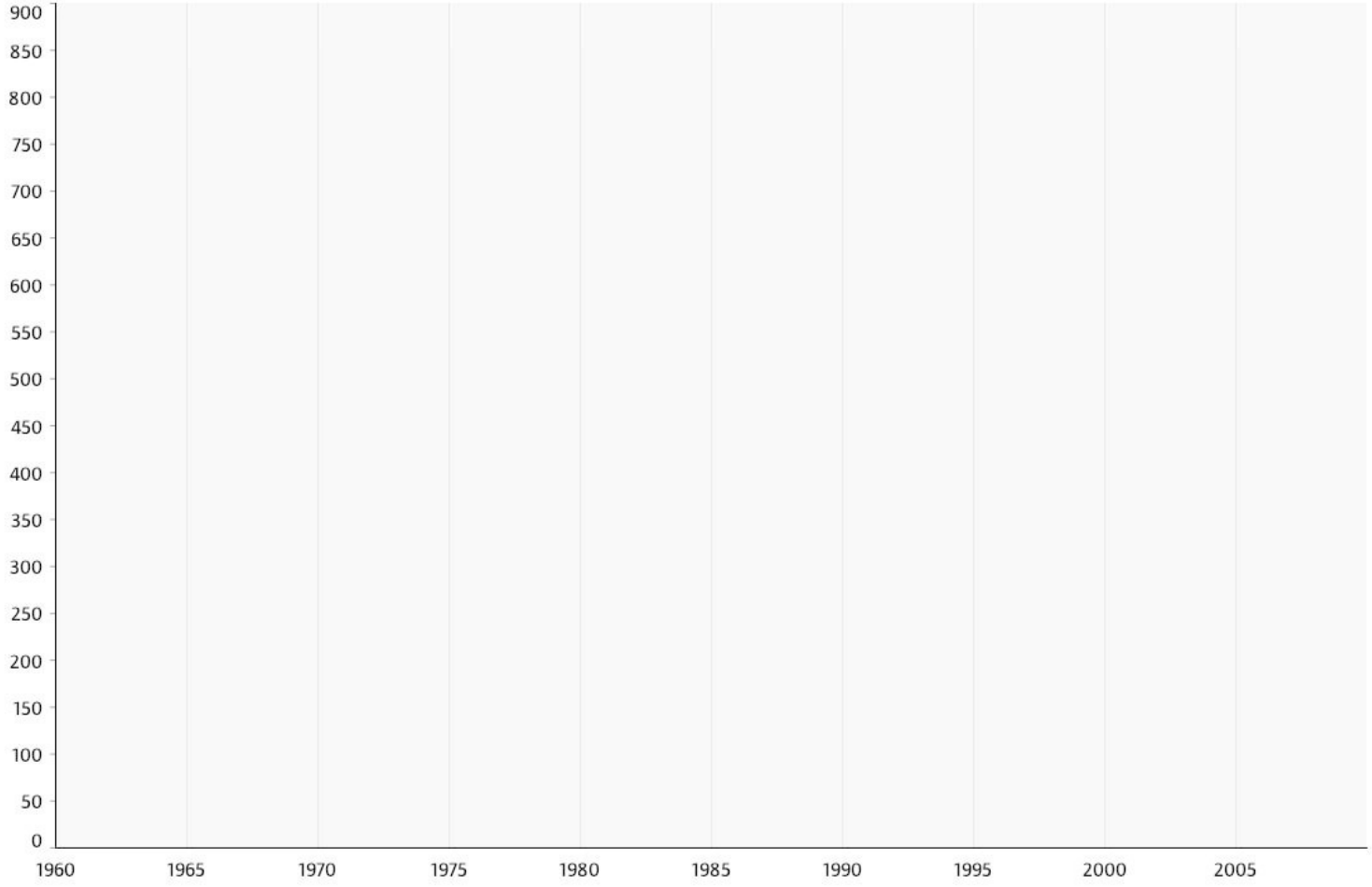

OLE


OLE


OLE


OLE


OLE


900
850
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1960
1965
1970
1975
1980
1985
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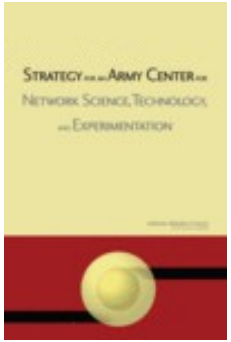
2000
2005

WHAT IS “NETWORK SCIENCE”?

THE NATIONAL ACADEMIES
Advisers to the Nation on Science, Engineering, and Medicine

NRC Panel on “Network Science”



An attempt to understand networks emerging in nature, technology and society using a unified set of tools and principles.

What is new here?

Despite the apparent differences, many networks emerge and evolve driven by a fundamental set of laws and mechanism.

BONUS: WHY KEVIN BACON?

Did he make the most movies, perhaps? List of actors with the most movie credits.

Kevin Bacon

No. of movies : 46
No. of actors : 1811
Average separation: 2.79

Is Kevin Bacon the most connected actor?

BONUS: WHY KEVIN BACON?

Measure the average distance between Kevin Bacon and all other actors.

Kevin Bacon

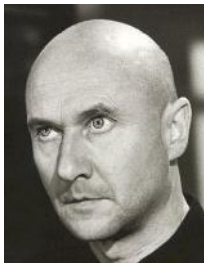
No. of movies : 46
No. of actors : 1811
Average separation: 2.79

Is Kevin Bacon the most connected actor?

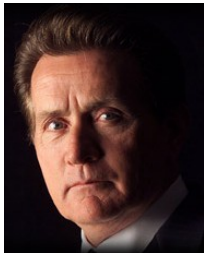
Rank	Name	Average distance	# of movies	# of links
1	Rod Steiger	2.537527	112	2562
2	Donald Pleasence	2.542376	180	2874
3	Martin Sheen	2.551210	136	3501
4	Christopher Lee	2.552497	201	2993
5	Robert Mitchum	2.557181	136	2905
6	Charlton Heston	2.566284	104	2552
7	Eddie Albert	2.567036	112	3333
8	Robert Vaughn	2.570193	126	2761
9	Donald Sutherland	2.577880	107	2865
10	John Gielgud	2.578980	122	2942
11	Anthony Quinn	2.579750	146	2978
12	James Earl Jones	2.584440	112	3787
…				
876	Kevin Bacon	2.786981	46	1811
…				

KEVIN BACON MAP

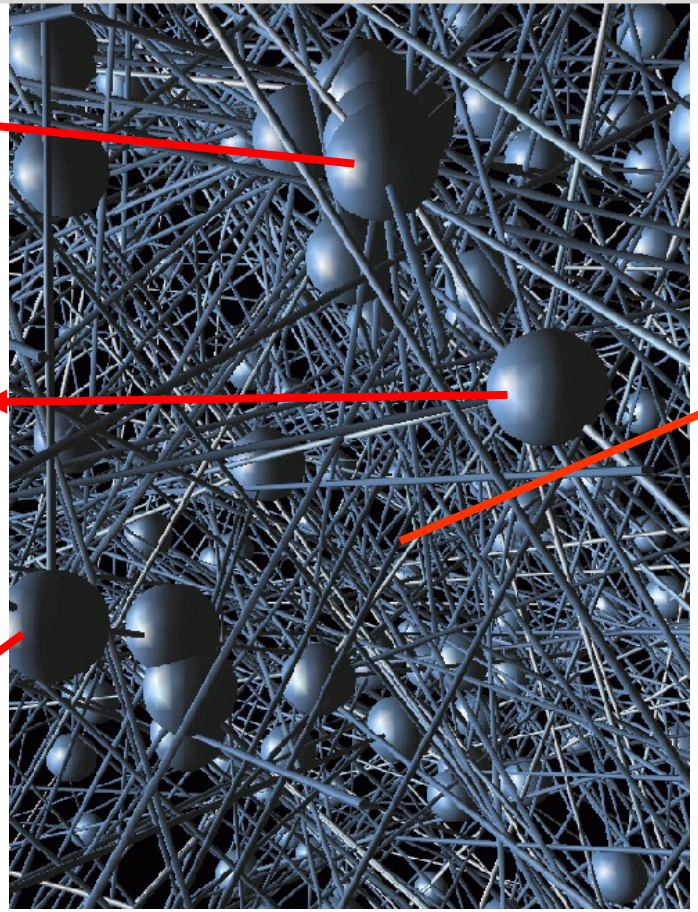
1
Rod Steiger



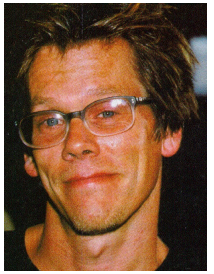
2
Donald Pleasence



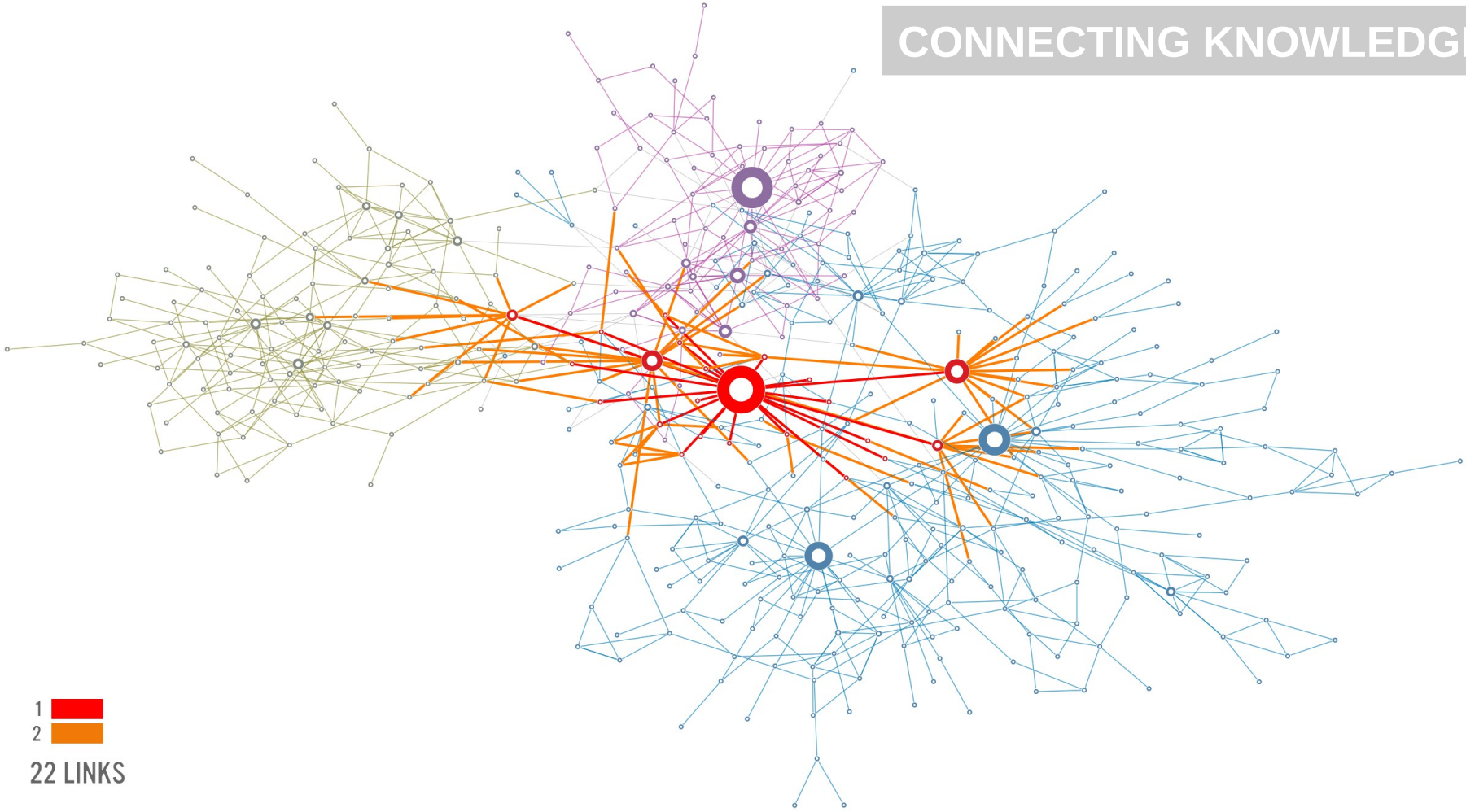
3
Martin Sheen



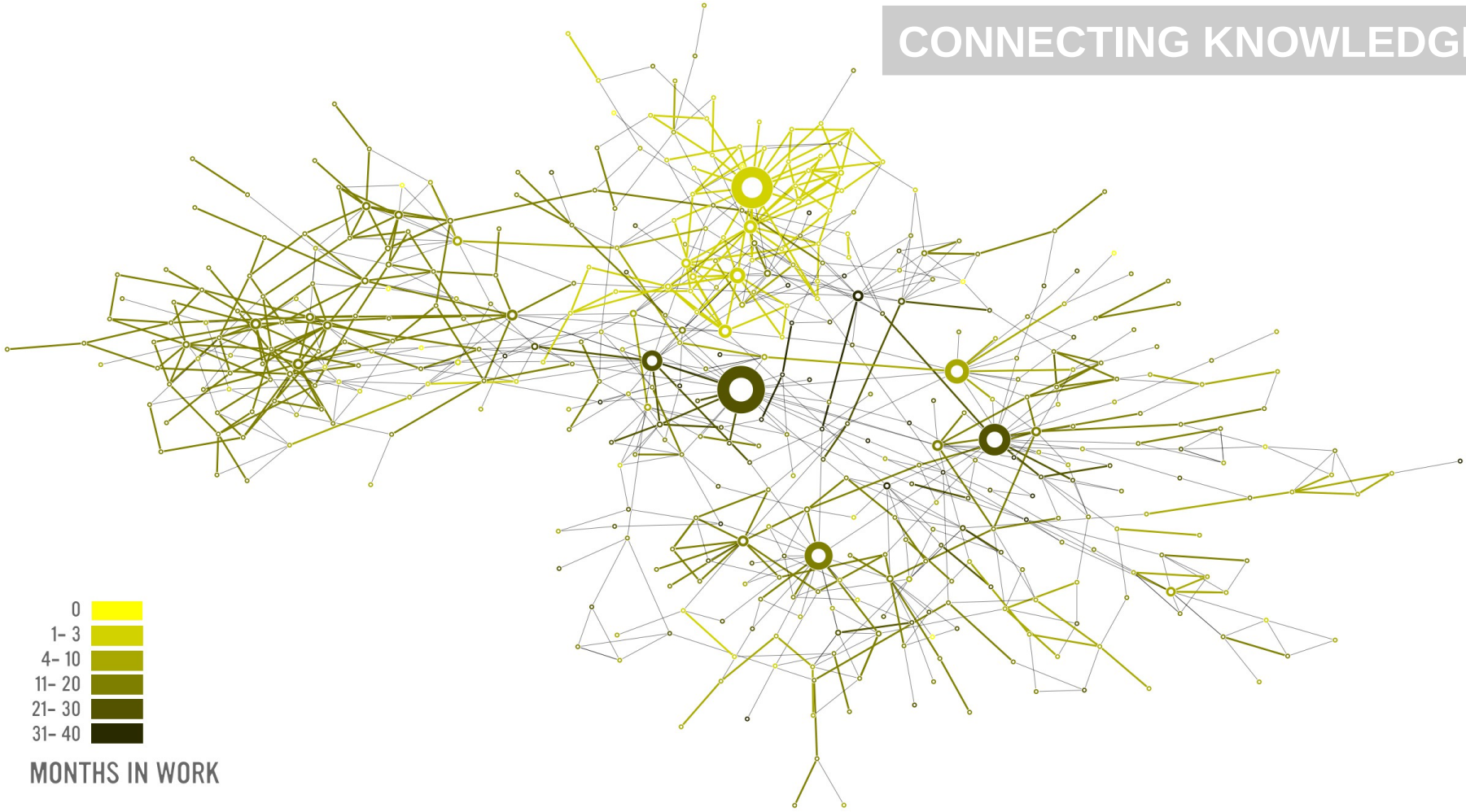
876
Kevin Bacon



CONNECTING KNOWLEDGE



CONNECTING KNOWLEDGE



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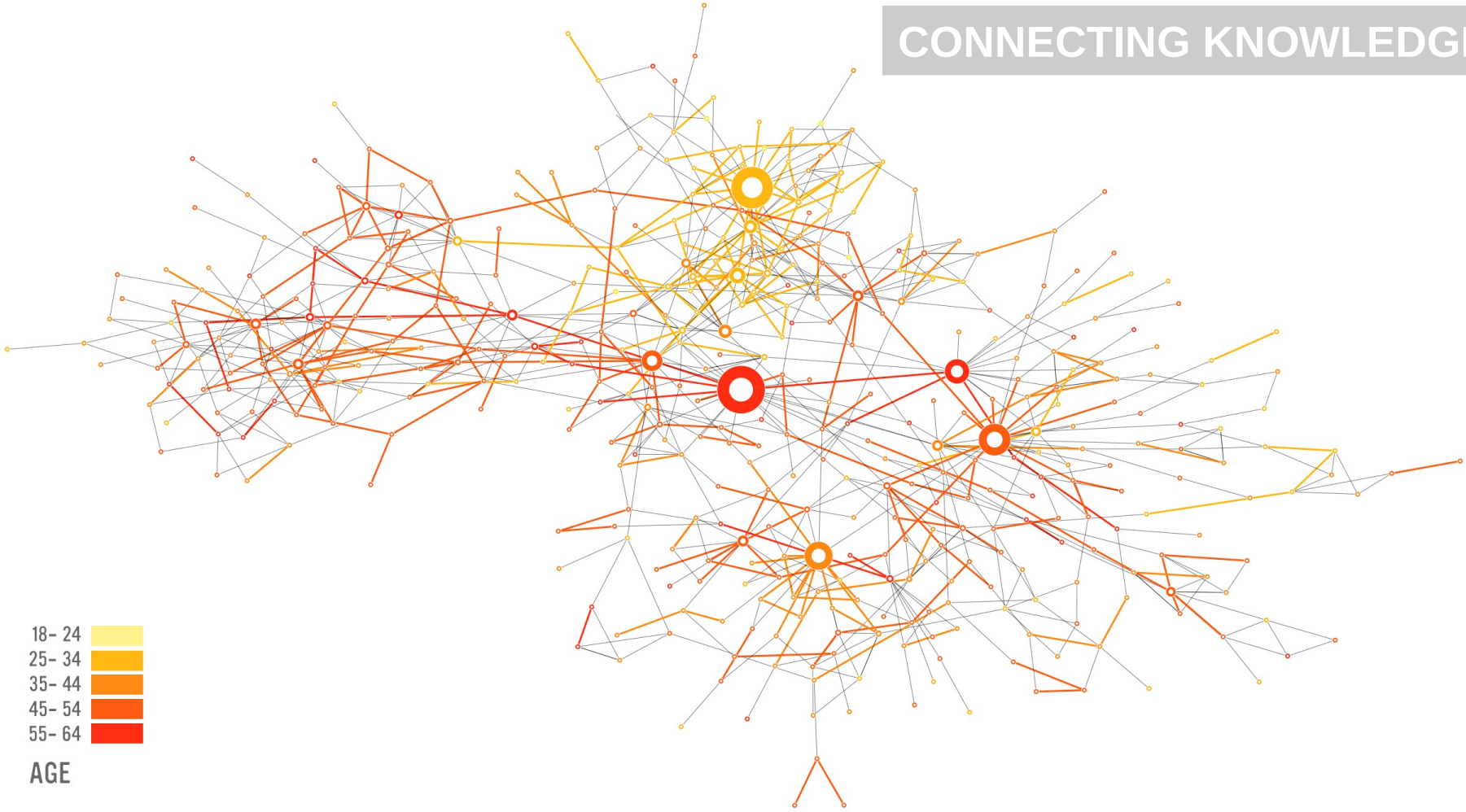
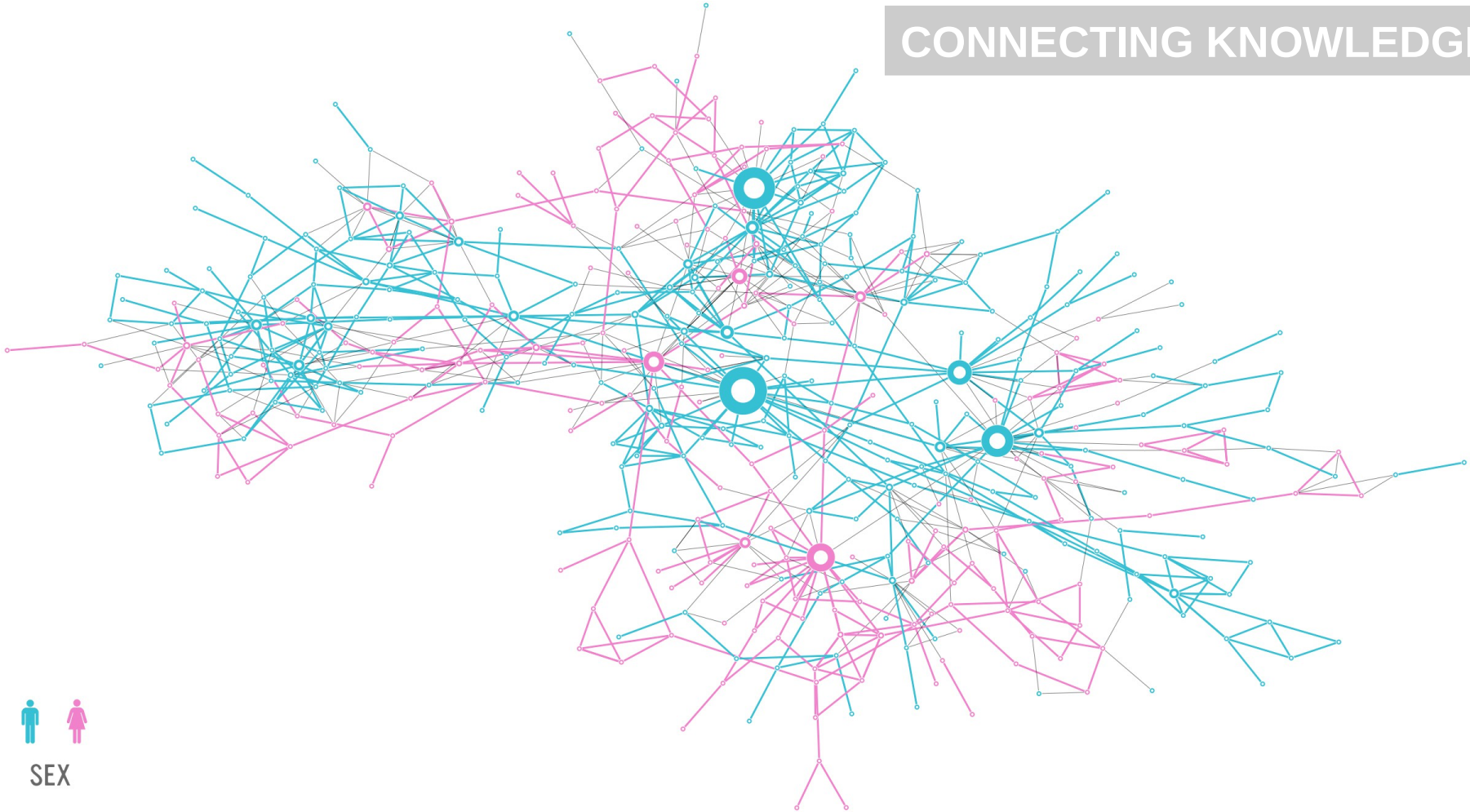
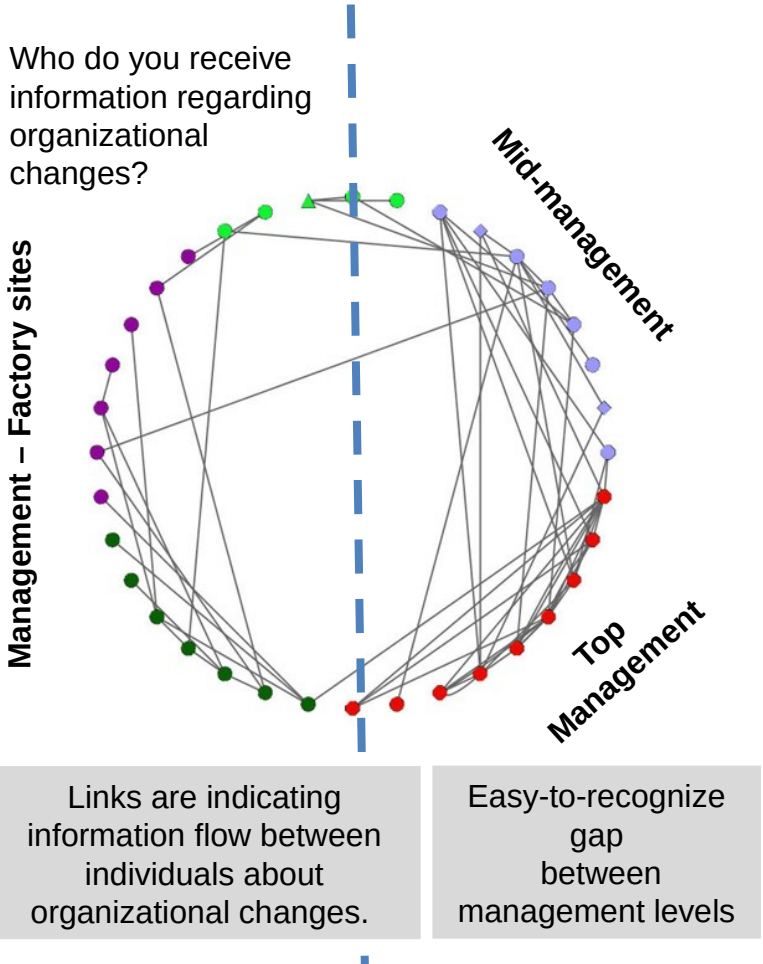


CONNECTING KNOWLEDGE



SEX

IMPROVING INFORMATION FLOW



Links are indicating information flow between individuals about organizational changes.

Easy-to-recognize gap between management levels

Manufacturing company with about 800 employees

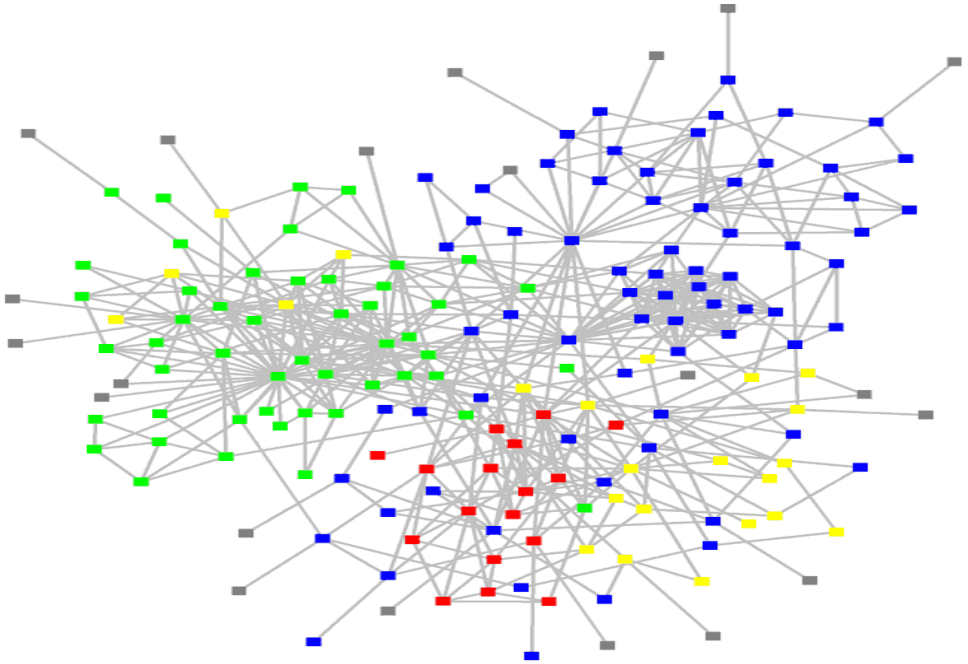
Issues: (1) Information gaps and gossip about organizational changes; (2) Strategic decisions miss-understood; (3) Lack of trust in management.

Aim: Reduce time for accepting changes; Gossip management; Build trust.

Findings: Robust communication between mid and senior management *BUT* Lack of information flow between mid-management and management of manufacturing sites.

Main source of information for Factory Management: EHS Manager – no connection to management, no career plan and frustrated about own possibilities.

STRUCTURE OF AN ORGANIZATION



www.orgnet.com

: departments

: consultants

: external experts

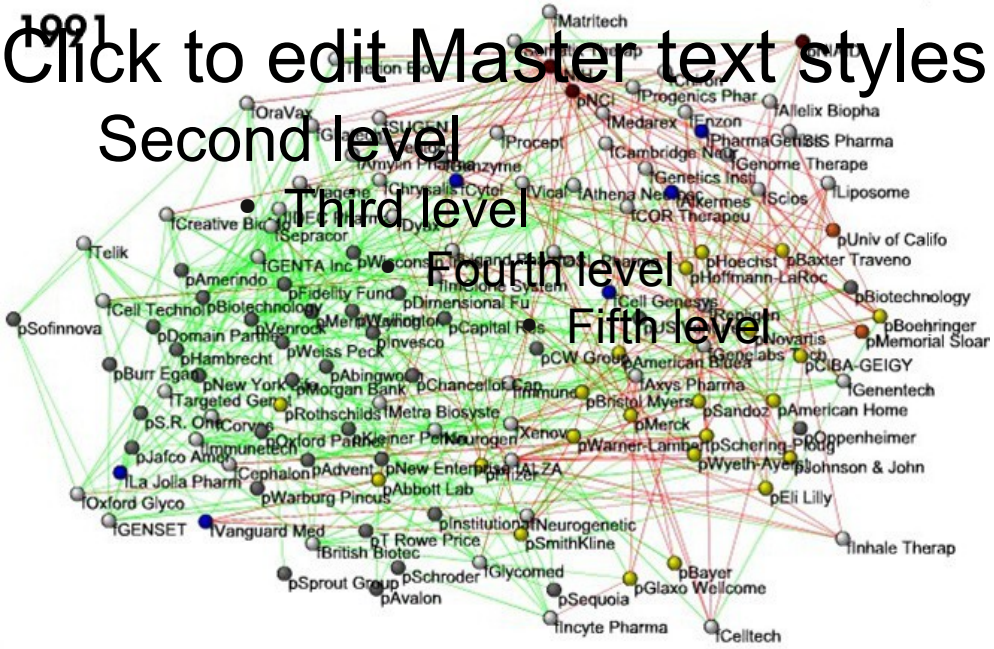
BUSINESS TIES IN US BIOTECH-INDUSTRY

Nodes:

- Companies
- Investment
- Pharma
- Research Labs
- Public
- Biotechnology

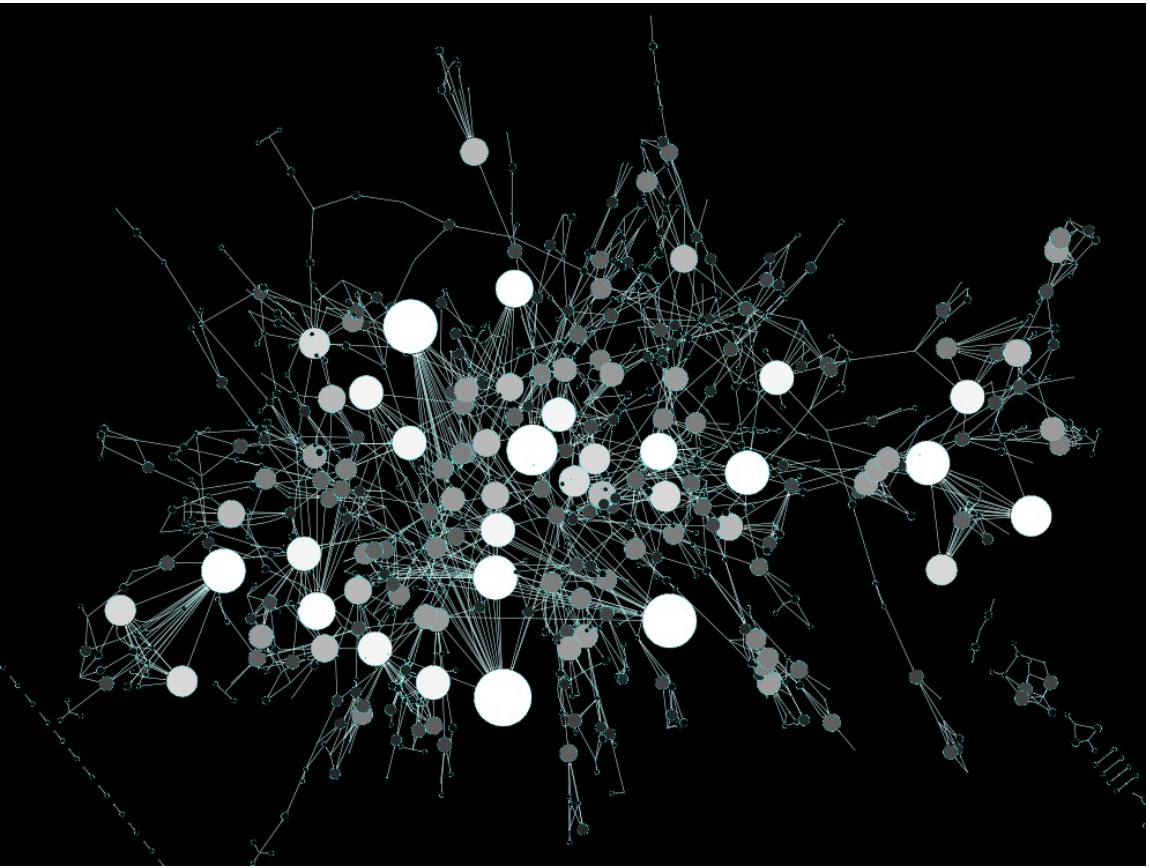
Links:

- Collaborations
- Financial
- R&D



http://ecclectic.ss.uci.edu/~drwhite/Movie

OPINION LEADERS IN ORGANIZATIONS



Question visualized:

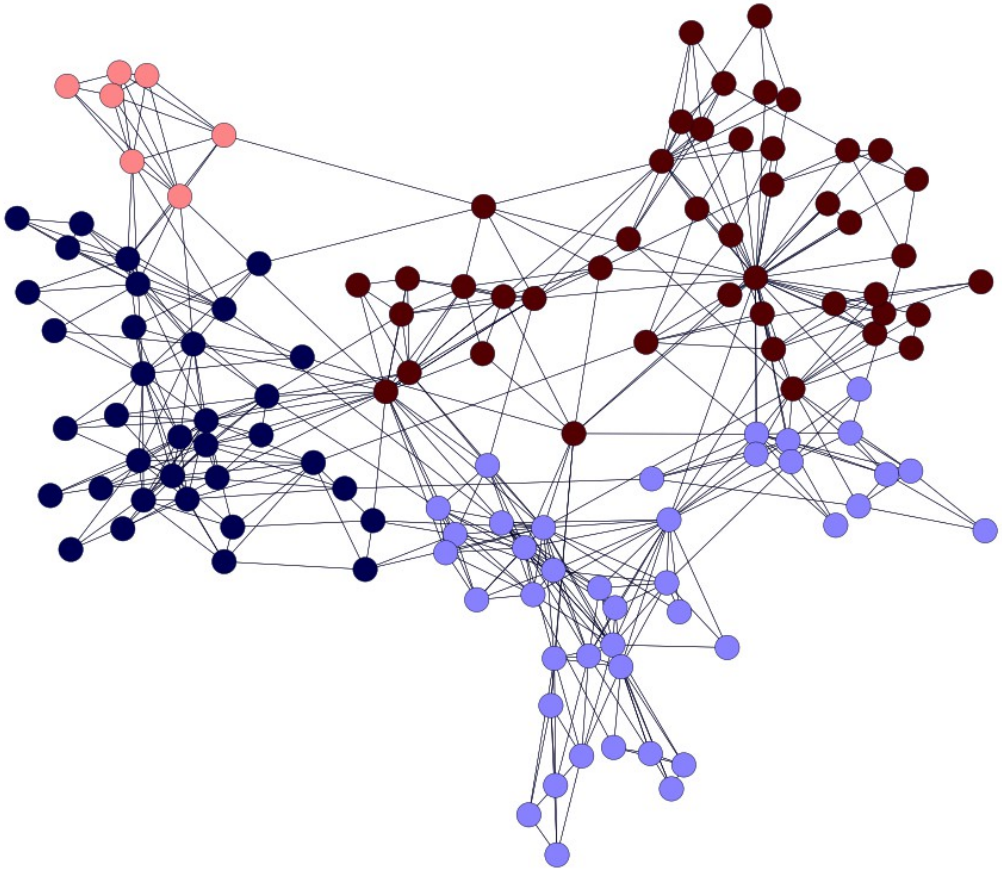
Who's opinion do you trust the most when there is a change process at the company?

Nodes = employees

Size = Numer of mentions (in-degree)

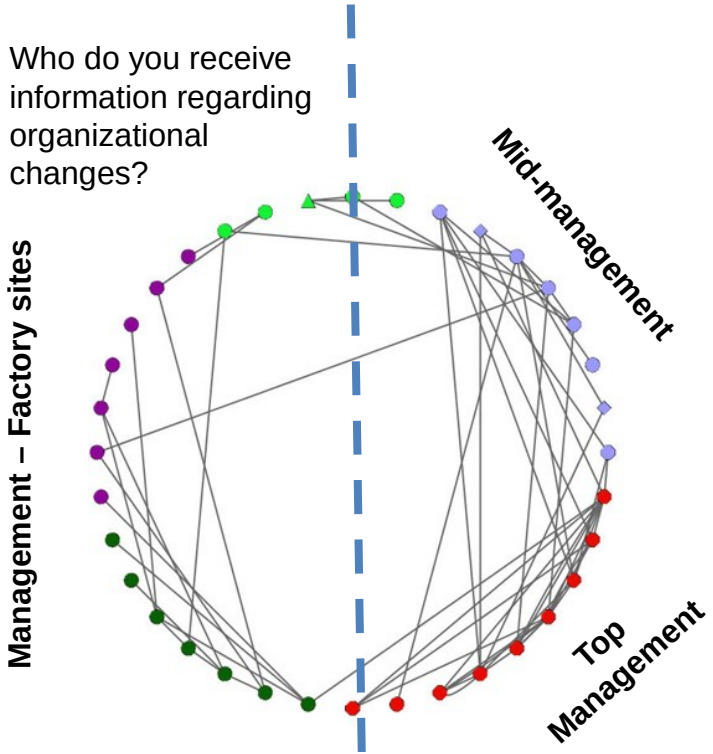
The white nodes are the opinion leaders who the company involved in future shaping forums after the survey.

ZOOM



IMPROVING INFORMATION FLOW

Who do you receive information regarding organizational changes?



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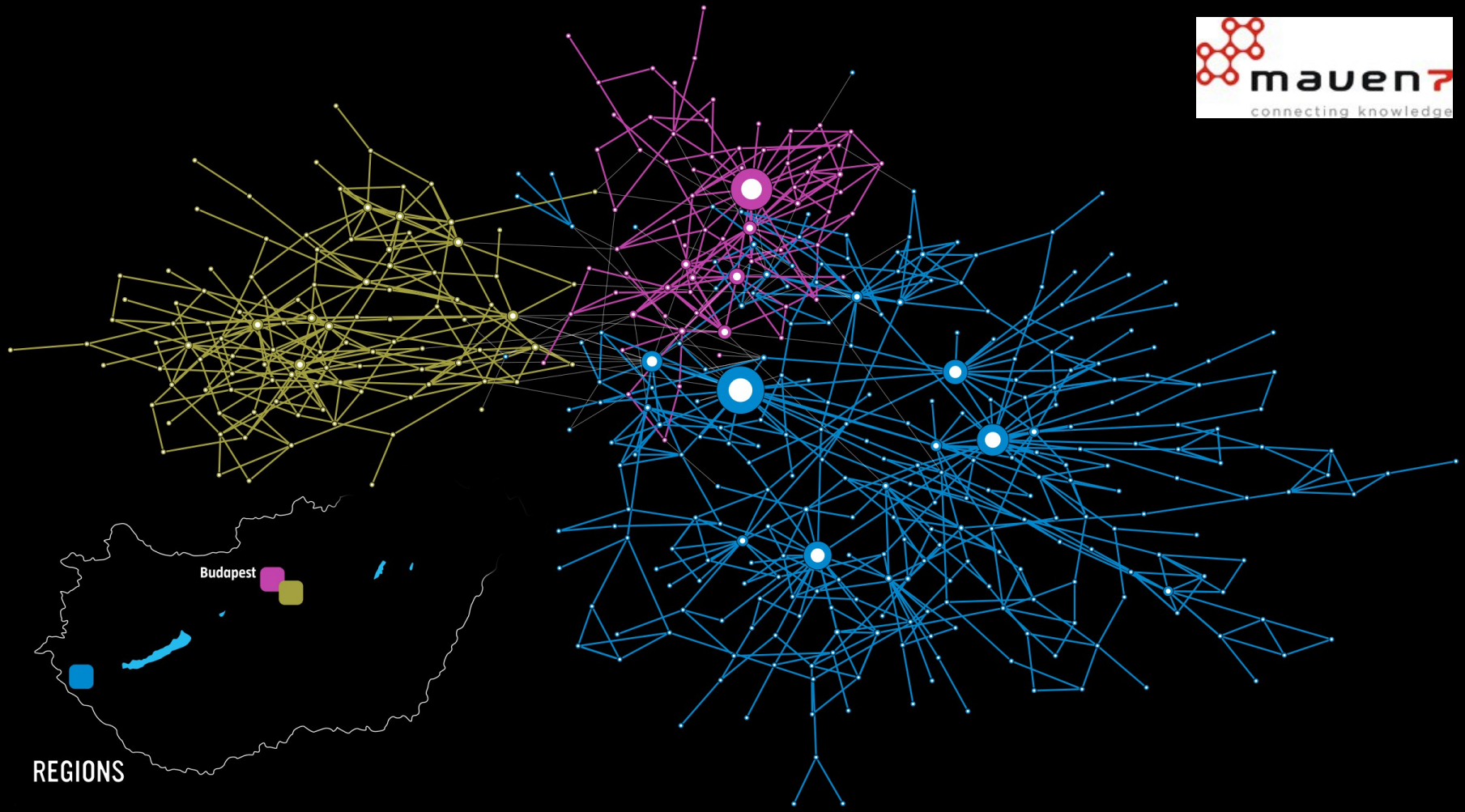
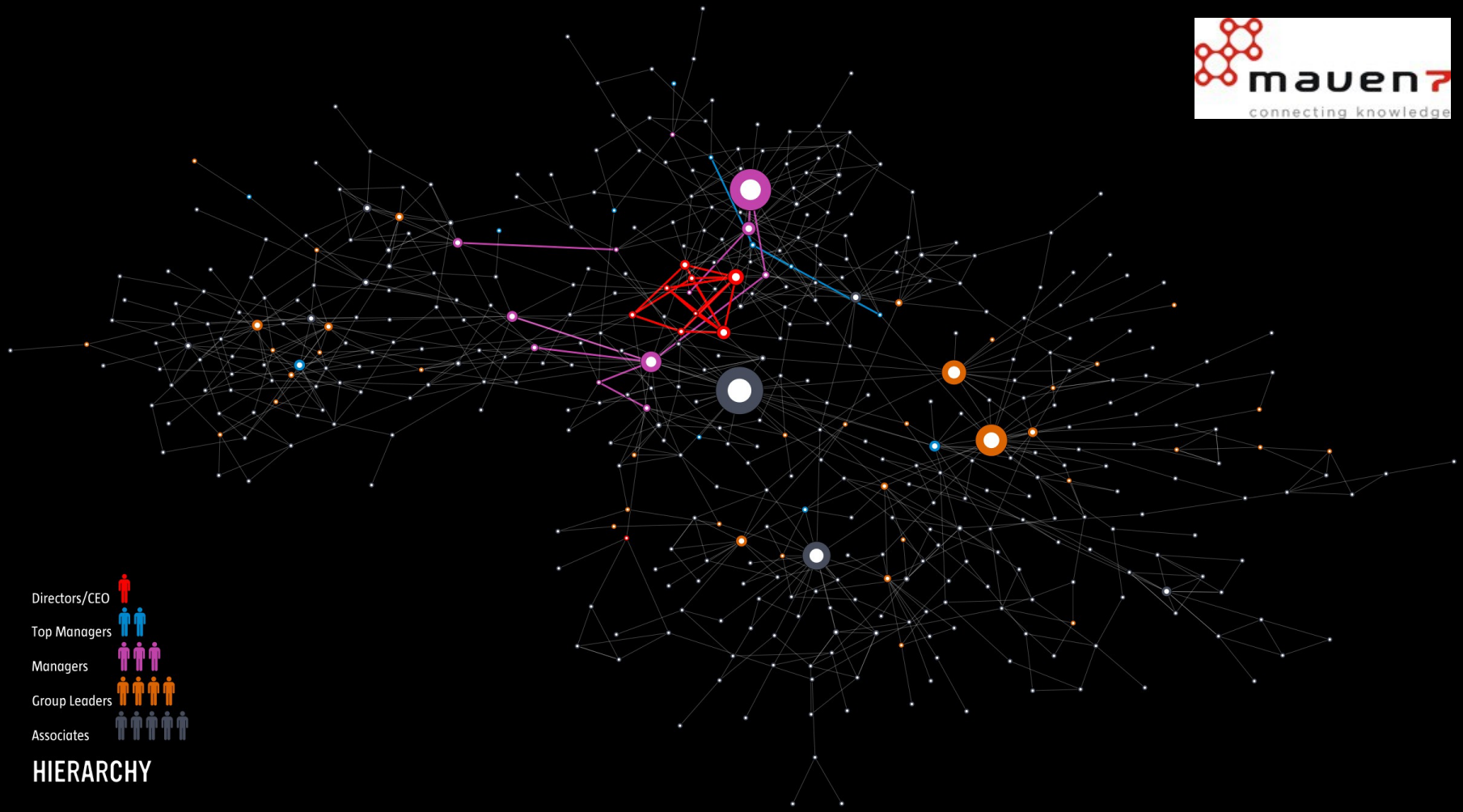
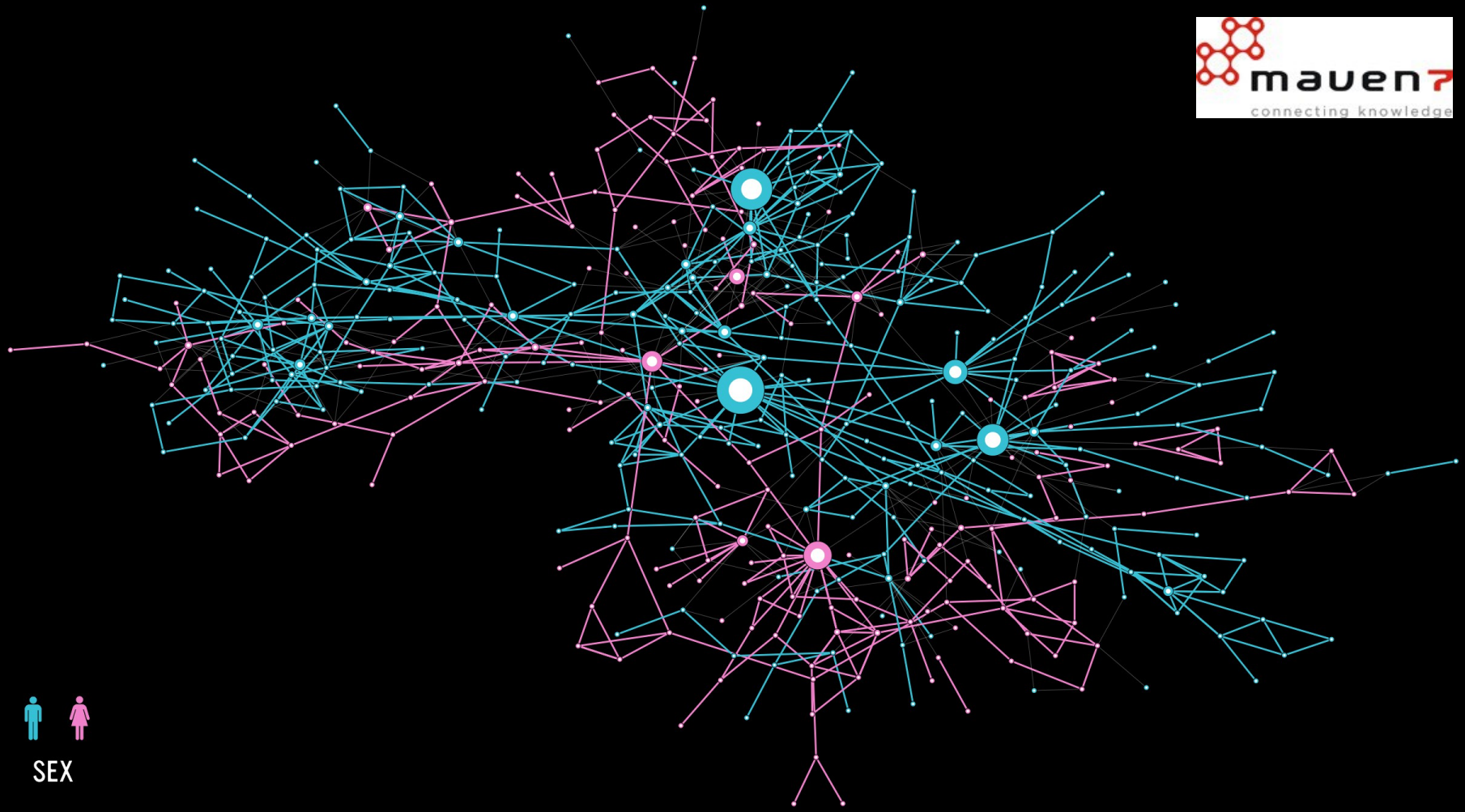
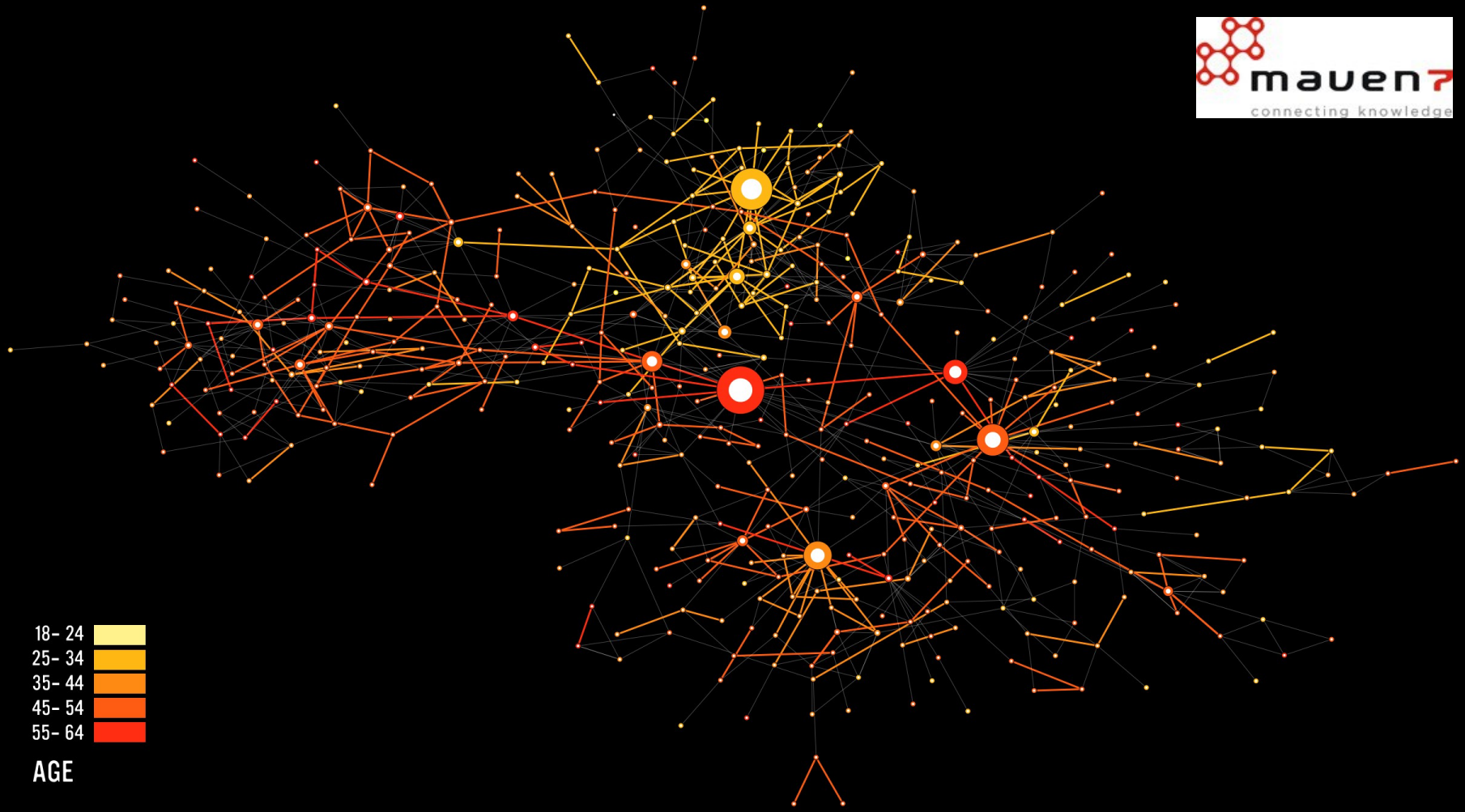
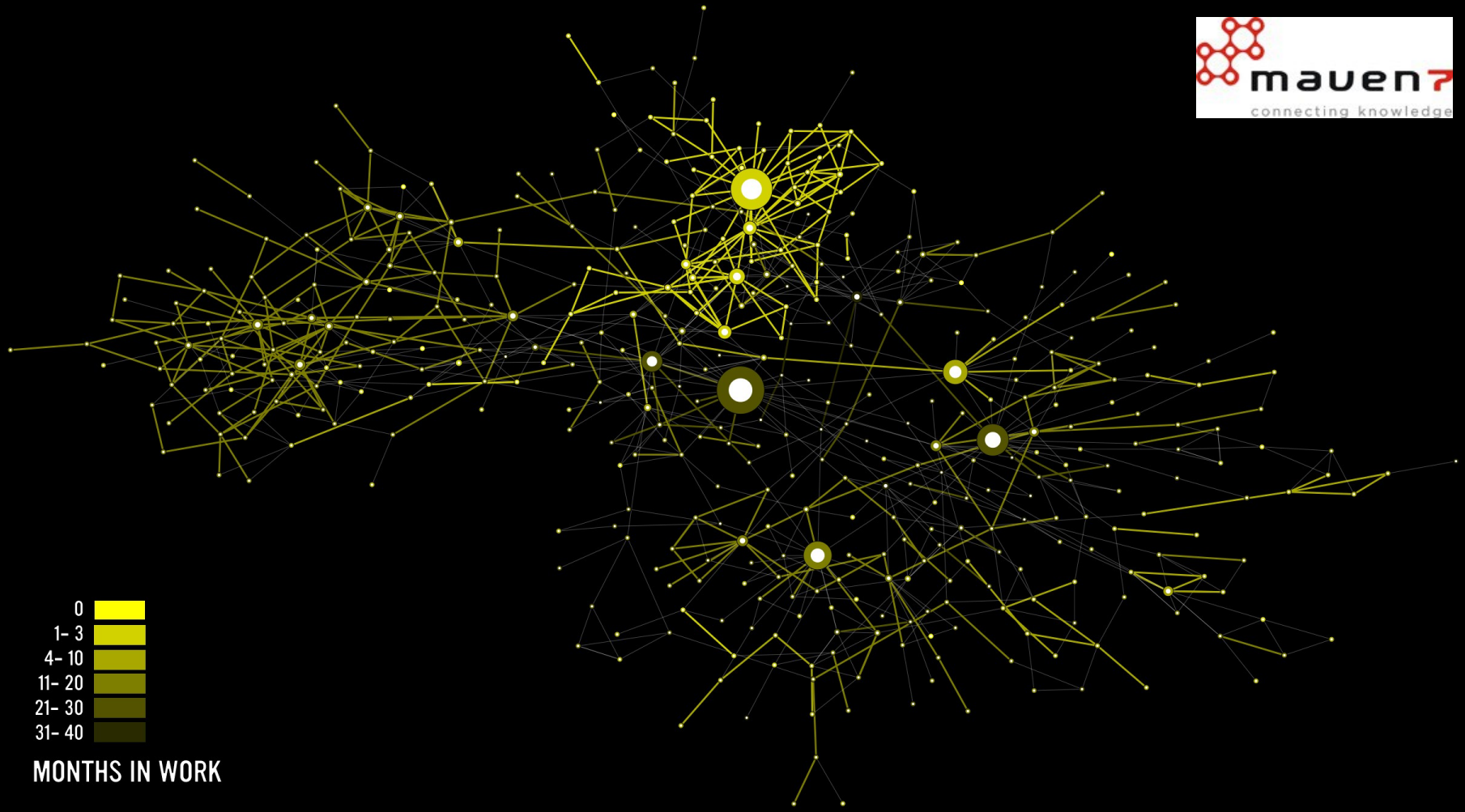
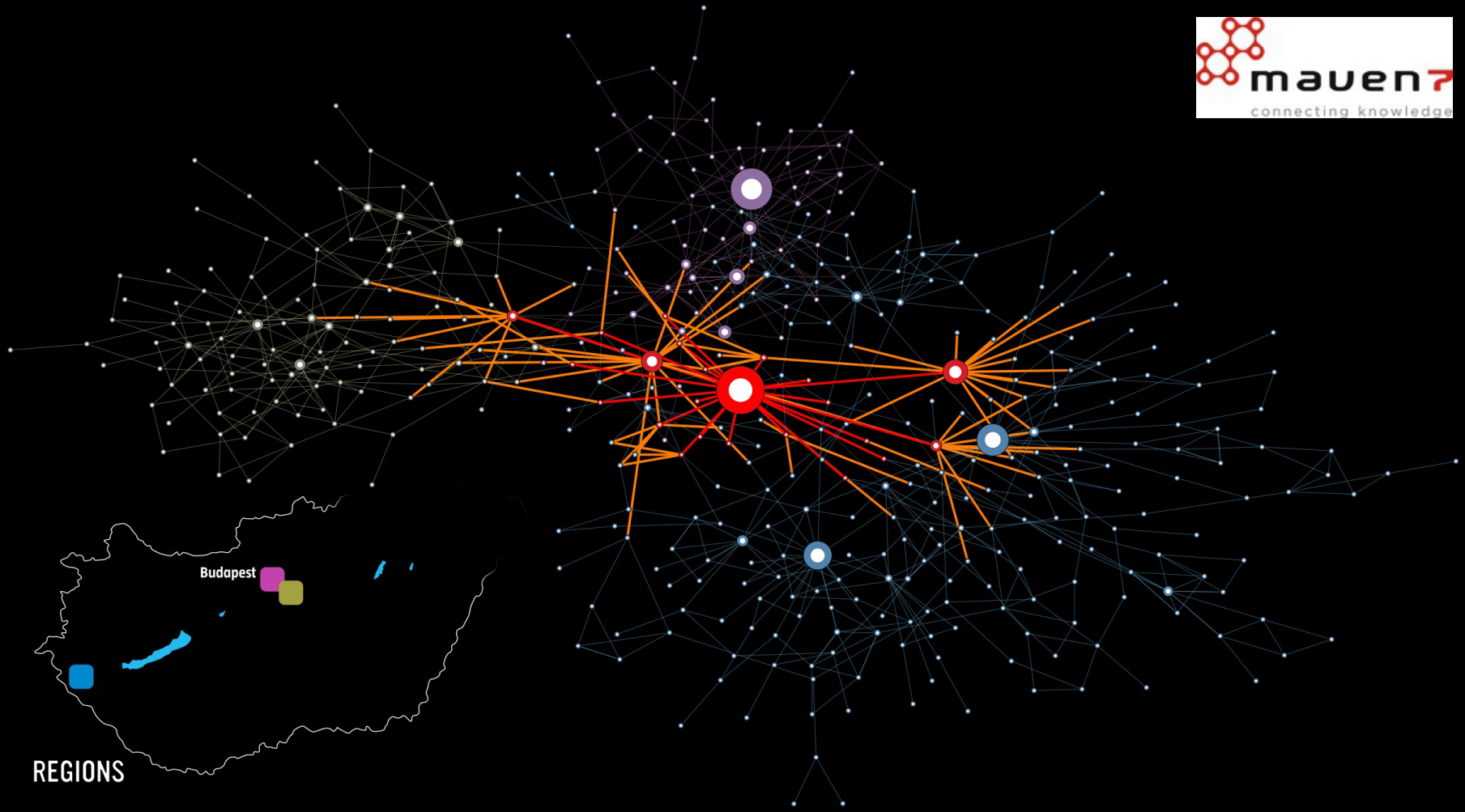

Budapest
REGIONS


Directors/CEO
Top Managers
Managers
Group Leaders
Associates
HIERARCHY

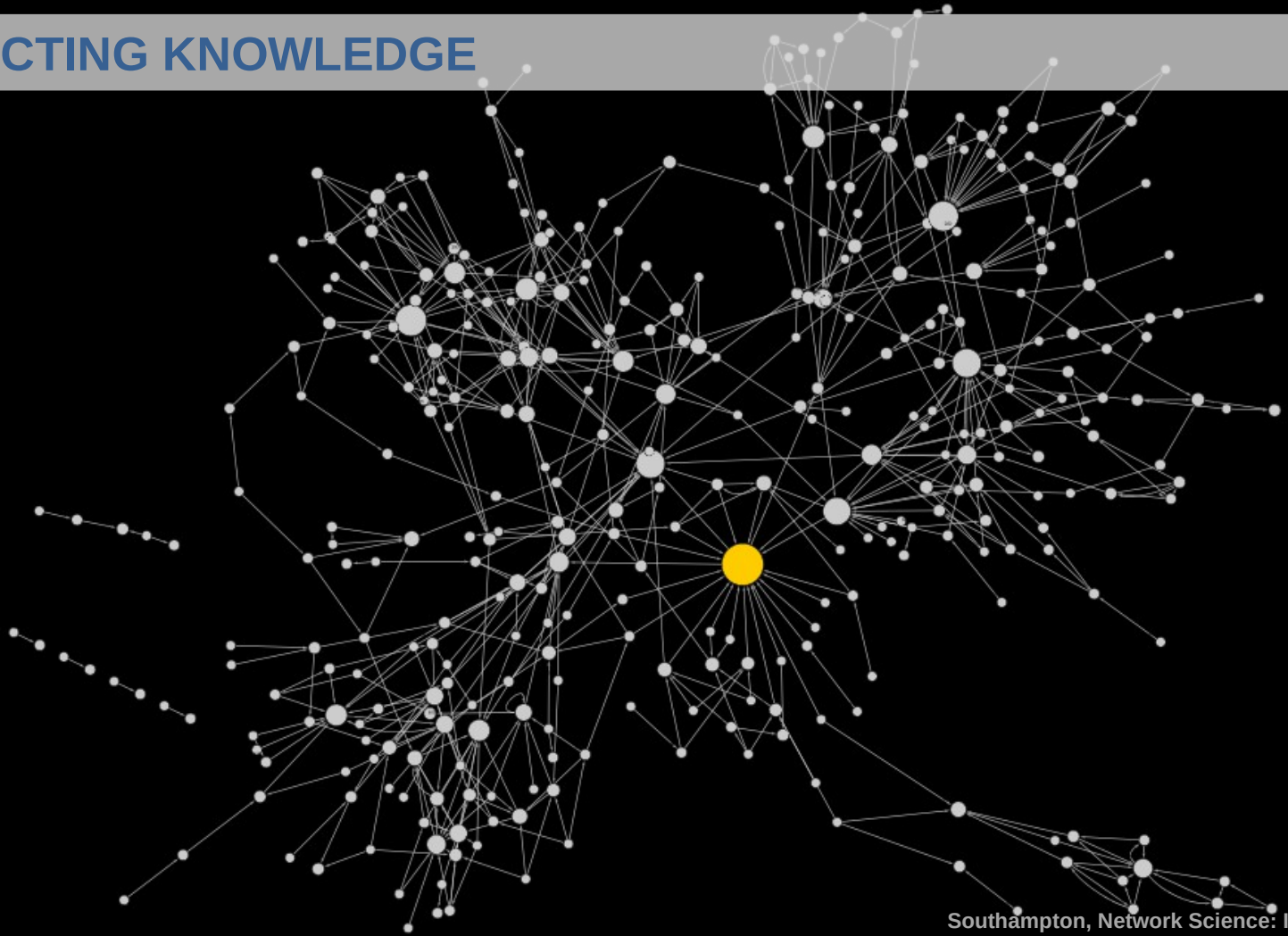

maven7
connecting knowledge
SEX


maven7
connecting knowledge
18- 24
25- 34
35- 44
45- 54
55- 64
AGE

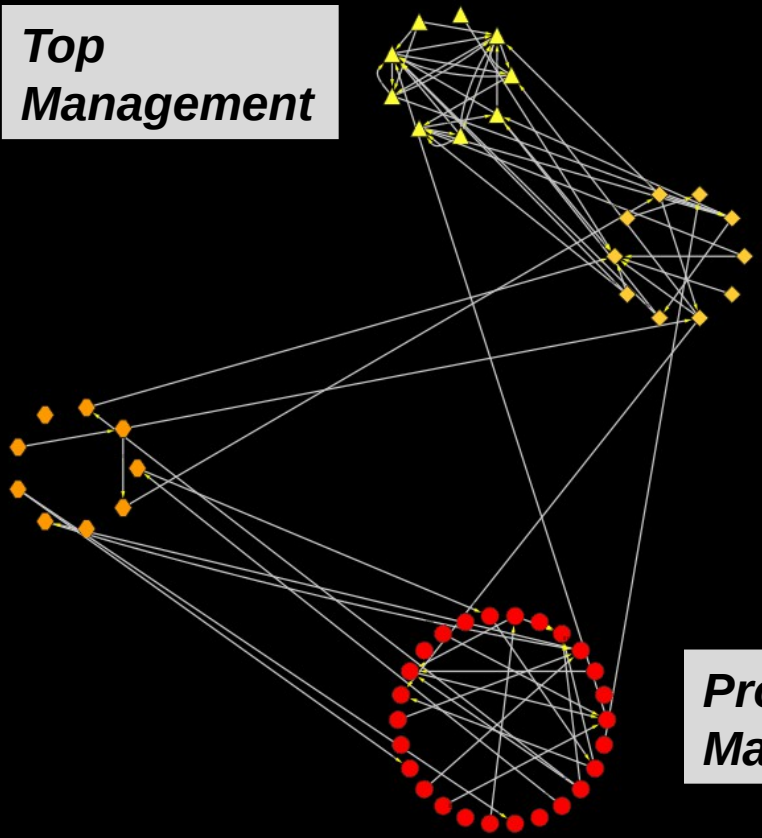

0
1- 3
4- 10
11- 20
21- 30
31- 40
MONTHS IN WORK


mavenz
connecting knowledge
Budapest
REGIONS

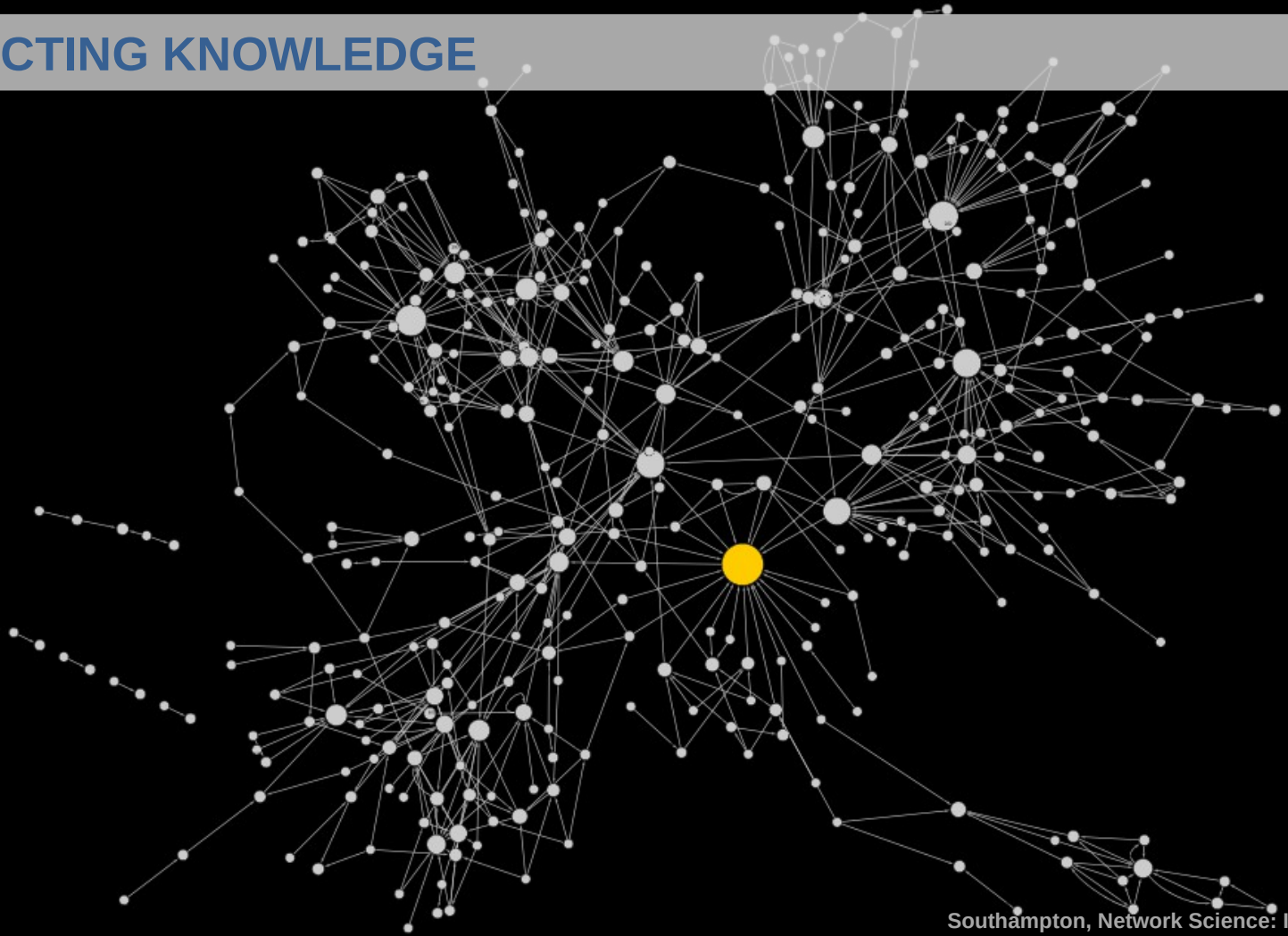
CONNECTING KNOWLEDGE



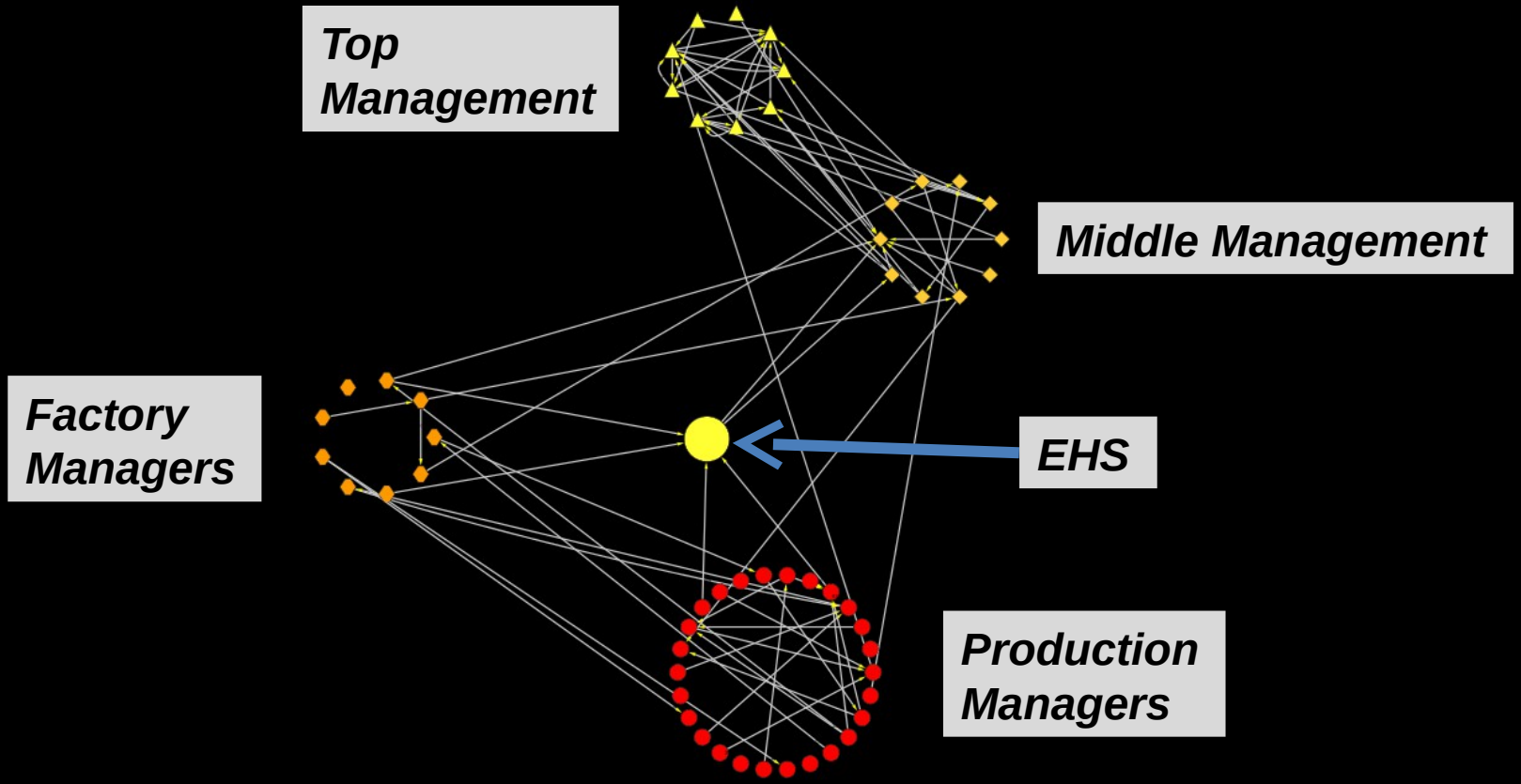
CONNECTING KNOWLEDGE



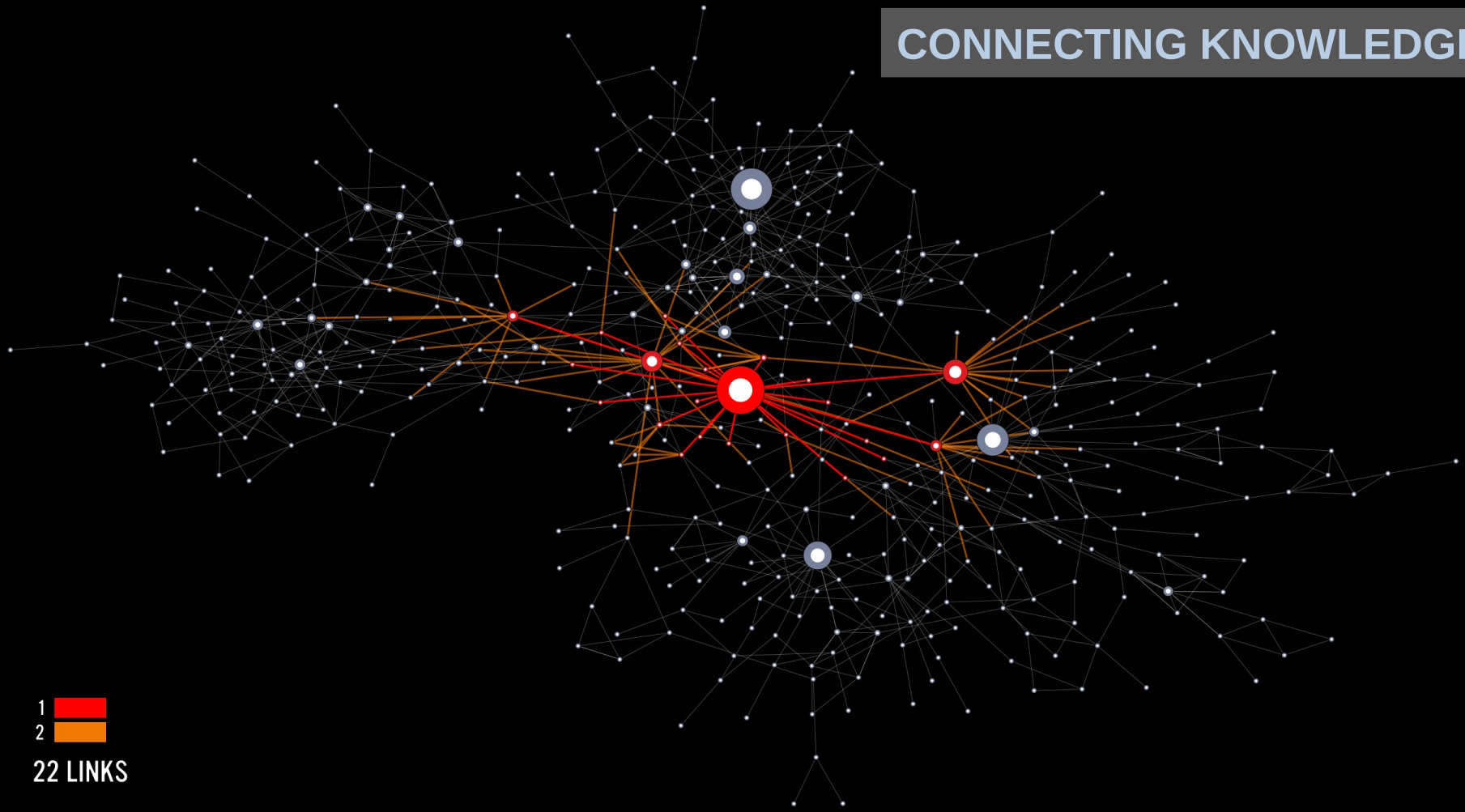
CONNECTING KNOWLEDGE



CONNECTING KNOWLEDGE

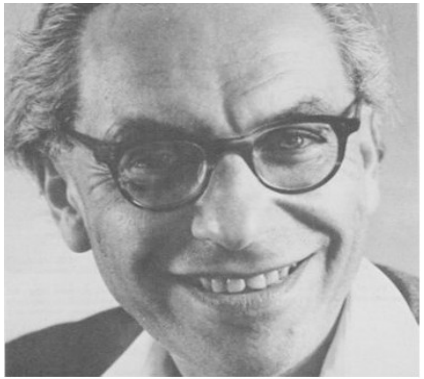


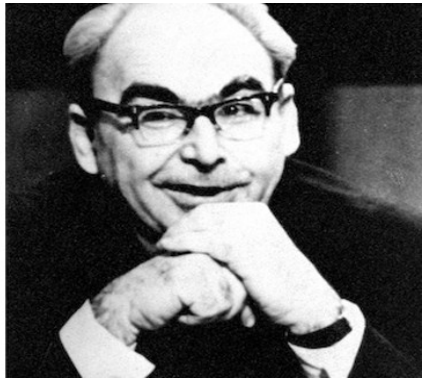
CONNECTING KNOWLEDGE

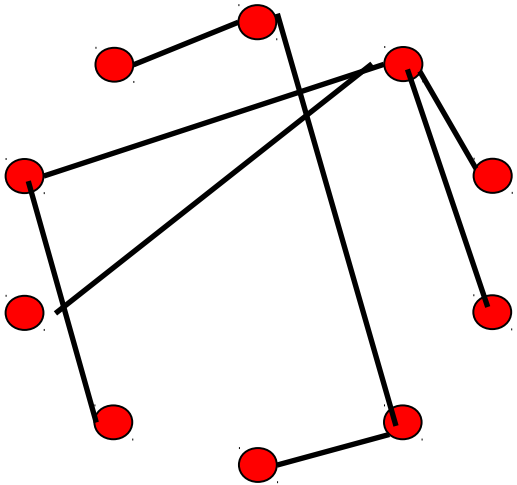


RANDOM NETWORK MODEL

Pál Erdös
(1913-1996)







Erdös-Rényi model (1960)

Connect with probability p

p=1/6 N=10

$\langle k \rangle \sim 1.5$