

Fourth Lecture: Density Matrix Renormalization Group et al.

Andreas Läuchli, "New states of quantum matter" MPI für Physik komplexer Systeme - Dresden

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Lecture Notes at http:www.pks.mpg.de/~aml/LesHouches

"Modern theories of correlated electron systems" - Les Houches - 21/5/2009

Outline

- Tower of States (from yesterday's ED lecture)
- Wilson's RG
- White's DMRG
- What can you do with DMRG?
	- Ground states, correlation functions
	- Dynamical correlation functions
	- Real-time evolution
	- Classical systems in higher dimension
	- **Finite Temperature properties**
- DMRG++, MPS, PEPS, MERA, iTEBD, iPEPS

Continuous symmetry breaking and "Tower of States"

"Tower of States" spectroscopy

- What are the finite size manifestations of a continuous symmetry breaking ?
- Low-energy dynamics of the order parameter Theory: P.W. Anderson 1952, Numerical tool: Bernu, Lhuillier and others, 1992 -

- Dynamics of the free order parameter is visible in the finite size spectrum. Depends on the continuous symmetry group.
- $U(1)$: $(S^z)^2$ SU(2): S(S+1)
- Symmetry properties of levels in the Tower states are crucial and constrain the nature of the broken symmetries.

Tower of States S=1 on triangular lattice

#!!"

"

 \overline{a}

Bilinear-biquadratic S=1 model on the triangular lattice (model for NiGaS₄).

AML, F. Mila, K. Penc, PRL '06

Tower of States S=1 on triangular lattice

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"

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Bilinear-biquadratic S=1 model on the triangular lattice (model for NiGaS₄).

$$
H = \sum_{\langle i,j \rangle} \cos(\theta) \mathbf{S}_i \cdot \mathbf{S}_j + \sin(\theta) \left(\mathbf{S}_i \cdot \mathbf{S}_j\right)^2
$$

AML, F. Mila, K. Penc, PRL '06

Tower of States S=1 on triangular lattice: Antiferromagnetic phase

?

!!&

SU(3)

"!!&

SU(3)

 $9=0$: coplanar magnetic order, 120 degree structure

- Breaks translation symmetry. Tree site unit cell \Rightarrow nontrivial momenta must appear in TOS
- **•** non-collinear magnetic structure \Rightarrow SU(2) is completely broken,

number of levels in TOS increases with S

Quantum number are identical to the S=1/2 case

Tower of States S=1 on triangular lattice: Ferroquadrupolar phase

?

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SU(3)

"!!&

SU(3)

 $\theta = -\pi/2$: ferroquadrupolar phase, finite quadrupolar moment, no spin order

- \bullet No tr u
E nly trivial momer No translation symmetry breaking. **⇒ only trivial momentum appears in TOS**
	- Ferroquadrupolar order parameter, only even S
	- all directors are collinear

 $S_{\rm{eff}}$

 \Rightarrow SU(2) is broken down to U(1),

number of states in TOS is independent of S.

 $S_{\rm{eff}}$

d

 \mathbf{y}

 x y

z

 $\overline{}$

Tower of States S=1 on triangular lattice: Antiferroquadrupolar phase

02 6 12 20 30 42

?

!!&

SU(3)

"!!&

SU(3)

 θ =3 π /8 : antiferroquadrupolar phase, finite quadrupolar moment, no spin order, three sublattice structure.

d

 \mathbf{y}

 x y

z

- Breaks translation symmetry. Tree site unit cell \Rightarrow nontrivial momenta must appear in TOS
- Antiferroquadrupolar order parameter, complicated S dependence. Can be calculated using group theoretical methods.

Density Matrix Renomalization Group et al.

● K. Wilson introduced the numerical renormalization group as a powerful numerical tool to solve the Kondo problem (Wilson RMP 75).

$$
\frac{\psi_1}{m} \bigcirc \longrightarrow \boxed{\psi_2} \longrightarrow \boxed{\psi_2} \bigcirc \longrightarrow \boxed{\psi_3}
$$

 Many people tried to apply this idea in a straightforward way to quantum L L many body problems and failed. The reason for this failure was understood in a study of the tight-binding problem on a chain (White & Noack PRL 92):

$\psi[2L] \neq \psi[L] \otimes \psi[L]$

 $\frac{1}{2}$ at $\frac{1}{2}$ of the la The groundstate of the large system is not at all well approximated by the tensor product of the groundstates of the smaller systems

 The next crucial step was to realize how one has to choose the states to be kept in a partition of the universe:

System description with *m* states:

$$
|\tilde{\psi}\rangle=\sum_{n}^{m}\sum_{\alpha}\tilde{\psi}_{n,\alpha}|\phi_{n}\rangle_{S}\otimes|\alpha\rangle_{E}
$$

How to choose the *m* states in order to approximate $|\psi\rangle$ best ?

The answer is given by the subsystem density matrix

$$
\rho_{i,j}=\text{Tr}_E|\psi\rangle\langle\psi|
$$

 With the help of the Schmidt decomposition one can show that the m eigenfunctions of ρ associated with the largest eigenvalues w_n give the best approximation of $|\psi\rangle$. (and not the lowest eigenstates of H_S)

The error can be estimated to be

$$
\left| \tilde{\psi} \right\rangle - \left| \psi \right\rangle \right|^2 \approx 1 - \sum_{n=0}^{m} w_n = P_m
$$

● Based on these condsiderations, S.R. White came up with the DMRG S.R. White, PRL 69, 2863 (1992); PRB 48,10345 (1993).

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- **Infinite System Algorithm**

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• Finite System Algorithm

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Applicability of ground state DMRG

- Very efficient groundstate simulations for many 1D systems. Frustration or fermions pose no particular problem.
- **•** Typical system sizes and number of states: simple spin systems: 100-1000 sites, lower hundred states complicated spin systems: hundred sites, larger hundred to thousand states fermionic systems, wide systems, many low energy states, they all increase *m*
- Variational, quasi-exact method.
- Wavefunction method, therefore many observables are easily available.
- Drawbacks: it is not yet possible to simulate very large 2D systems, open boundaries might not always be what we desire, especially because of the lack of spatial quantum numbers.

1. Form a superblock containing L sites which is small enough to be exactly diagonalized

m | () () | m

m m

- 2. Diagonalize the superblock Hamiltonian H_L^{super} numerically, obtaining only the ground state eigenvalue and eigenvector $|\psi\rangle$ using Lanczos or Davidson.
- 3. Form the reduced density matrix $\rho_{ii'}$ for the new system block from $\ket{\psi}$ using $\rho_{ii'} = \sum \psi_{ij}^* \psi_{i'j}$. *j* $\psi_{ij}^* \psi_{i'j}$
- 4. Diagonalize $\rho_{ii'}$ with a dense matrix diagonalization routine to obtain the m eigenvectors with the largest eigenvalues
- 5. Construct H_{l+1} and other operators in the new system block and transform them to the reduced density matrix eigenbasis using $\bar{H}_{l+1} = O_L^{\dagger} H_{l+1} O_L$ where the columns of O_L contain the the m highest eigenvectors of $\rho_{ii'}$.
- 6. Form a superblock of size L+2 using \bar{H}_{l+1} , two single sites and $\bar{H}^{R}_{l+1}.$
- 7. Repeat starting with step 2, substituting H_{L+2}^{super} for H_{L}^{super} .

Efficiency

- Efficient multiplication of H_L^{super} with a vector is important. Wave function transformation yields significant speedup.
- Computational cost: L m³ Memory cost: (L) m²
- Abelian quantum numbers Np and Sz can be implemented. They render the matrices block sparse, but require book keeping effort.
- Nonabelian quantum numbers [e.g. S(S+1)] can be implemented, but gives more complex code.
- Writing to disk. Information not needed at the current step can be stored on disk.

Extension to ladder systems

- 1D algorithm folded into 2D
- finite size algorithm needed
- Convergence depends strongly on width of system
- Exponential effort in width for spinless fermions

 $A_{\rm{max}}$ and $A_{\rm{max}}$ are the set of $A_{\rm{max}}$

system block environment block

• convergence depends strongly on width of system

 \sim exponential in width for spinless fermions fermions fermions fermions (Liang \sim

DMRG Applications The S=1/2 Kagome Strips gap to the lowest-lying state, with the modified ends, as a state α state, with the modified ends, as a state α a function of the system of the system of the system α very small triplet gap of D . J . Details of the D . D – 172 Nayune ol

1st Kagome Strip (Azaria et al, 1998) and strengths and strengths of the local bond strengths of the local bon

in the bulk have identical values *J*. In Fig. 2 we show the

 $2D$ epartment of P

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M. Indergand, AML, S. Capponi, M.Sigrist, PRB (2006)

DMRG Applications Triangular lattice Heisenberg model

 Very accurate determination of order parameter using special aspect ratio to suppress leading finite size corrections. ory accurate actumnimation or oracr parameter asing special aspect
I sunnress leading finite size corrections

 $E_{\rm eff}$ order parameters parameter parameters μ , $E_{\rm eff}$, $E_{\rm eff}$

See White & Chernyshev, PRL 99, 127004 (2007)

 $\Delta E \sim 0.3\%, \ \Delta S_z > 0.01$

Extensions: Dynamical DMRG (D-DMRG)

Dynamical correlation function

$$
G(\mathbf{k}, \omega = \langle \psi_0 | A_{\mathbf{k}}^{\dagger} (\omega + i\eta - H)^{-1} A_{\mathbf{k}} | \psi_0 \rangle
$$

additional density matrix eigenstates must be "targeted"

Lanczos vector method: target Krylov vectors (Hallberg 1995) $|\psi_0\rangle$, $A^{\dagger}_{\mathbf{k}}|\psi_0\rangle$, $HA^{\dagger}_{\mathbf{k}}|\psi_0\rangle$, $H^2A^{\dagger}_{\mathbf{k}}|\psi_0\rangle$, ...

Correction vector method: target vectors: (White & Kuehner 1999) $|\psi_0\rangle$, $A^{\dagger}_{\mathbf{k}}|\psi_0\rangle$, $(\omega + i\eta - H)^{-1}A^{\dagger}_{\mathbf{k}}|\psi_0\rangle$

more accurate than Lanczos vector method, but requires new run for each ω

Minimization method (DDMRG) (Jeckelmann 2002) correction vector minimizes certain functional and value of functional at minimum is the requested spectral weight. more accurate than correction vector

D-DMRG: Example for photoemission spectrum

Comparison with ARPES on TTF-TCNQ

Time dependent DMRG (t-DMRG)

 $\textsf{Time}\ \textsf{evolution}\ \ |\psi(t)\rangle = \exp[-iHt]|\psi(0)\rangle.$

typically $H = H_0 + H_1 \Theta(t)$, $|\psi(0)\rangle = |\psi_0\rangle$ or $A^{\dagger}|\psi_0\rangle$.

DMRG Approaches to time evolution

- Runge Kutta integration of $|\psi_0\rangle_{\text{DMRG}}$ (Cazallila & Marston 2002) only good at small times due to static Hilbert space
- Division of exp[-iHt] into two site parts which are exactly applied (Vidal; White et al; Daley et al 2003/4) Suzuki-Trotter decomposition, quantum gates.
- Expansion of exp[-iHt] in Krylov Basis (Schmitteckert; Manmana et al, 2004) multi-target method
- Applications: Tunnel current between Luttinger liquids Transport current in a quantum dot Quench dynamics Dynamical correlation functions (Fourier transform)

Application of t-DMRG Quench in the Bose Hubbard model

Start with 1D superfluid initial state, then quench to large U/J value !

Finite Temperature DMRG

Many ideas

- Use Boltzmann weights to weight target states in mixed density matrix Works only at low T, but that's also the region where finite size effects are important (Moukouri & Caron 1996)
- Transfer Matrix DMRG (Bursill et al '1996; Wang & Xiang 1997) Uses Suzuki-Trotter decomposition to map onto 1+1 classical system Perform DMRG on the Suzuki-Trotter lattice for the transfer matrix Imaginary time discrete, but directly for infinite system.
- Purification approach: (e.g. Feiguin & White, 2005) Infinite temperature density matrix of a system can be represented as a pure state of enlarged system. Perform imaginary time evolution to lower temperature T and measure desired quantities (also real-time simulations).

More extensions

- Classical transfer matrices (2D stat mech problem)
- Non-equilibrium classical systems (reaction-diffusion)
- Momentum space DMRG (accurate at small U/t)
- Quantum chemistry

Beyond DMRG

Tensor Networks

A nice graphical way to represent tensors and their contractions

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A nice graphical way to represent tensors and their contractions

Tensor Networks as wave functions

Matrix Product State (MPS)

 $A[\sigma]_{\alpha,\beta}$

Tensor Product States (TPS), Projected Entangled Pair States (PEPS) (Verstraete & Cirac)

Multiscale Entanglement Renormalization Ansatz (MERA) (Vidal)

Tensor Networks as wave functions

- Instead of exponential number of coefficients in the wave functions only $O(N d X^{p})$ coefficients are needed (p=2 for MPS, p=4 for PEPS)
- These tensor networks can be seen as variational wave functions
- The proposed algorithms (PEPS, MERA, ...) contain recipes on how to optimize the network to give the lowest energy for a chosen Hamiltonian
- Not all networks are equally easy to handle.

parameterizes how many correlations the latter can ac-

count for. We refer to¹² for a detailed explanation of the

iPEPS algorithm. In what follows we briefly comment on \mathbb{R}^n

 $W_{\rm eff}$ distinguish three main sources of errors in the sim-

 $u_{\rm{min}}$

lying TPS/PEPS ansatz and two that originate in the

 $\mathcal{L}(\mathcal{L})$ bond dimension $\mathcal{L}(\mathcal{L})$

limits the amount of correlations the amount of $\mathcal{P}_\mathcal{R}$

 $c_{\rm eff}$ typical state of interest \sim typical state of interest \sim

action is defined of D, say \mathbb{R}^2 for D, say D \mathbb{R}^2 for D, say D \mathbb{R}^2

 $s_{\rm H}$ that depends on μ that depends on μ , or μ

 τ

(ii) MPS bond dimension \mathcal{L}^{∞} and dimension \mathcal{L}^{∞} field a field and dimension \mathcal{L}^{∞}

 $\tau_{\rm eff}$ the correlations/entanglement in the target state $\tau_{\rm eff}$

particular way the iPEPS algorithm operates:

 $\tau_{\rm eff}$ sources of errors and on the simulation costs.

i-TEBD / i-PEPS

Assume a translationally invariant ansatz with two matrices in the case of MPS (i-TEBD) or two different tensors in the case of PEPS (i-PEPS)

Perform imaginary time evolution in order to anneal to the ground state state of a local Hamiltonian, requires in general a very

i-TEBD is already quite popular as an alternative to DMRG in 1D, and is simple to implement (I saw a Matlab code with 71 lines for the 1D Q-Ising model) ent (I saw a Matlab code with 71 lines for the 1D Q-Ising model) \qquad | Γ and in Γ \mathcal{D} , and is may differ significantly from the exact values, indicating

First (finite size) PEPS Applications

- PEPS: Frustrated J₁-J₂-J₃ Square lattice (Murg et al., arXiv:0901.2019)
- Square lattices with open boundary conditions

First (finite size) PEPS Applications The results were obtained for a 14 × 14–lattice and virtual $\mathbf{F}_{\mathbf{S}}$, structure factors $\mathbf{F}_{\mathbf{S}}$ (upper plot) and $\mathbf{F}_{\mathbf{S}}$ (upper plot) and $\mathbf{F}_{\mathbf{S}}$ (upper plot) and $\mathbf{F}_{\mathbf{S}}$

- PEPS: Frustrated J₁-J₂-J₃ Square lattice (Murg et al., arXiv:0901.2019) $S(\mathbf{r}, \mathbf{r})$ (see Tauris ploting of \mathbf{r}) as \mathbf{r} p
- Square lattices with open boundary conditions to the thermodynamic limit (see text).

2

-5 0 5

-5 0 5

 $\mathcal{L}_{\mathcal{M}}$

J /J =0.0 3 1 J /J =0.2 3 1

 $J_{\rm eff}$

First (finite size) PEPS Applications

- PEPS: Frustrated J₁-J₂-J₃ Square lattice (Murg et al., arXiv:0901.2019)
- Square lattices with open boundary conditions

diagonalizing within the SRVB subspace (solid line) and by

 \mathcal{P}_max calculations with D $=$

dots). lower plot: Overlap between the SRVB ground state

-0.66 -0.11 -0.64 -0.18 -0.63 -0.15 -0.64

-0.58 -0.13 -0.57 -0.20 -0.54 -0.17 -0.55

-0.19 -0.19 0.05 0.05 0.05 0.05 -0.15 -0.19

-0.24 -0.18 -0.25 -0.23 -0.24 -0.25 -0.20 -0.26

-0.30 -0.24 -0.25 -0.24 -0.24 -0.26 -0.23 -0.27

-0.18 -0.10 0.01 0.02 -0.02 -0.02 -0.11 -0.22

-0.17 -0.15 0.08 0.08 0.11 0.11 0.02 -0.03

-0.18 -0.18 -0.01 -0.01 -0.13 -0.16 -0.24 -0.35

-0.14 -0.10 0.08 0.09 0.11 0.11 -0.04 -0.11

-0.51 -0.24 -0.46 -0.29 -0.45 -0.23 -0.50

-0.51 -0.27 -0.50 -0.30 -0.48 -0.24 -0.51

-0.60 -0.11 -0.61 -0.24 -0.55 -0.24 -0.57

-0.59 -0.13 -0.61 -0.23 -0.54 -0.20 -0.53

-0.64 -0.03 -0.63 -0.21 -0.55 -0.19 -0.51

-0.62 -0.22 -0.57 -0.27 -0.56 -0.23 -0.62

and the D = 3–PEPS ground state.

FIG. 9: upper plot: Ground state energy of the J¹ − J³ First (finite size) PEPS Applications

- diagonalizing with the SRVB subspace (solid line) and by the SRVB subspace (solid line) and by the SRVB subspace (PEPS: Frustrated J₁-J₂-J₃ Square lattice (Murg et al., arXiv:0901.2019)
- Square lattices with open boundary conditions

 ~ 0.5

nian.

 0.5

 0.22

1

 $\frac{\Lambda}{V}$ -0.55 -0.54

 -0.5 ~
七

 C° 0.21

යි 0.21 0.22

 0.4

(i) Bond dimension D.— A finite bond dimension D limits the amount of correlations the TPS/PEPS can carry. A typical state of interest |Ψ!, e.g. the ground state of a local Hamiltonian, requires in general a very $\overline{\mathcal{C}}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\begin{array}{ccc} \n\vee & & \nearrow \ \n\end{array}$ $\begin{array}{ccc} \n\searrow & \searrow \n\end{array}$ to a good approximate \mathcal{M} \mathcal{Y}) and \mathcal{Y} $\frac{1}{10}$ -0.55 $\frac{1}{\sqrt{1-\frac{1$ $\begin{bmatrix} 0.566 \\ 0.566 \end{bmatrix}$ the correlations \mathbb{Z} $\frac{1}{\sqrt{2}}$ MPS bond dimension $\frac{1}{\sqrt{2}}$ $\begin{matrix} 0 \\ 0 \end{matrix}$ implies that the contraction $\begin{matrix} 0 \\ 0 \end{matrix}$ 0.22 $\overline{}$ $\overline{\text{O}_{0.21}}$ pute expected values of local observables) is only approx- $\overline{0.2\frac{1}{0}}$ or in the expected value of local observables even when the TPS/PEPS was an accurate representation of the in-

 \sim $\frac{1}{\sqrt{2}}$ imaginary time) is simulated by using a Suzuki-Trotter expansion of the evolution operator (e Γ $\begin{bmatrix} 2 & 0.21 \end{bmatrix}$ introduces an error in the evolution that scales as some \overline{a}

 $\begin{array}{c} 0 \\ 0 \end{array}$

 \int_0^∞

 -0.55 0

 -0.56 0

 $\overline{\mathbf{0}}$

 $\begin{array}{c} \n\hline\n\end{array}$ $\begin{bmatrix} 0 \\ 0 \\ 22 \end{bmatrix}$ invariant perturbation \mathbb{R}^n

 $0.2 \frac{1}{0}$ 1

 $^{\circ}$ $^{\circ}$

 $\begin{array}{c} 0.2 \ \hline 0 \end{array}$

 $\mathcal{F}_{\mathcal{A}}$ for $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}}$ the density ρ, and condensate fraction ρ⁰ after a translation invariant perturbation V is suddenly added to the Hamilto-

 w ithin the separable Mott-Insulator phase ($\frac{1}{2}$ \mathcal{L}^2 \mathcal{L}^2 is matrix different from that between that between that between that between that between the set $g_{\rm eff}$ states in the superfluid region (4 where the properties of the system vary continuously. Moreover, similarly to what has been observed for the 2D quantum Ising model¹⁶ or in the 2D quantum XYX model19, the presence of a continuous quantum phase transition between insulating and superfluid phases in the $2D$ HCBH model is signaled by pinch points of \mathcal{L} at μ \sim μ ground state properties across the critical point is evidenced by a rapid, continuous change in the fidelity per lattice site as one considers two ground states on opposite

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An attractive feature of the algorithms based on tensor networks is the possibility to simulate (real) time (real) times (real) times (

sides of the critical point and moves away from it.

 $T_{\rm eff}$ of the simulations scales as $T_{\rm eff}$ (here we indicate only the leading orders in χ and D; the cost of the simulation is also roughly proportional to the inverse of the time step). This scaling implies that only small values of the bond dimensions D and χ can be used in practice. In our simulations, given a value of D $\mathcal{L} = \{ \mathcal{L} \mid \mathcal{L} \in \mathcal{L} \}$ range 10 − 40) and sufficiently small time step (δt or δτ) such that the results no longer depend significantly on these two parameters. In this way the bond dimension D is the only parameter on which the accuracy of our

On a 2.4 GHz dual core desktop with 4 Gb of RAM, computing a superfluid ground state (e.g. µ = 0) with D = 2, χ = 20 and with δτ decreasing from 10−¹ to 10−⁴ requires about 12 hours. Computing the same ground state with D = 3 and χ = 40 takes of the order of two

duced by simply diminishing the time step.

First i-PEPS Applications SSE 4

Andersen

MF

In this section we present the numerical results ob-

 $W_{\rm eff}$

In this section we present the numerical results ob-

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 \vert \mathbb{R}^n . Then we use the resulting \mathbb{R}^n

 \vert .

 \mathbb{R}

the expected value of local observables, analyze ground

 \vert \mathbb{R}^n

state entanglement, compute two-point correlators and

fidelities, or as the starting point for an evolution in real

the expected value of local observables, analyze ground

state entanglement, compute two-point correlators and

fidelities, or as the starting point for an evolution in real

 \mathbb{I}

 \mathbb{R}^n

 $\mathcal{L}_{\mathcal{A}}$

 $| \cdot |$ is vacant. Our results are in remarkable agreement with those obtained in Ref.⁶ with stochastic series expansions

 $\mathcal{L}_{\mathcal{A}}$

 \mathcal{L} is vacant. Our results are in remarkable agreement with \mathbb{R}^n

Particle density ρ.— Fig. 2 shows the density ρ as a

 \vert mean field calculations plus spin wave corrections (SW).

 \mathbb{R}^n

 \vert mean field calculations plus spin wave corrections (SW).

 \mathbb{R}^n

site " as a function of the density ρ. This is obtained

by computing "(µ) and then replacing the dependence

Energy per site ".— Fig. 2 also shows the energy per

 $\overline{}$

 $\overline{}$ as $\overline{}$

Energy per site ".— Fig. 2 also shows the energy per

 \vert

 \mathbb{R}^n

 \vert

 \mathbb{R}^n

 \mathcal{L}

• iPEPS: Hardcore bosons on the square lattice $\begin{array}{ccc} \begin{array}{c} 1 & \rightarrow & \end{array} & \begin{array}{ccc} 0 & \rightarrow & \$ equivalent to S=1/2 XY model $\overline{}$ \overline{c} Andersen ' + D = 3

responding to several other techniques. Our results follow

tice site "(ρ) and condensate fraction ρ0(ρ) for a TPS/PEPS

First MERA Applications

fect hexagons (green). Dotted arrows indicate the axis where

ners of three neighboring blocks. Then disentanglers *v*

 \mathbb{R}^n

 \mathbf{r}

 $\overline{}$

 \Box

MERA: Kagome lattice Heisenberg model (Evenbly and Vidal, arXiv:0904.3383)

Confirms the hypothesis of a 36 sites VBC on the kagome lattice

Conclusion

- DMRG is a highly successful method to treat 1D quantum mechanical many body systems at T=0, based on density matrix driven truncation.
- Many extension have been put forward which allow to calculate spectral functions, real time evolution, classical systems in 2D, finite temperature properties, dissipation,
- Quantum information theory provided deep insights in the possibilities and limitations of DMRG and proposes new setups to extend DMRG to 2D. However the efficiency of 1D DMRG has not yet been achieved in 2D.

DMRG Literature

Original papers:

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Thank you !