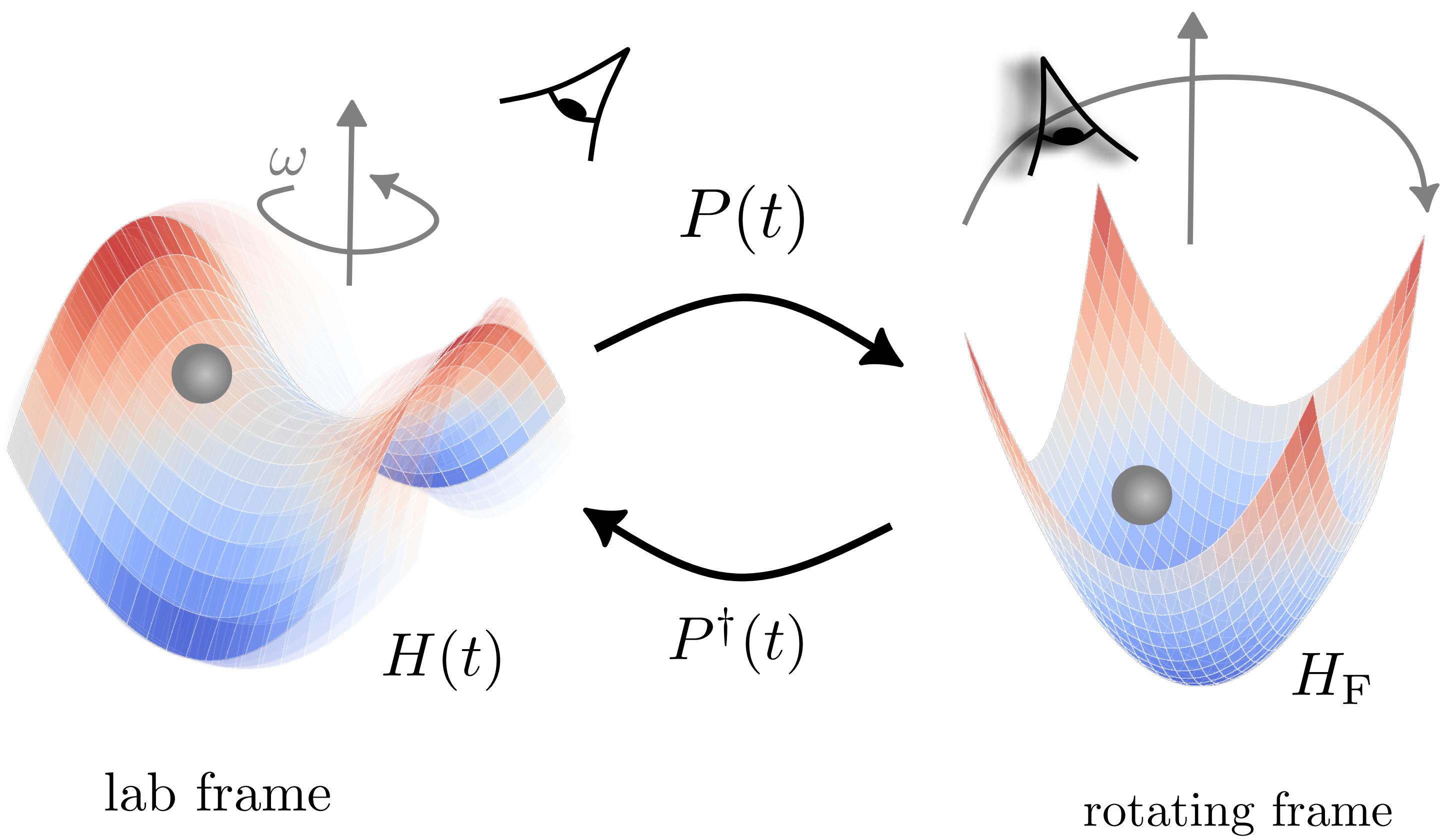
MAX PLANCK INSTITUTE
FOR THE PHYSICS OF COMPLEX SYSTEMS

Floquet engineering for quantum simulation



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MAX PLANCK GESELLSCHAFT
Funded by the European Union
European Research Council
Established by the European Commission
DFG
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Marín Bukov

MPI-PKS (Dresden)

Quantum technologies

Quantum Communication

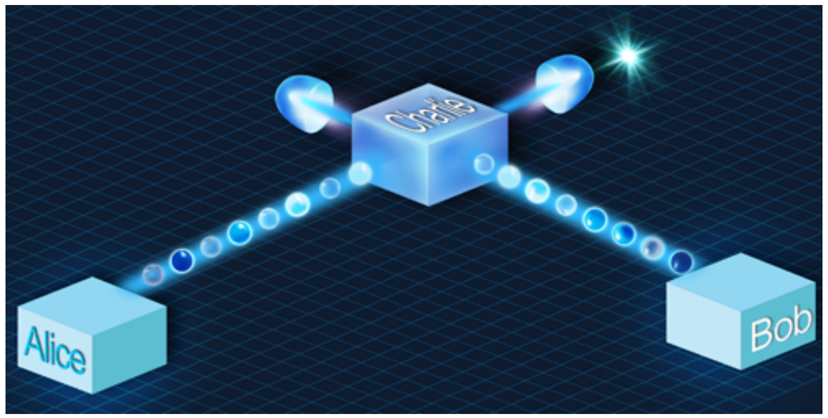


image: PRL 123 100506 (2019)

- process info with unprecedented security

Quantum Sensing & Metrology

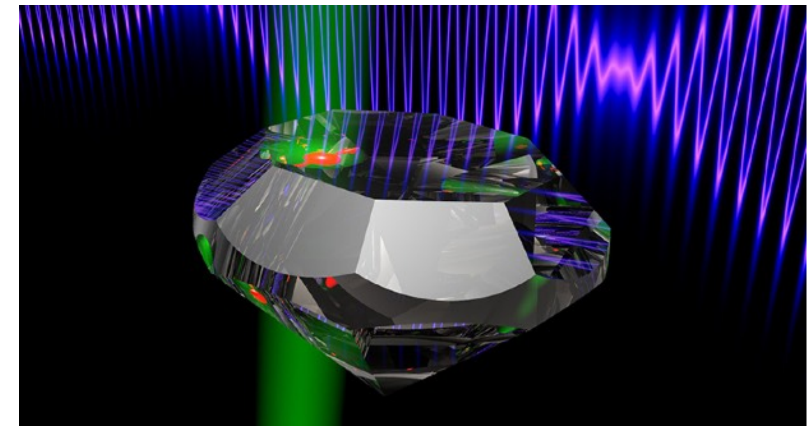


image: ETH Zurich

- measure weakest of fields

Quantum Computing

image: IBM

- speed up essential algorithms

Quantum Simulation

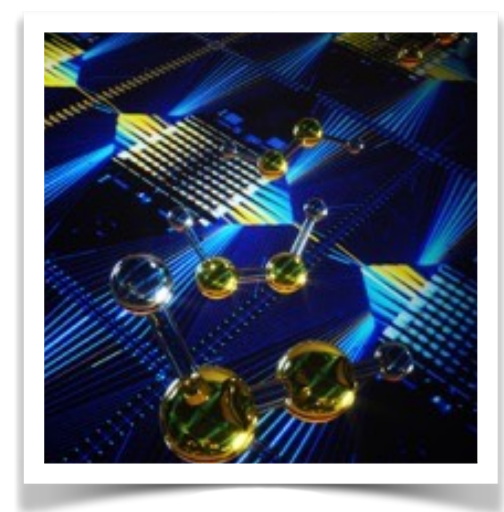


image: KITP

- understand properties of quantum matter, complex molecules, drug discovery *neutral atoms*

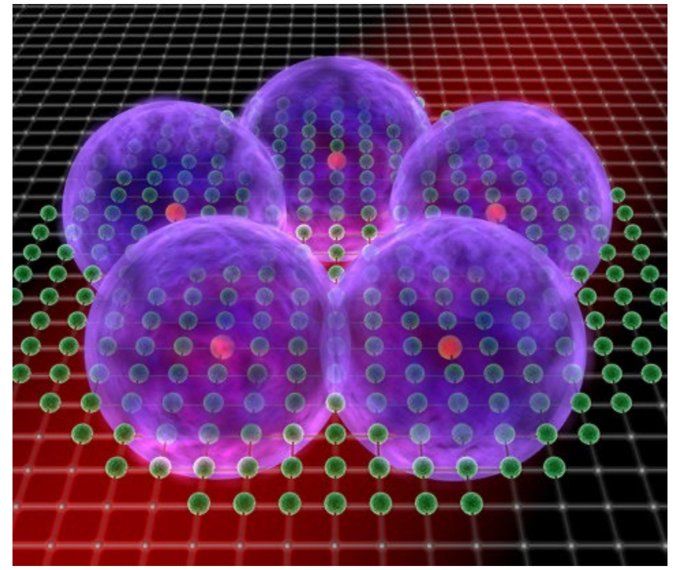


image: MPQ

superconducting qubits

image: IBM

trapped ions

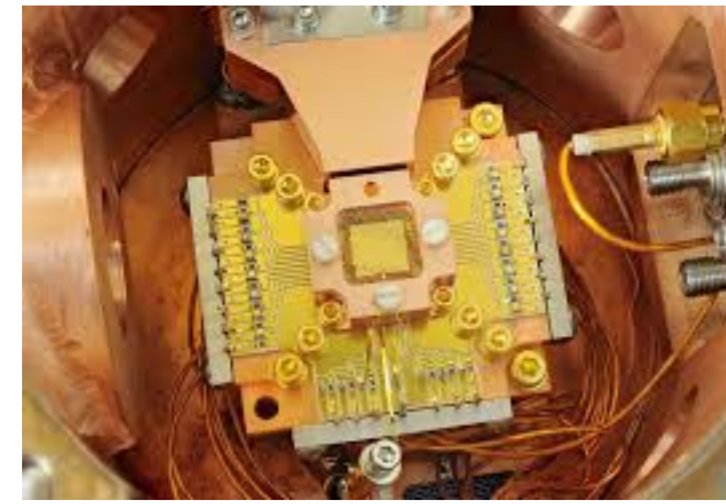


image: Wikipedia

platforms for quantum simulation

nitrogen-vacancy (NV) centers

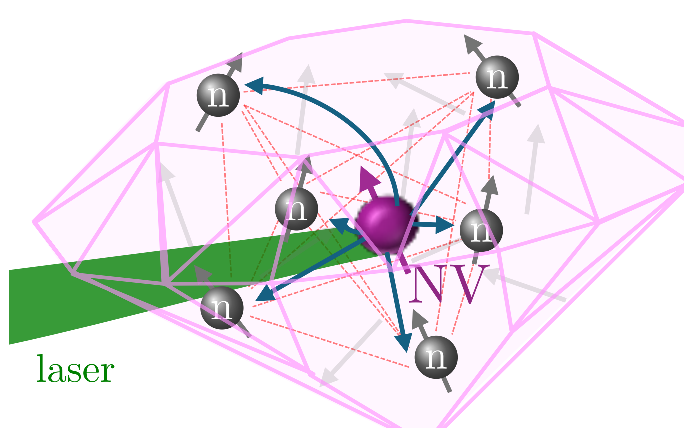


image: arXiv:2404.05620

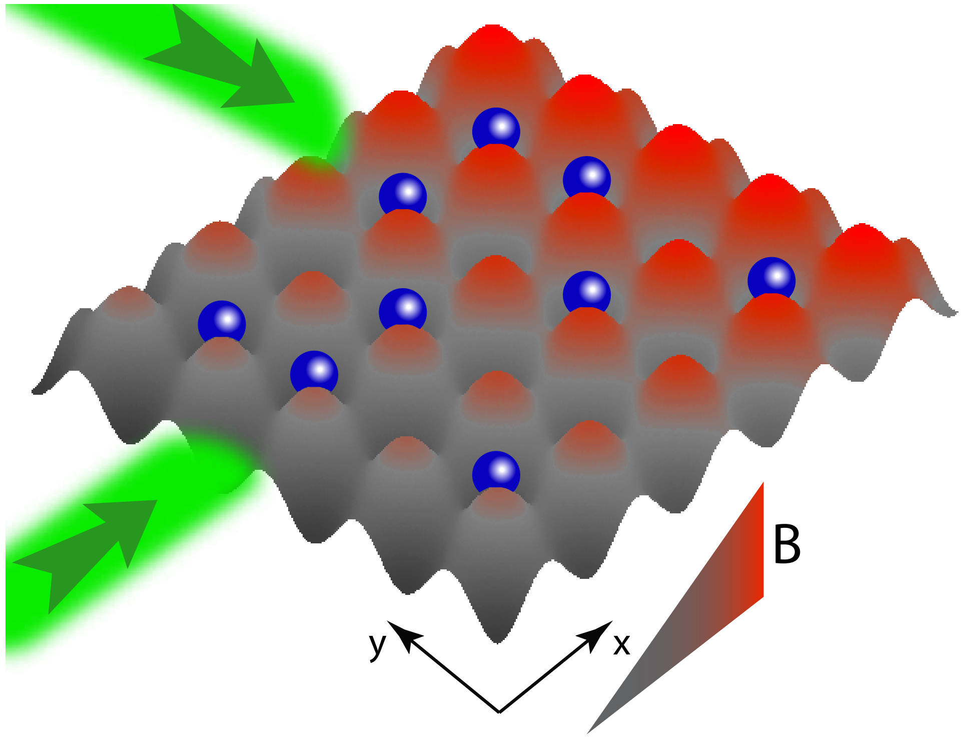
photons

image: ParityQC

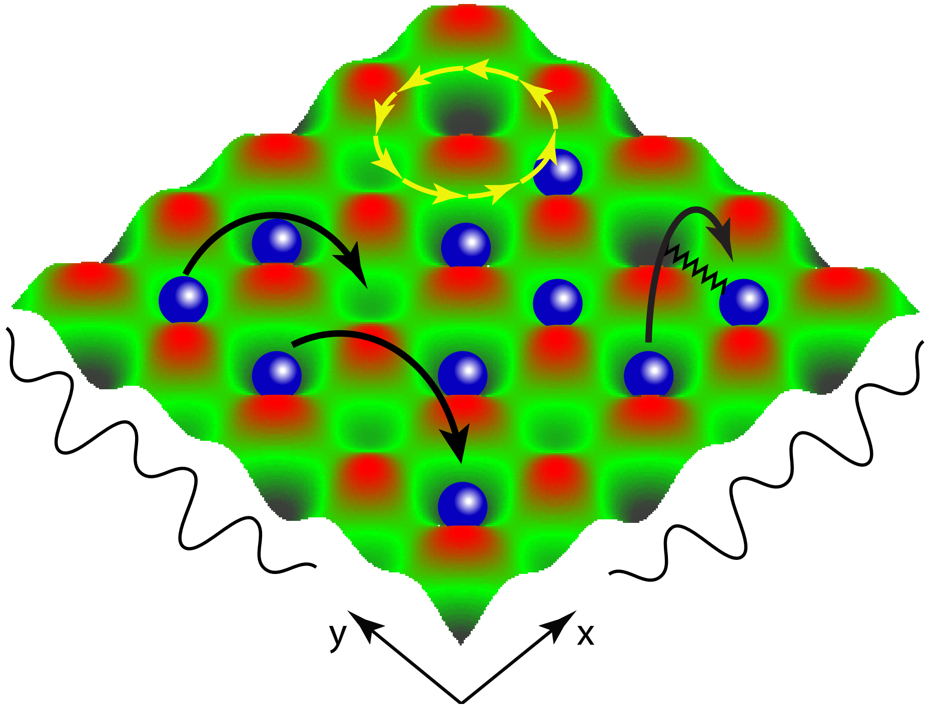
Quantum Simulators

Richard P Feynman

quantum simulator



quantum system of interest



"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy." (1982)

- use one quantum system to emulate the behavior of another
- restrictions: not all quantum systems can be simulated

Q: how can we expand the range of systems we can simulate?

Gross & Bloch, Science 357, 6355 (2017)

Periodically driven systems

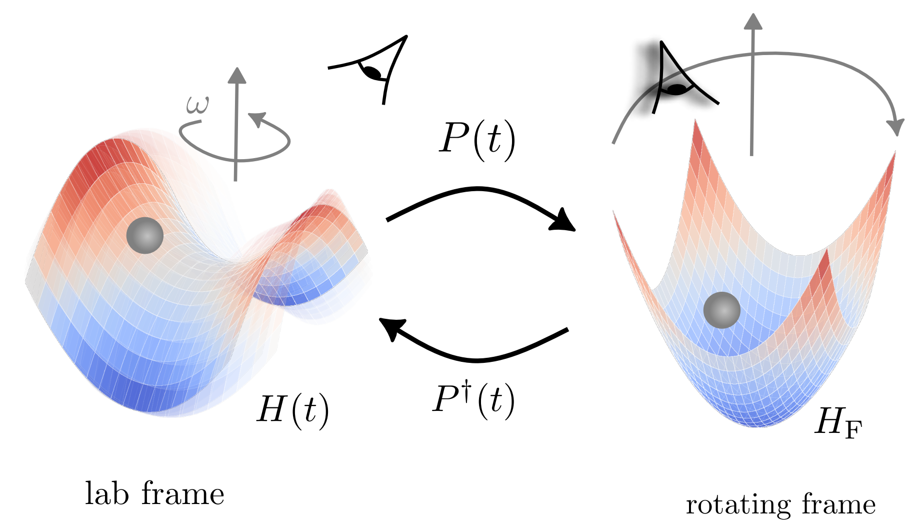
video: YouTube (bluedwarf1127)

video: YouTube (Harvard Nat Sci)

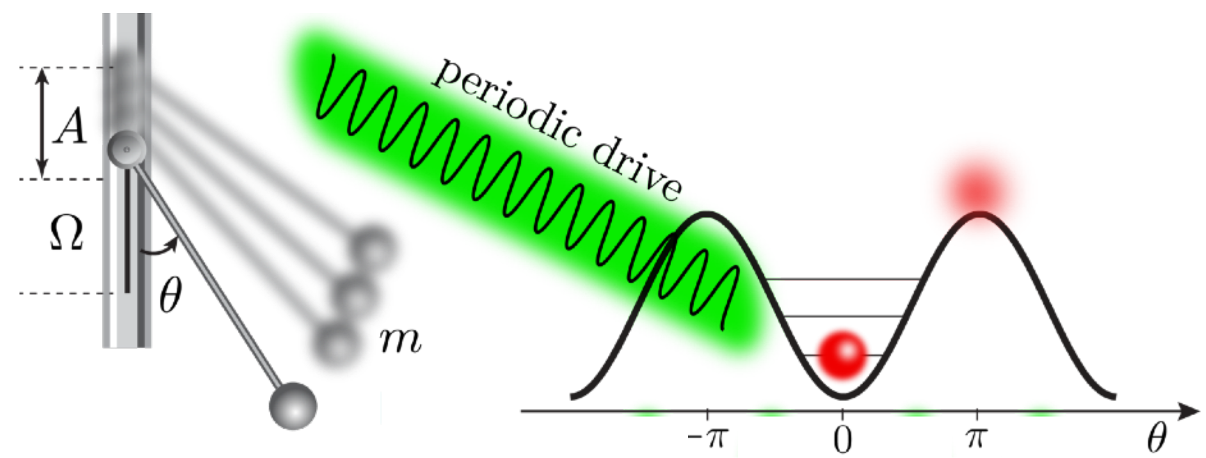
High-frequency periodic drives
can change drastically
the fundamental properties of physical systems

Outline

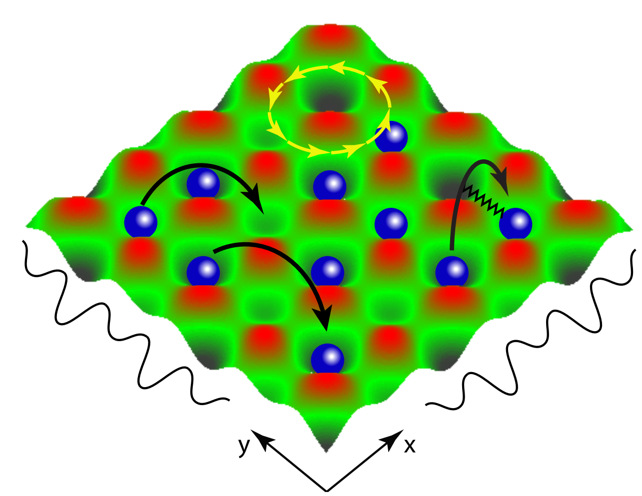
- Rotating reference frames
  - classical systems: fictitious forces
  - quantum systems



- Periodically driven quantum systems
  - Floquet theorem
  - Floquet engineering

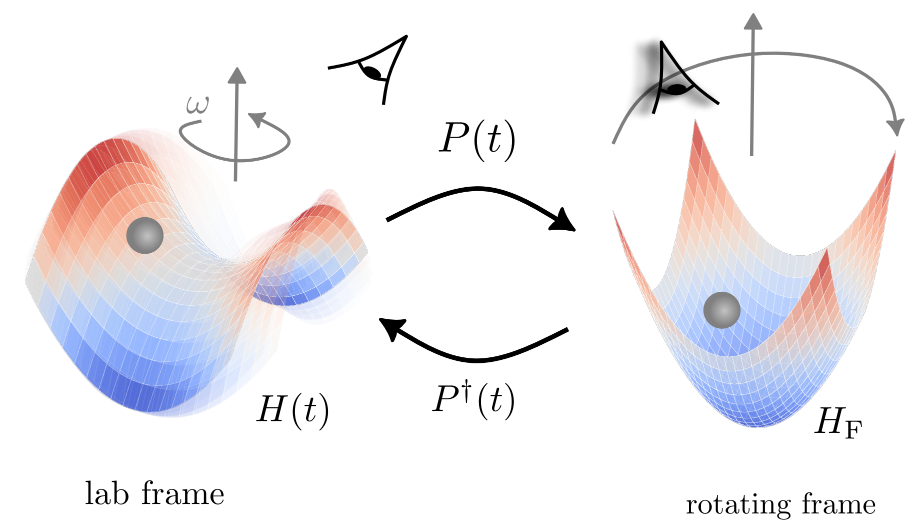


- Examples
  - spin-1 particle in a circularly polarized drive
  - quantum Kapitza oscillator
  - artificial gauge fields



Outline

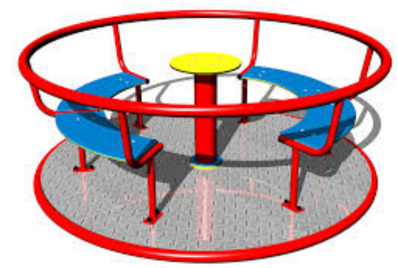
- Rotating reference frames
    - classical systems: fictitious forces

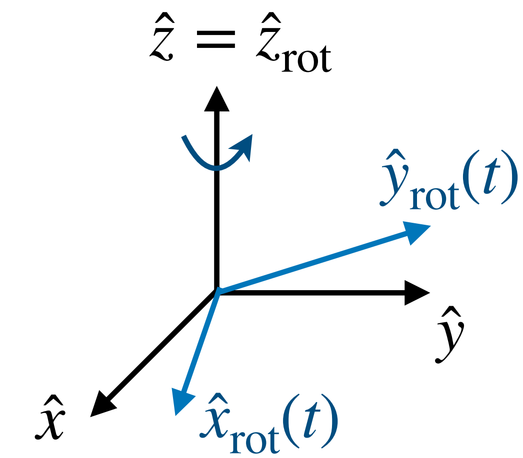


Classical mechanics

- rotating reference frame
  - not inertial
  - fictitious forces

Merry-go-round

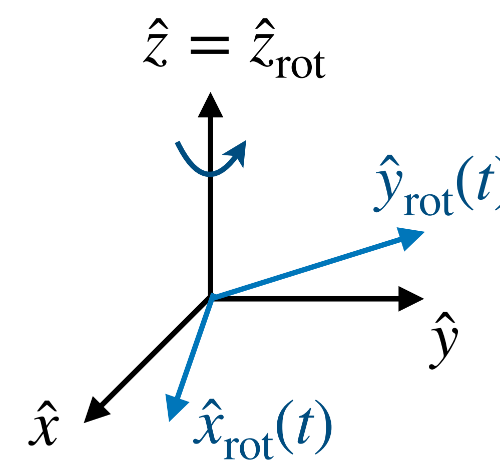




Classical mechanics

- rotating reference frame
  - not inertial
  - fictitious forces

e.g., $P(t) = \begin{pmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$



- transformation between lab and rotating frames

$$\text{lab frame} \underset{\text{rotation matrix: } P(t)}{\overset{P^{-1}(t) = P^{\dagger}(t)}{\rightleftarrows}} \text{rot frame}$$

Classical mechanics

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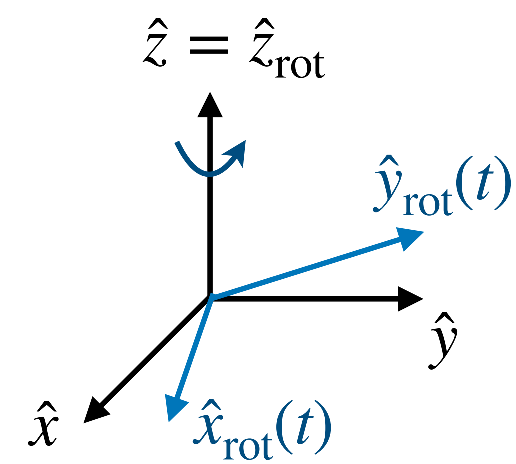

- **transformation between lab and rotating frames**

$P^{-1}(t) = P^{\dagger}(t)$

lab frame

rot frame

rotation matrix: $P(t)$

position: $\vec{r}_{\rm lab}(t)$

$\vec{r}_{\rm rot}(t)$

$$\vec{r}_{\rm lab}(t) = P(t)\vec{r}_{\rm rot}(t)$$

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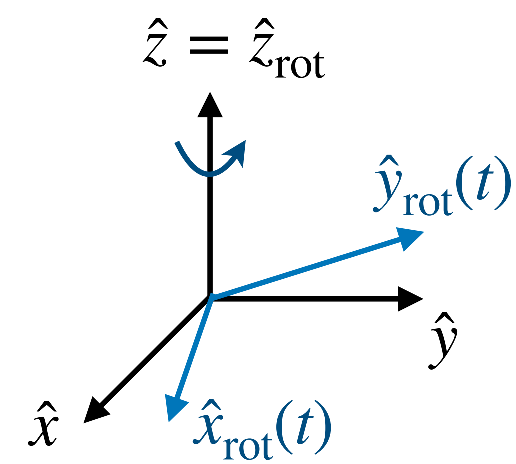

- **transformation between lab and rotating frames**

$$P^{-1}(t) = P^{\dagger}(t)$$

lab frame $\quad\rightleftarrows\quad$ **rot frame**

rotation matrix: $P(t)$

position: $\vec{r}_{\rm lab}(t)$		$\vec{r}_{\rm rot}(t)$
velocity: $\dot{\vec{r}}_{\rm lab}(t)$		$\dot{\vec{r}}_{\rm rot}(t)$
acceleration: $\ddot{\vec{r}}_{\rm lab}(t)$		$\ddot{\vec{r}}_{\rm rot}(t)$

$$\vec{r}_{\rm lab}(t) = P(t)\vec{r}_{\rm rot}(t)$$

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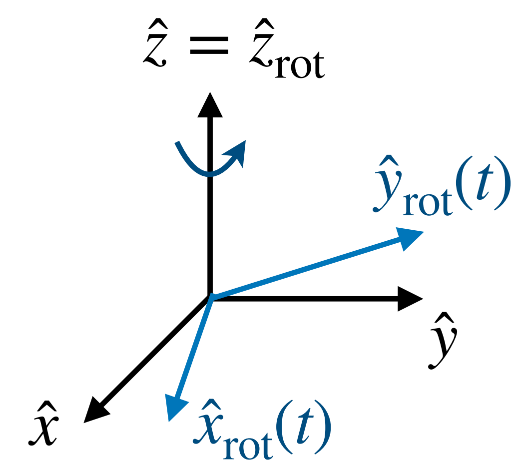


- **transformation between lab and rotating frames**

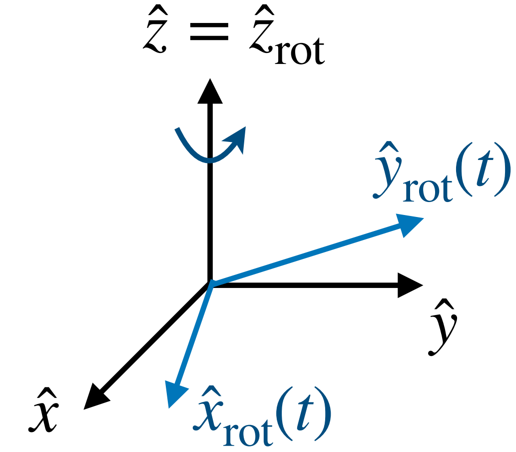
$$\text{lab frame} \underset{\text{rotation matrix: } P(t)}{\overset{P^{-1}(t) = P^{\dagger}(t)}{\rightleftarrows}} \text{rot frame}$$

lab frame	rot frame
position: $\vec{r}_{\mathrm{lab}}(t)$	$\vec{r}_{\mathrm{rot}}(t)$
velocity: $\dot{\vec{r}}_{\mathrm{lab}}(t)$	$\dot{\vec{r}}_{\mathrm{rot}}(t)$
acceleration: $\ddot{\vec{r}}_{\mathrm{lab}}(t)$	$\ddot{\vec{r}}_{\mathrm{rot}}(t)$
force: $\vec{F}_{\mathrm{lab}}$	$\vec{F}_{\mathrm{rot}}$

$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

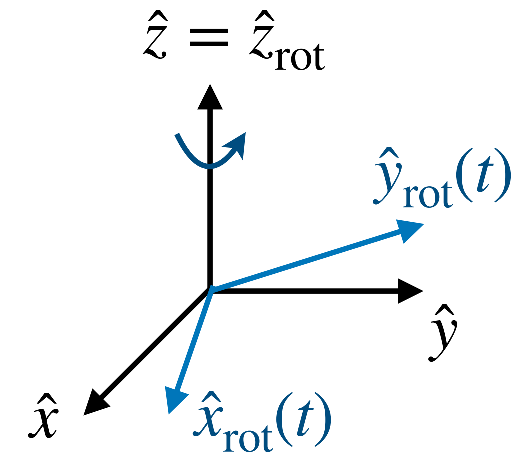
$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

Rotations



- time-dependent rotation matrix

$$P(t) = \begin{pmatrix} \cos \omega t & -\sin \omega t & 0 \\ \sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Rotations

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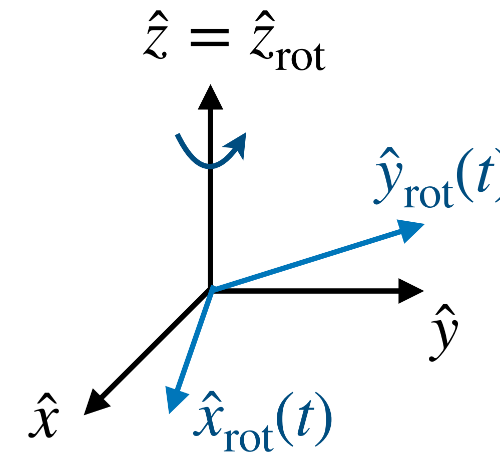
  - inverse transformation

$$P^{-1}(t) = P^{\dagger}(t) = P(t; -\omega) = \begin{pmatrix} \cos\omega t & +\sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotations



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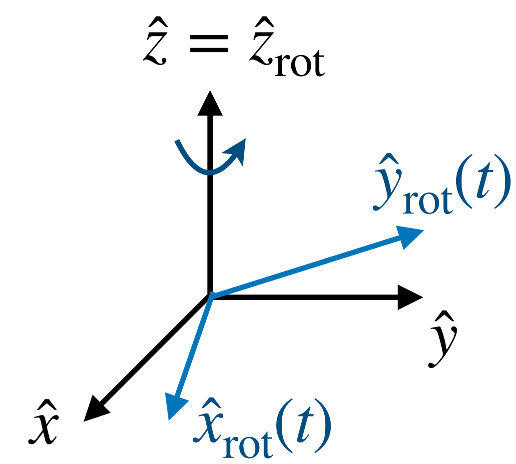
  - element-wise derivatives $$\dot{P}(t) = \omega \begin{pmatrix} -\sin\omega t & -\cos\omega t & 0 \\ \cos\omega t & -\sin\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotations



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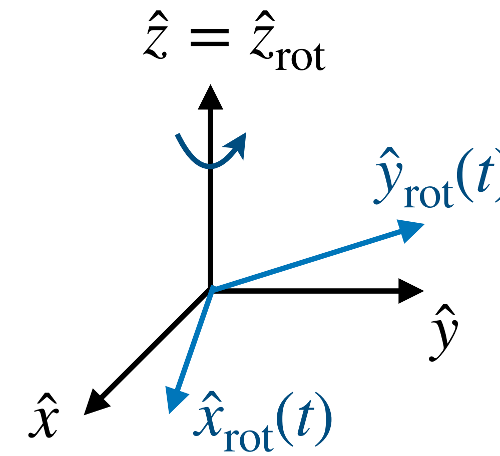
$$P^{\dagger}(t)\dot{P}(t) = \begin{pmatrix} \cos\omega t & +\sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \omega \begin{pmatrix} -\sin\omega t & -\cos\omega t & 0 \\ \cos\omega t & -\sin\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix} = \omega \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \omega \hat{z} \times$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$



Rotations

- **time-dependent rotation matrix**

$$P(t) = \begin{pmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$\dot{P}(t) = \omega \begin{pmatrix} -\sin\omega t & -\cos\omega t & 0 \\ \cos\omega t & -\sin\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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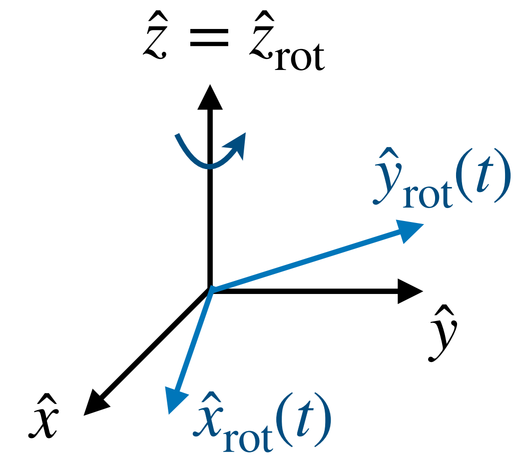
- **general time-dependent rotation axis $\vec{\omega}(t)$**

$$P^{\dagger}(t)\dot{P}(t) = \vec{\omega}(t) \times$$

Newton's equations in rotating frame

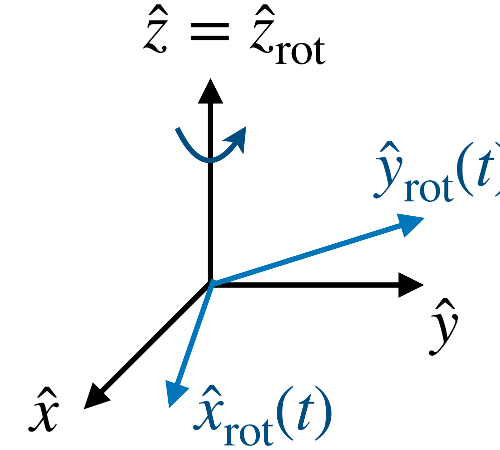


- **Newton's law**
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\text{lab}}(t) = \overrightarrow{F}_{\text{lab}}(t) \quad / \quad P^{\dagger}(t)\,\cdot$

$$\vec{r}_{\text{lab}}(t) = P(t)\vec{r}_{\text{rot}}(t)$$

$$\overrightarrow{F}_{\text{lab}}(t) = P(t)\overrightarrow{F}_{\text{rot}}(t)$$

Newton's equations in rotating frame



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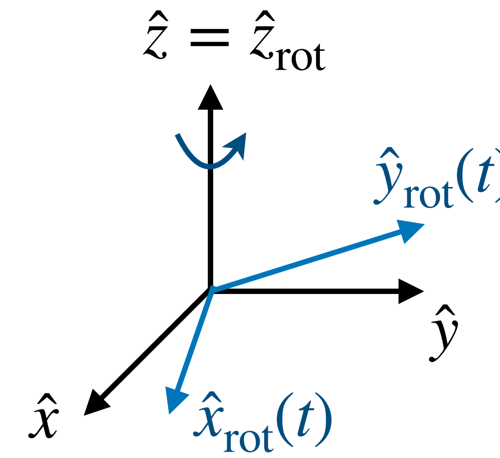
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\rm lab}(t) = \overrightarrow{F}_{\rm lab}(t) \quad / \quad P^\dagger(t)\,\cdot$
  - rot frame: $mP^\dagger(t)\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[P(t)\underline{P^\dagger(t)\vec{r}_{\rm lab}(t)}\right] = P^\dagger(t)\overrightarrow{F}_{\rm lab}(t)$

$$\vec{r}_{\rm lab}(t) = P(t)\vec{r}_{\rm rot}(t)$$

$$\overrightarrow{F}_{\rm lab}(t) = P(t)\overrightarrow{F}_{\rm rot}(t)$$

Newton’s equations in rotating frame



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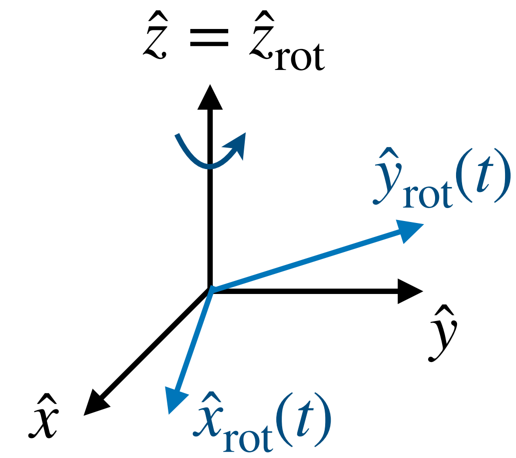
Newton's equations in rotating frame



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$$m\left(P^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}P(t)\right)^2\vec{r}_{\text{rot}}(t) = P^\dagger(t)\vec{F}_{\text{lab}}(t)$$

$$\vec{r}_{\text{lab}}(t) = P(t)\vec{r}_{\text{rot}}(t)$$

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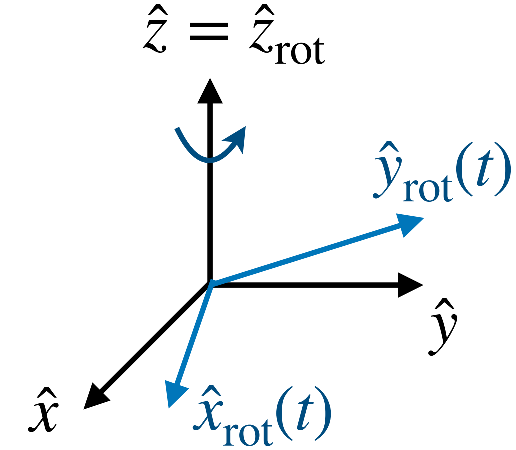
$$P^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}P(t)\vec{f}(t) = P^\dagger(t)\dot{P}(t)\vec{f}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\vec{f}(t)$$
$$= \left(P^\dagger(t)\dot{P}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\right)\vec{f}(t)$$

Newton's equations in rotating frame



- Newton's law
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$$m\left(\underline{P^\dagger(t)\dot{P}(t)} + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2\vec{r}_{\rm rot}(t) = P^\dagger(t)\vec{F}_{\rm lab}(t)$$

$$\vec{r}_{\rm lab}(t) = P(t)\vec{r}_{\rm rot}(t)$$

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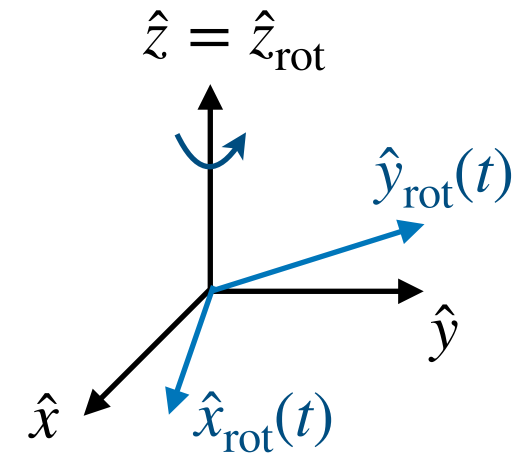
$$= \left(P^\dagger(t)\dot{P}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\right)\vec{f}(t)$$

$$P^\dagger(t)\dot{P}(t) = \vec{\omega}(t)\times$$

Newton's equations in rotating frame



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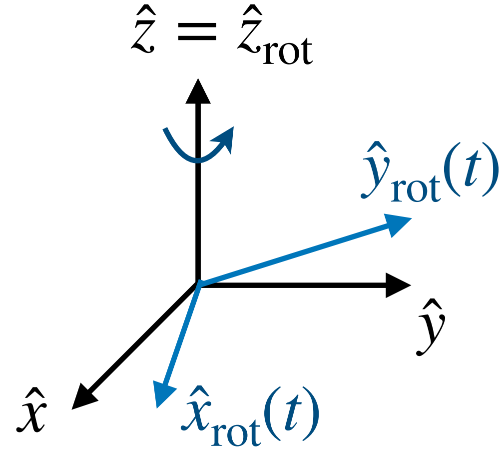
$$P^\dagger(t)\dot{P}(t) = \vec{\omega}(t)\times$$

$$m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)^2\vec{r}_{\mathrm{rot}}(t) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$



Newton's equations in rotating frame

- **Newton's law**
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t) \quad / \quad P^\dagger(t)\,\cdot$
  - rot frame: $mP^\dagger(t)\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[P(t)P^\dagger(t)\vec{r}_{\mathrm{lab}}(t)\right] = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$

$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

$$mP^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}\left(P(t)P^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}\left[P(t)\vec{r}_{\mathrm{rot}}(t)\right]\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$m\left(P^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}P(t)\right)^2\vec{r}_{\mathrm{rot}}(t) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$P^\dagger(t)\frac{\mathrm{d}}{\mathrm{d}t}P(t)\vec{f}(t) = P^\dagger(t)\dot{P}(t)\vec{f}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\vec{f}(t)$$
$$= \left(P^\dagger(t)\dot{P}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\right)\vec{f}(t)$$

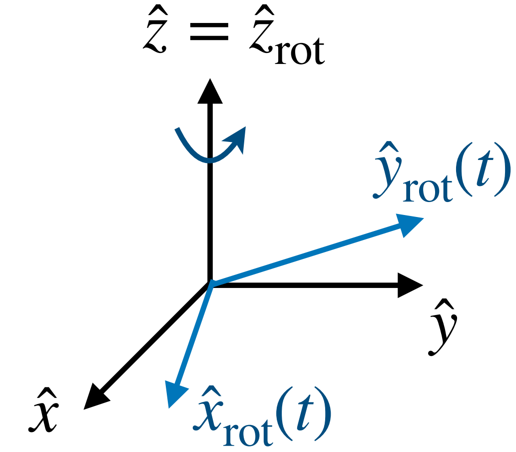
$$m\left(P^\dagger(t)\dot{P}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2\vec{r}_{\mathrm{rot}}(t) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$P^\dagger(t)\dot{P}(t) = \vec{\omega}(t)\times$$

$$m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)^2\vec{r}_{\mathrm{rot}}(t) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

Newton's equations in rotating frame

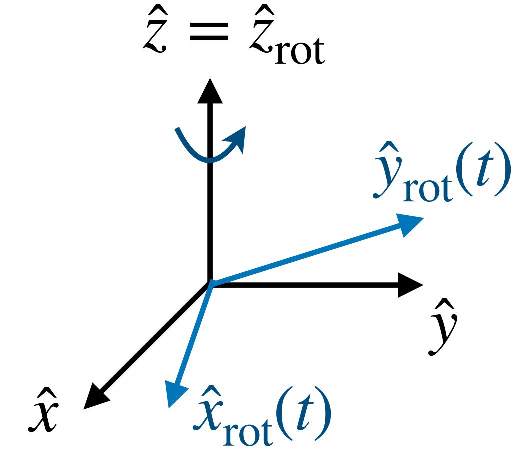


$$\vec{r}_{\text{lab}}(t) = P(t)\vec{r}_{\text{rot}}(t)$$

$$\vec{F}_{\text{lab}}(t) = P(t)\vec{F}_{\text{rot}}(t)$$

- **Newton's law**
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\text{lab}}(t) = \vec{F}_{\text{lab}}(t)$
  - rot frame: $m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\text{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\text{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\text{lab}}(t)$

Newton's equations in rotating frame



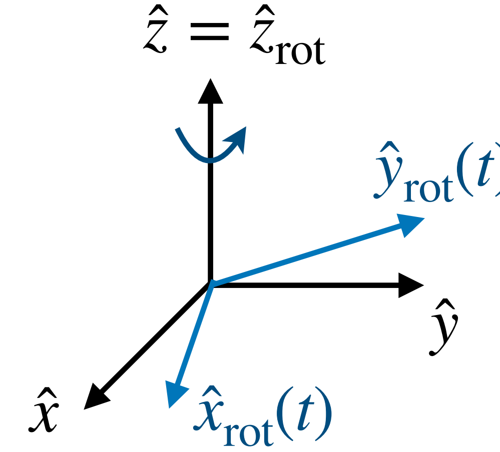
$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

- Newton's law
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t)$
  - rot frame: $$m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

HW: $$m\ddot{\vec{r}}_{\mathrm{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

Newton's equations in rotating frame



$$\vec{r}_{\rm lab}(t) = P(t)\vec{r}_{\rm rot}(t)$$

$$\vec{F}_{\rm lab}(t) = P(t)\vec{F}_{\rm rot}(t)$$

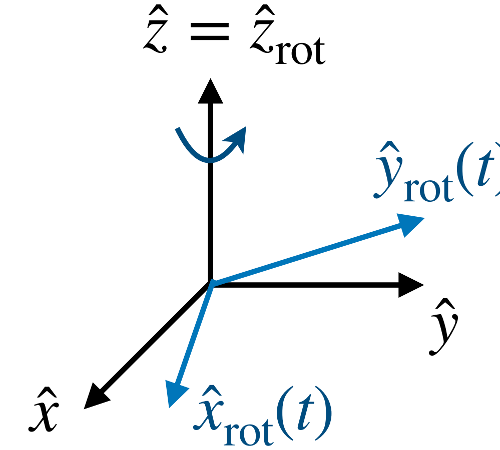
- Newton's law
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\rm lab}(t) = \vec{F}_{\rm lab}(t)$
  - rot frame: $m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\rm rot}(t) + \vec{\omega}(t)\times\vec{r}_{\rm rot}(t)\right) = P^\dagger(t)\vec{F}_{\rm lab}(t)$

$$m\ddot{\vec{r}}_{\rm rot}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\rm rot}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\rm rot}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\rm rot}(t)\right) = P^\dagger(t)\vec{F}_{\rm lab}(t)$$

$$m\ddot{\vec{r}}_{\rm rot}(t) = P^\dagger(t)\vec{F}_{\rm lab}(t) - m\dot{\vec{\omega}}(t)\times\vec{r}_{\rm rot}(t) - 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\rm rot}(t) - m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\rm rot}(t)\right)$$

transformed
original force

Newton’s equations in rotating frame



$$\vec{r}_{\text{lab}}(t) = P(t)\vec{r}_{\text{rot}}(t)$$

$$\vec{F}_{\text{lab}}(t) = P(t)\vec{F}_{\text{rot}}(t)$$

- **Newton’s law**
  - lab frame: $m\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\text{lab}}(t) = \vec{F}_{\text{lab}}(t)$
  - rot frame: $m\left(\dfrac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\text{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\text{rot}}(t)\right) = P^{\dagger}(t)\vec{F}_{\text{lab}}(t)$

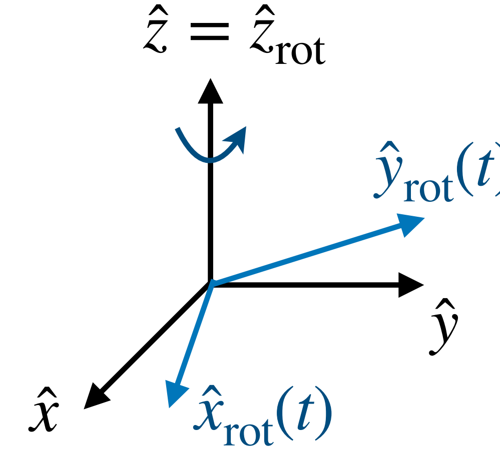
$$m\ddot{\vec{r}}_{\text{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\text{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\text{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\text{rot}}(t)\right) = P^{\dagger}(t)\vec{F}_{\text{lab}}(t)$$

$$m\ddot{\vec{r}}_{\text{rot}}(t) = P^{\dagger}(t)\vec{F}_{\text{lab}}(t) - m\dot{\vec{\omega}}(t)\times\vec{r}_{\text{rot}}(t) - 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\text{rot}}(t) - m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\text{rot}}(t)\right)$$

transformed original force

Euler force

Newton's equations in rotating frame



$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

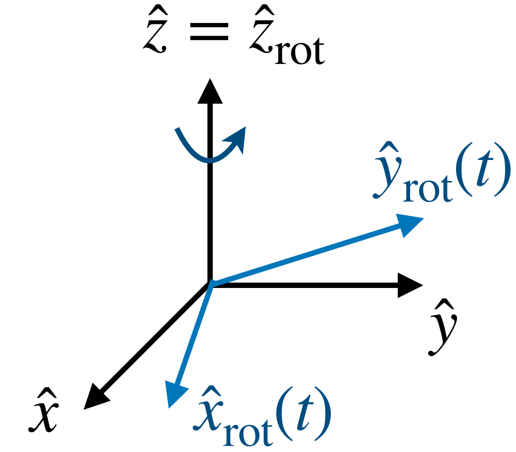
- Newton's law
  - lab frame: $m\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t)$
  - rot frame: $$m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^{\dagger}(t)\vec{F}_{\mathrm{lab}}(t)$$

$$m\ddot{\vec{r}}_{\mathrm{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^{\dagger}(t)\vec{F}_{\mathrm{lab}}(t)$$

$$m\ddot{\vec{r}}_{\mathrm{rot}}(t) = P^{\dagger}(t)\vec{F}_{\mathrm{lab}}(t) - m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) - 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) - m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right)$$

transformed original force | Euler force | Coriolis force

Newton's equations in rotating frame



$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

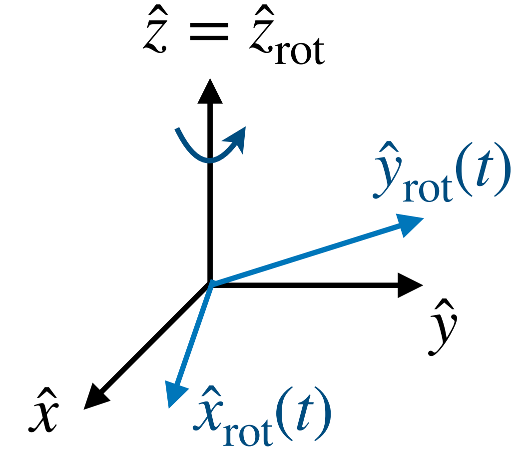
$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

- Newton's law
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t)$
  - rot frame: $m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$

$$m\ddot{\vec{r}}_{\mathrm{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$\boxed{m\ddot{\vec{r}}_{\mathrm{rot}}(t) = \underbrace{P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)}_{\text{transformed original force}} - \underbrace{m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t)}_{\text{Euler force}} - \underbrace{2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t)}_{\text{Coriolis force}} - \underbrace{m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right)}_{\text{centrifugal force}}}$$

Newton's equations in rotating frame



$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

- Newton's law
  - lab frame: $m\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t)$
  - rot frame: $m\left(\dfrac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$

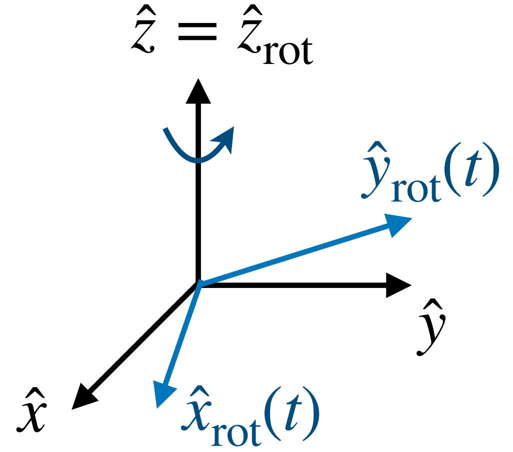
$$m\ddot{\vec{r}}_{\mathrm{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$\boxed{m\ddot{\vec{r}}_{\mathrm{rot}}(t) = \underbrace{P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)}_{\text{transformed original force}} - \underbrace{m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t)}_{\text{Euler force}} - \underbrace{2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t)}_{\text{Coriolis force}} - \underbrace{m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right)}_{\text{centrifugal force}}}$$

- fictitious forces arise from Galilean term $P^\dagger(t)\dot{P}(t)$

$$P^\dagger(t)\dot{P}(t) = \vec{\omega}(t)\times$$

Newton's equations in rotating frame



$$\vec{r}_{\mathrm{lab}}(t) = P(t)\vec{r}_{\mathrm{rot}}(t)$$

$$\vec{F}_{\mathrm{lab}}(t) = P(t)\vec{F}_{\mathrm{rot}}(t)$$

- Newton's law
  - lab frame: $m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}_{\mathrm{lab}}(t) = \vec{F}_{\mathrm{lab}}(t)$
  - rot frame: $m\left(\frac{\mathrm{d}}{\mathrm{d}t} + \vec{\omega}(t)\times\right)\left(\dot{\vec{r}}_{\mathrm{rot}}(t) + \vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$

$$m\ddot{\vec{r}}_{\mathrm{rot}}(t) + m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t) + 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t) + m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right) = P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)$$

$$\boxed{m\ddot{\vec{r}}_{\mathrm{rot}}(t) = \underbrace{P^\dagger(t)\vec{F}_{\mathrm{lab}}(t)}_{\text{transformed original force}} \underbrace{- m\dot{\vec{\omega}}(t)\times\vec{r}_{\mathrm{rot}}(t)}_{\text{Euler force}} \underbrace{- 2m\vec{\omega}(t)\times\dot{\vec{r}}_{\mathrm{rot}}(t)}_{\text{Coriolis force}} \underbrace{- m\vec{\omega}(t)\times\left(\vec{\omega}(t)\times\vec{r}_{\mathrm{rot}}(t)\right)}_{\text{centrifugal force}}}$$

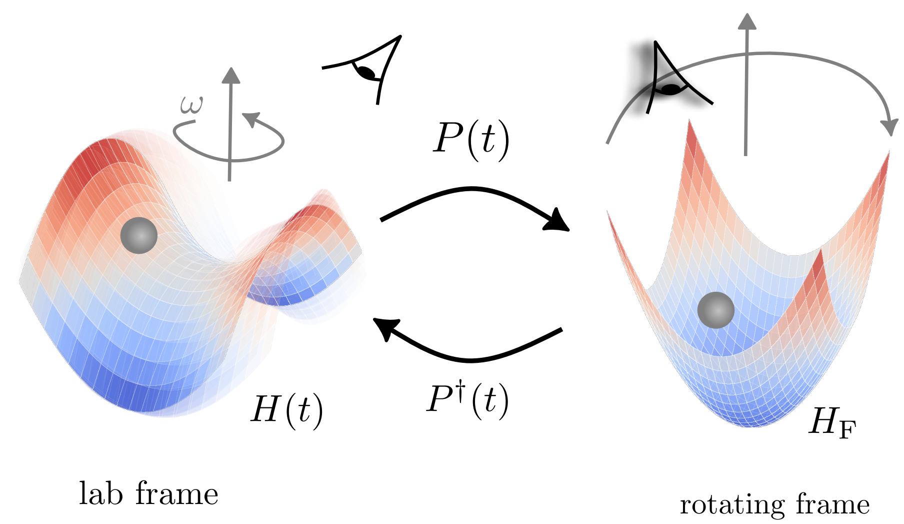
- fictitious forces arise from Galilean term $P^\dagger(t)\dot{P}(t)$

$$P^\dagger(t)\dot{P}(t) = \vec{\omega}(t)\times$$

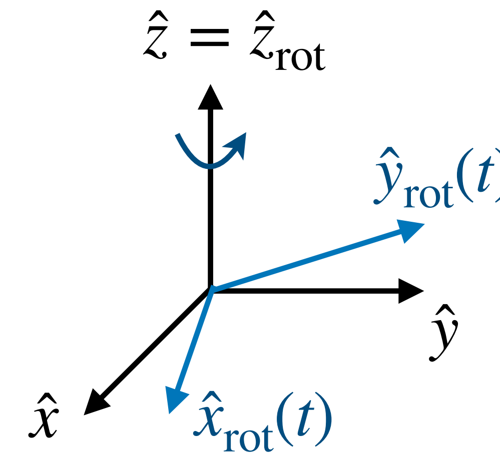
Q: can we understand dynamical stabilization as a fictitious force in some rotating frame?

Outline

- Rotating reference frames
  - quantum systems



Quantum mechanics

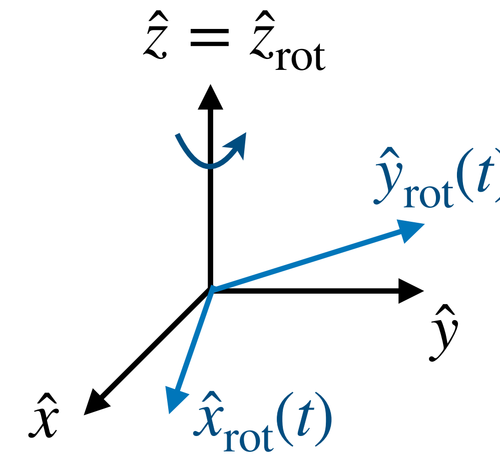


- rotating reference frame
  - not inertial
  - fictitious forces

- transformation between lab and rotating frames

$$\begin{tikzcd} \text{lab frame} \arrow[r, bend left, "P^{-1}(t) = P^{\dagger}(t)"] & \text{rot frame} \arrow[l, bend left, "P(t)"] \end{tikzcd}$$

Quantum mechanics



- rotating reference frame
  - not inertial
  - fictitious forces

- transformation between lab and rotating frames

$$P^{-1}(t) = P^{\dagger}(t)$$

lab frame | **rot frame**

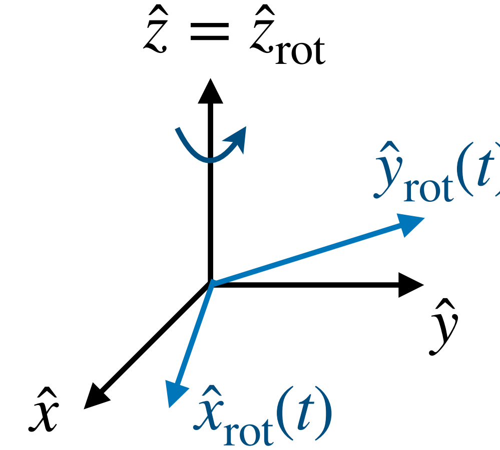
wave function: $|\psi(t)\rangle_{\mathrm{lab}}$ | $|\psi(t)\rangle_{\mathrm{rot}}$

$$P(t)$$

Hamiltonian: $H_{\mathrm{lab}}(t)$ | $H_{\mathrm{rot}}(t)$

$$|\psi(t)\rangle_{\mathrm{lab}} = P(t)\,|\psi(t)\rangle_{\mathrm{rot}}$$

Quantum mechanics



- rotating reference frame
  - not inertial
  - fictitious forces
- transformation between lab and rotating frames

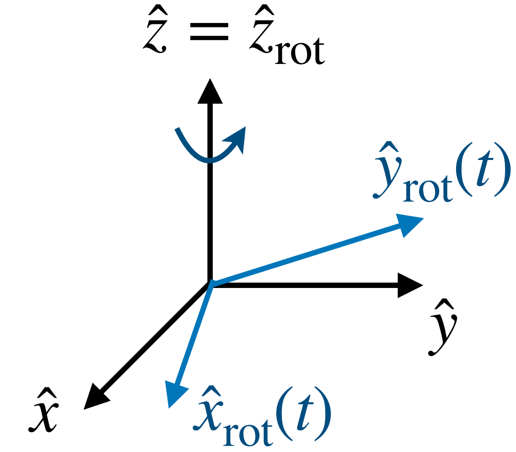
$$P^{-1}(t) = P^{\dagger}(t)$$

lab frame		rot frame
wave function: $\lvert\psi(t)\rangle_{\mathrm{lab}}$	$P(t)$	$\lvert\psi(t)\rangle_{\mathrm{rot}}$
Hamiltonian: $H_{\mathrm{lab}}(t)$		$H_{\mathrm{rot}}(t)$
evolution operator: $U_{\mathrm{lab}}(t,0)$		$U_{\mathrm{rot}}(t,0)$

$$\lvert\psi(t)\rangle_{\mathrm{lab}} = P(t)\lvert\psi(t)\rangle_{\mathrm{rot}}$$

$$U_{\mathrm{lab}}(t,0) = P(t)U_{\mathrm{rot}}(t,0)P^{\dagger}(0)$$

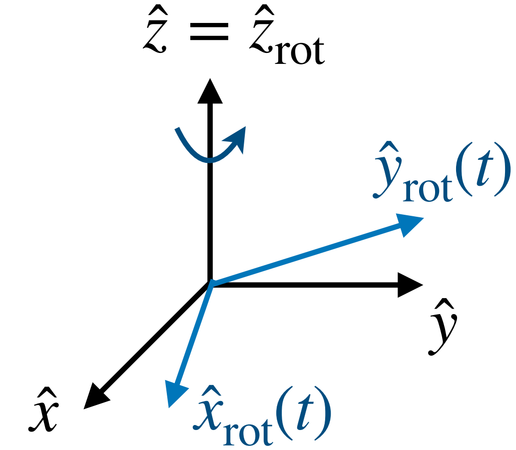
Quantum mechanics



$|\psi(t)\rangle_{\mathrm{lab}} = P(t)|\psi(t)\rangle_{\mathrm{rot}}$

$U_{\mathrm{lab}}(t,0) = P(t)U_{\mathrm{rot}}(t,0)P^{\dagger}(0)$

- Schrödinger's equation (set $\hbar = 1$)
  - lab frame: $i\partial_t |\psi(t)\rangle_{\mathrm{lab}} = H_{\mathrm{lab}} |\psi(t)\rangle_{\mathrm{lab}} \quad / \quad P^{\dagger}(t)\,\cdot$



Quantum mechanics

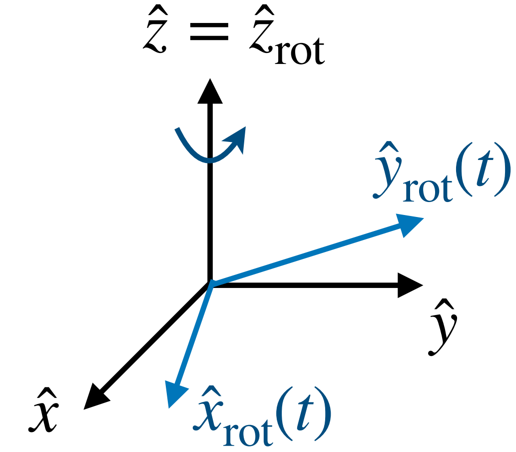
- Schrödinger's equation

$$|\psi(t)\rangle_{\text{lab}} = P(t)|\psi(t)\rangle_{\text{rot}}$$

$$U_{\text{lab}}(t,0) = P(t)U_{\text{rot}}(t,0)P^\dagger(0)$$

  - lab frame: $i\partial_t|\psi(t)\rangle_{\text{lab}} = H_{\text{lab}}|\psi(t)\rangle_{\text{lab}} \quad / \quad P^\dagger(t)\,\cdot$

  - rot frame: $iP^\dagger(t)\partial_t\left(P(t)\underline{P^\dagger(t)|\psi(t)\rangle_{\text{lab}}}\right) = P^\dagger(t)H_{\text{lab}}P(t)\underline{P^\dagger(t)|\psi(t)\rangle_{\text{lab}}}$



Quantum mechanics

- **Schrödinger's equation**

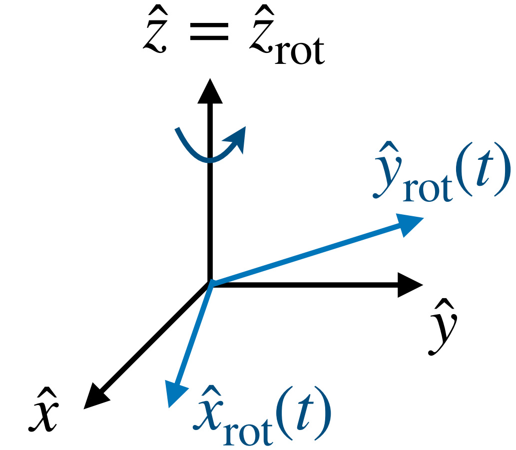
$$|\psi(t)\rangle_{\text{lab}} = P(t)|\psi(t)\rangle_{\text{rot}}$$

$$U_{\text{lab}}(t,0) = P(t)U_{\text{rot}}(t,0)P^\dagger(0)$$

  - lab frame: $i\partial_t|\psi(t)\rangle_{\text{lab}} = H_{\text{lab}}|\psi(t)\rangle_{\text{lab}} \quad / \quad P^\dagger(t)\,\cdot$

  - rot frame: $iP^\dagger(t)\partial_t\left(P(t)\underline{P^\dagger(t)|\psi(t)\rangle_{\text{lab}}}\right) = P^\dagger(t)H_{\text{lab}}P(t)\underline{P^\dagger(t)|\psi(t)\rangle_{\text{lab}}}$

$$iP^\dagger(t)\partial_t\left(P(t)|\psi(t)\rangle_{\text{rot}}\right) = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}}$$



Quantum mechanics

- **Schrödinger's equation**

$$|\psi(t)\rangle_{\text{lab}} = P(t)|\psi(t)\rangle_{\text{rot}}$$

$$U_{\text{lab}}(t,0) = P(t)U_{\text{rot}}(t,0)P^\dagger(0)$$

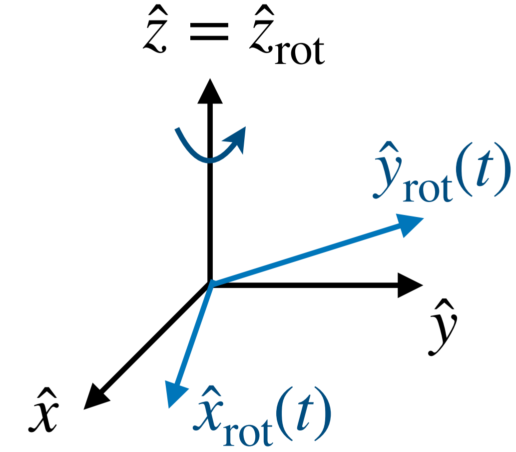
  - lab frame: $$i\partial_t|\psi(t)\rangle_{\text{lab}} = H_{\text{lab}}|\psi(t)\rangle_{\text{lab}} \quad / \quad P^\dagger(t)\,\cdot$$

  - rot frame: $$iP^\dagger(t)\partial_t\left(P(t)P^\dagger(t)|\psi(t)\rangle_{\text{lab}}\right) = P^\dagger(t)H_{\text{lab}}P(t)P^\dagger(t)|\psi(t)\rangle_{\text{lab}}$$

$$iP^\dagger(t)\underline{\partial_t\left(P(t)|\psi(t)\rangle_{\text{rot}}\right)} = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}}$$

$$iP^\dagger(t)\dot{P}(t)|\psi(t)\rangle_{\text{rot}} + i\partial_t|\psi(t)\rangle_{\text{rot}} = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}}$$

Quantum mechanics



- Schrödinger's equation

$$|\psi(t)\rangle_{\text{lab}} = P(t)|\psi(t)\rangle_{\text{rot}}$$

$$U_{\text{lab}}(t,0) = P(t)U_{\text{rot}}(t,0)P^\dagger(0)$$

  - lab frame: $$i\partial_t|\psi(t)\rangle_{\text{lab}} = H_{\text{lab}}|\psi(t)\rangle_{\text{lab}} \quad / \quad P^\dagger(t)\,\cdot$$

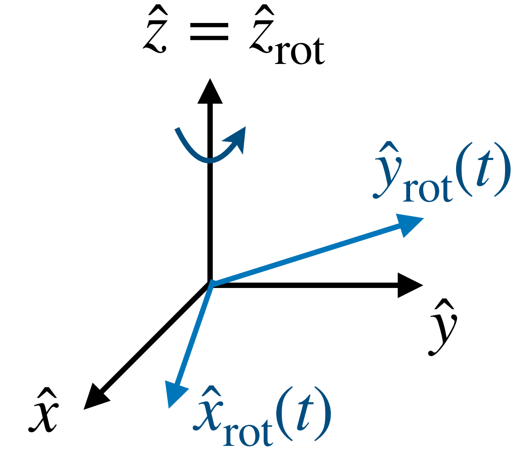
  - rot frame: $$iP^\dagger(t)\partial_t\left(P(t)P^\dagger(t)|\psi(t)\rangle_{\text{lab}}\right) = P^\dagger(t)H_{\text{lab}}P(t)P^\dagger(t)|\psi(t)\rangle_{\text{lab}}$$

$$iP^\dagger(t)\partial_t\left(P(t)|\psi(t)\rangle_{\text{rot}}\right) = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}}$$

$$iP^\dagger(t)\dot{P}(t)|\psi(t)\rangle_{\text{rot}} + i\partial_t|\psi(t)\rangle_{\text{rot}} = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}}$$

$$i\partial_t|\psi(t)\rangle_{\text{rot}} = P^\dagger(t)H_{\text{lab}}P(t)|\psi(t)\rangle_{\text{rot}} - iP^\dagger(t)\dot{P}(t)|\psi(t)\rangle_{\text{rot}}$$

Quantum mechanics



- Schrödinger's equation

$$|\psi(t)\rangle_{\rm lab} = P(t)|\psi(t)\rangle_{\rm rot}$$

$$U_{\rm lab}(t,0) = P(t)U_{\rm rot}(t,0)P^\dagger(0)$$

  - lab frame: $$i\partial_t|\psi(t)\rangle_{\rm lab} = H_{\rm lab}|\psi(t)\rangle_{\rm lab} \quad / \quad P^\dagger(t)\,\cdot$$
  - rot frame: $$iP^\dagger(t)\partial_t\left(P(t)P^\dagger(t)|\psi(t)\rangle_{\rm lab}\right) = P^\dagger(t)H_{\rm lab}P(t)P^\dagger(t)|\psi(t)\rangle_{\rm lab}$$

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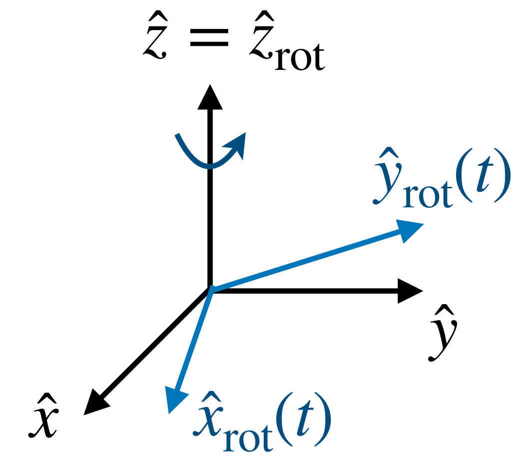
Quantum mechanics



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energy in the rot frame not the same as the transformed lab-frame Hamiltonian

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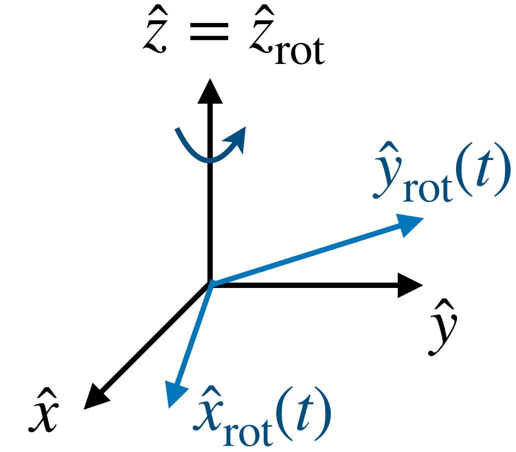

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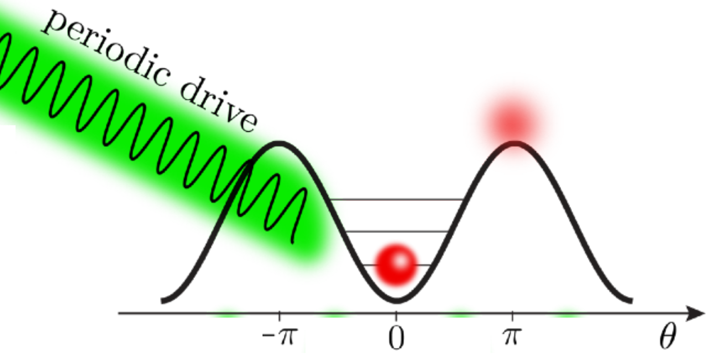
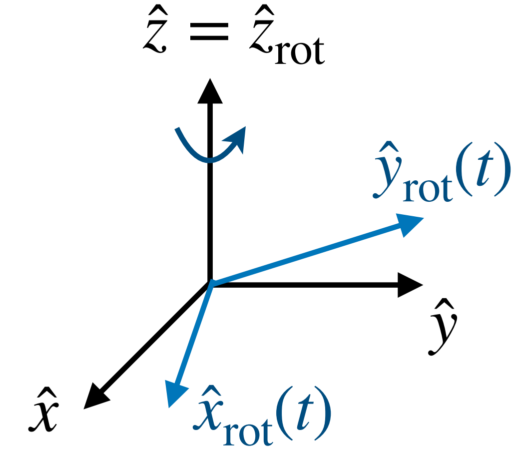


Quantum mechanics



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Q: can we understand dynamical stabilization as a fictitious force in a rotating frame?

Quantum dynamics

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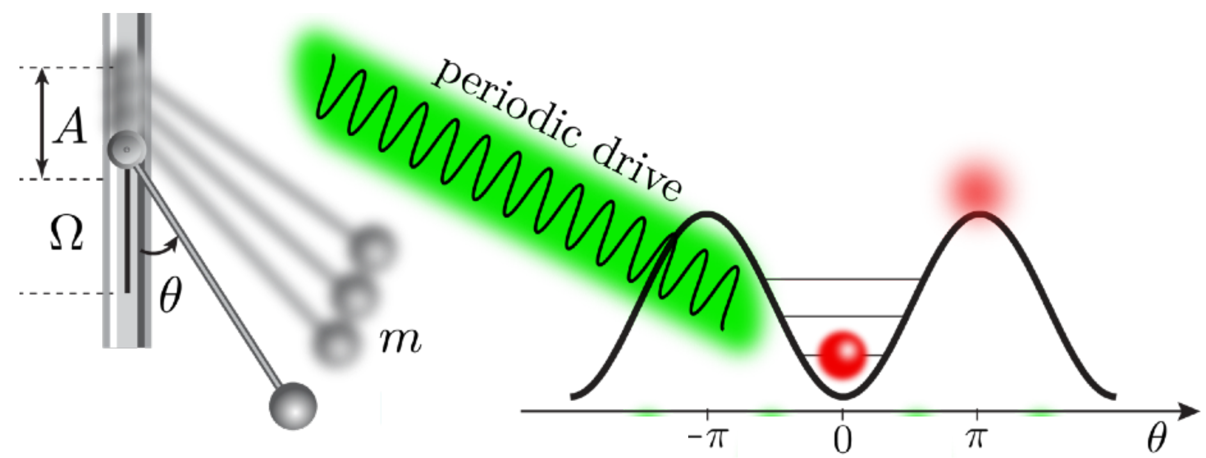
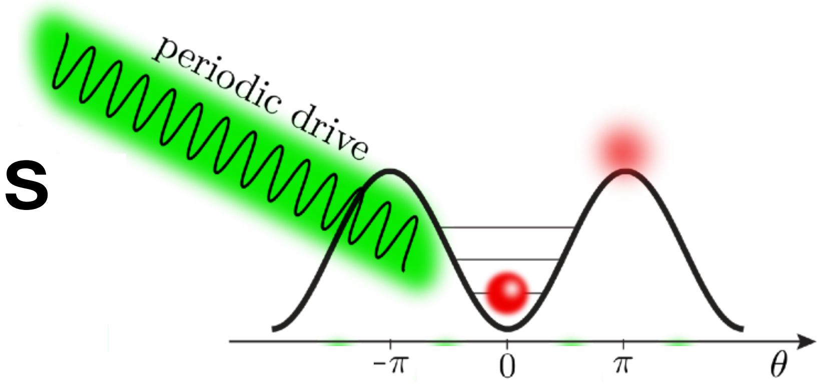
  - do not have meaningful stationary states (in general)

Outline

- Periodically driven quantum systems
  - Floquet theorem
  - Floquet engineering



Periodically driven systems



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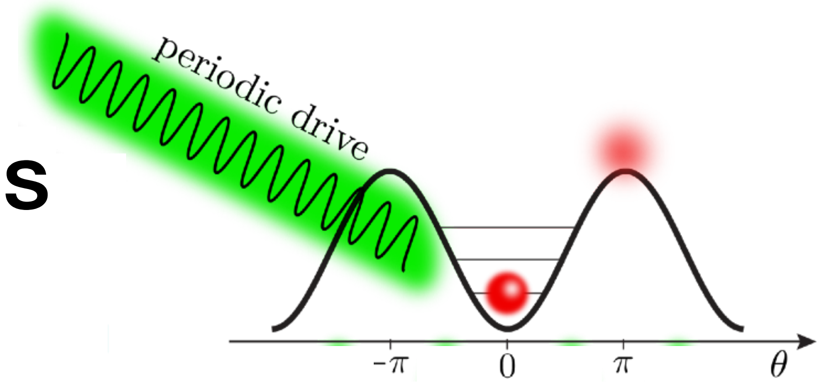

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micromotion Floquet Hamiltonian

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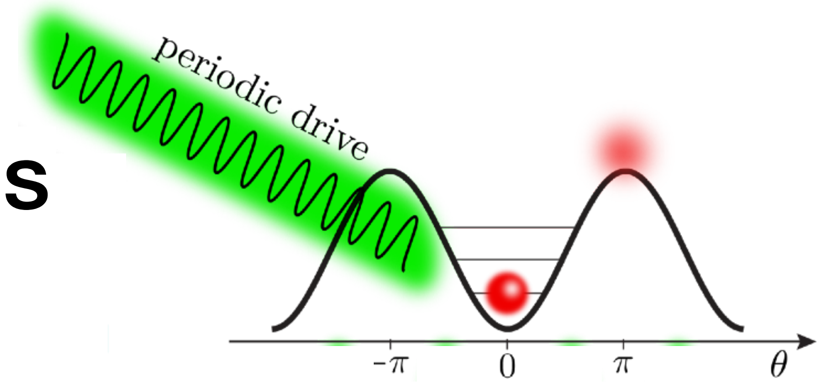

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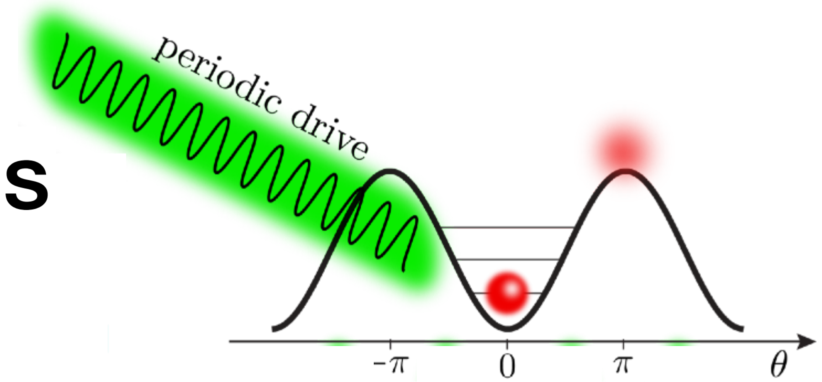

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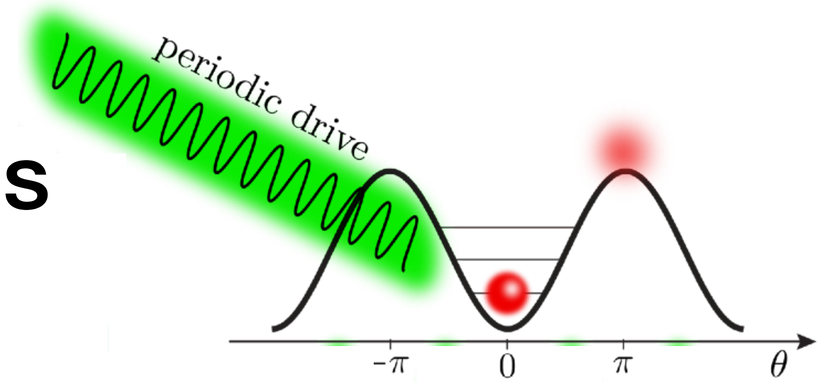

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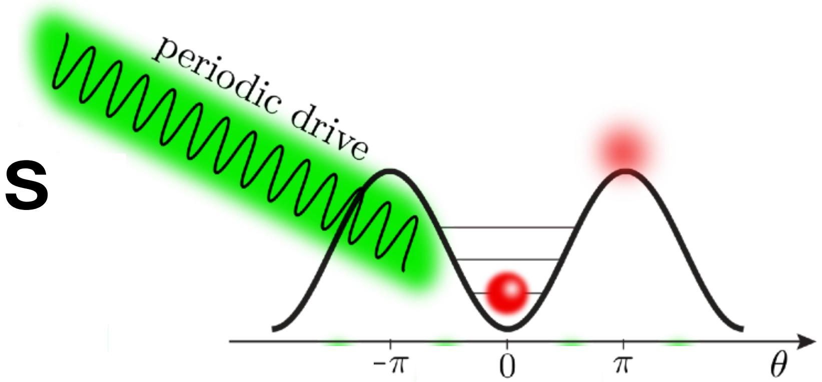

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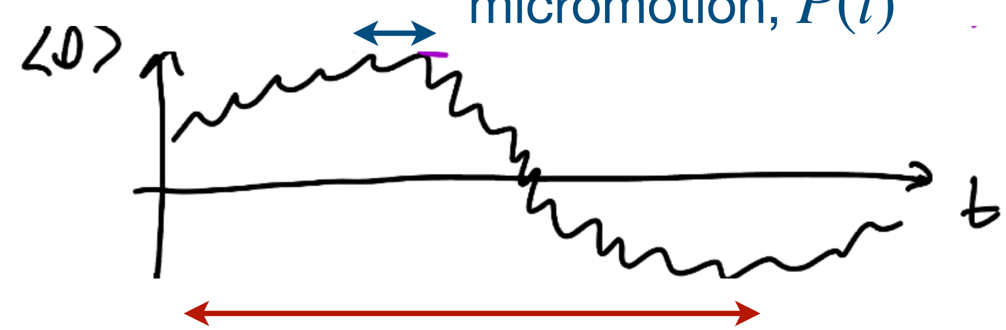
- **why useful?**
  - theory similar to static systems
  - time-scale separation in high-frequency limit



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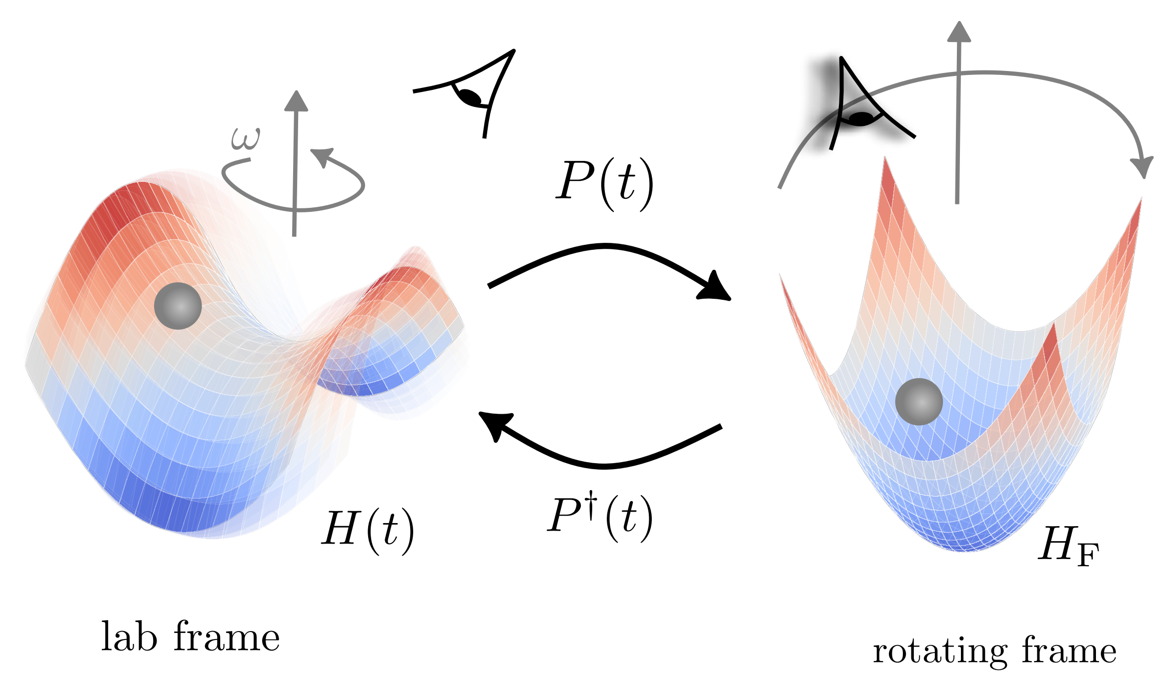

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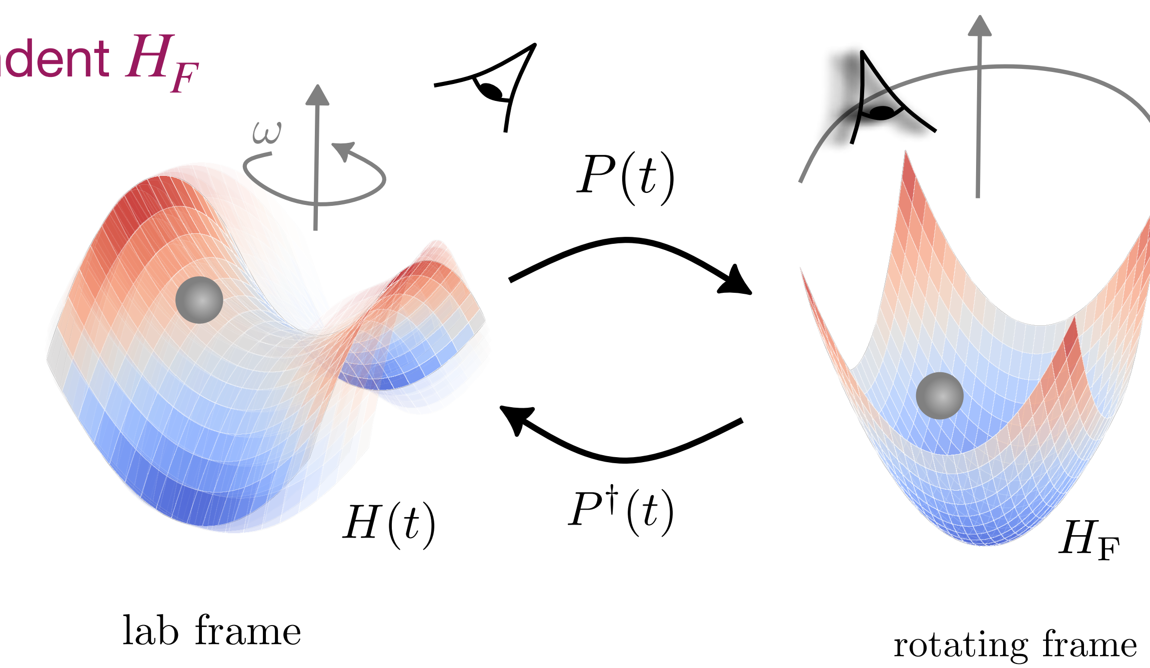

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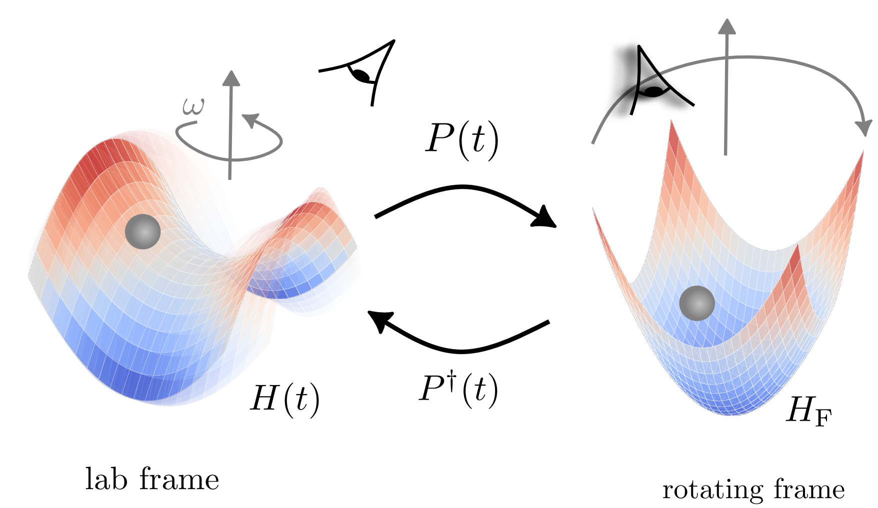
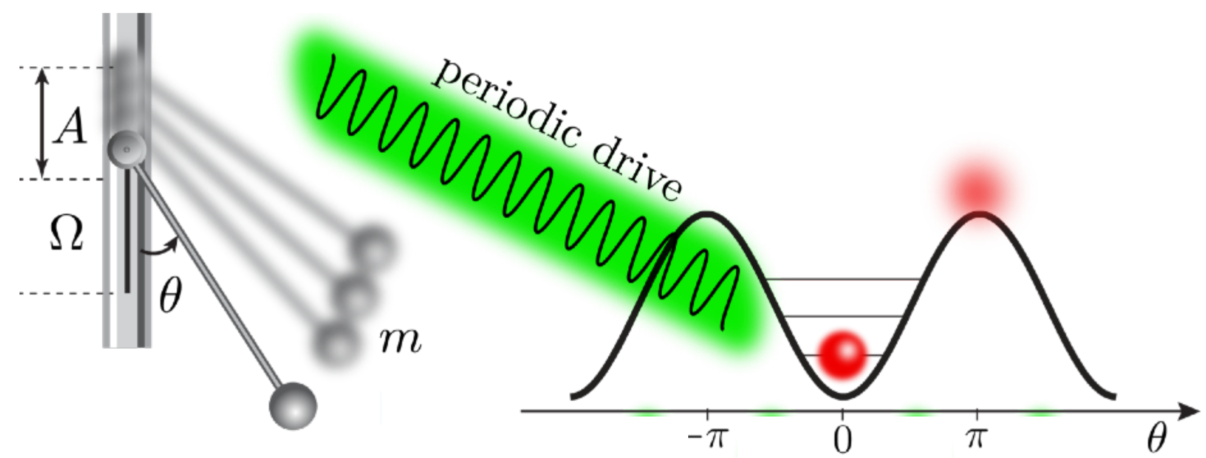
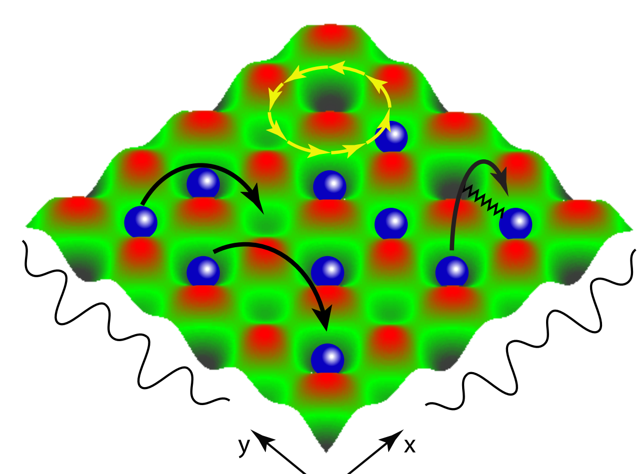


- **note: H_F contains fictitious force potential $iP^\dagger\dot{P}$!**
  - Floquet engineering: how do we choose the drive $H(t)$ to design properties to H_F?

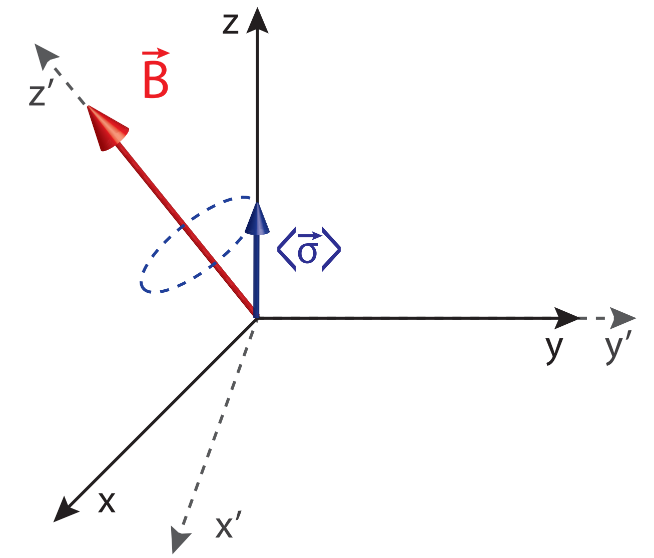
Outline



- Examples
  - spin-1 particle in a circularly polarized drive
  - quantum Kapitza oscillator
  - artificial gauge fields



Spin-1 particle in a circularly polarized magnetic field



- **Hamiltonian** $H(t) = \Delta S^z + g(\cos\omega t\, S^x + \sin\omega t\, S^y)$
  - spin-1 matrices

$$S^x = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} \qquad S^y = \frac{1}{i\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 0\end{pmatrix} \qquad S^z = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

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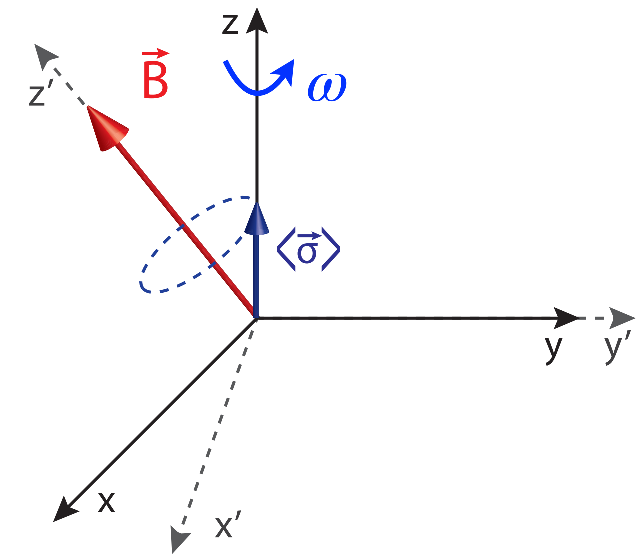

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$$S^x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S^y = \frac{1}{i\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad S^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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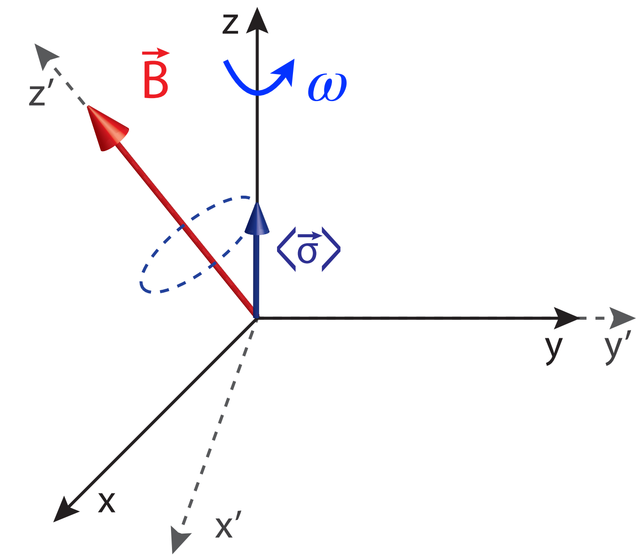
Spin-1 particle in a circularly polarized magnetic field



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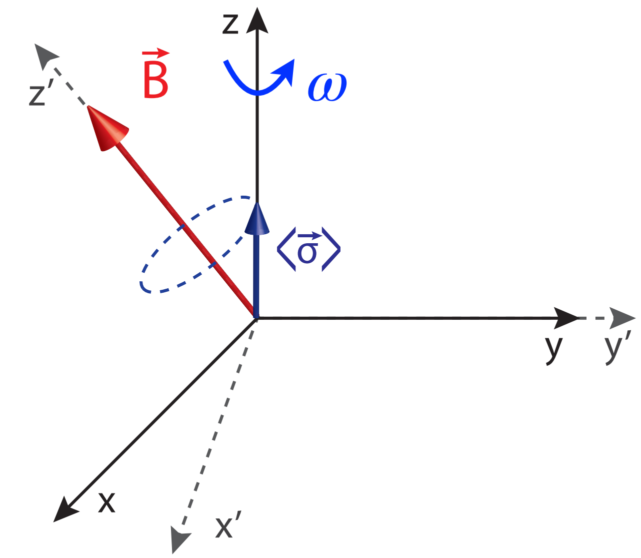

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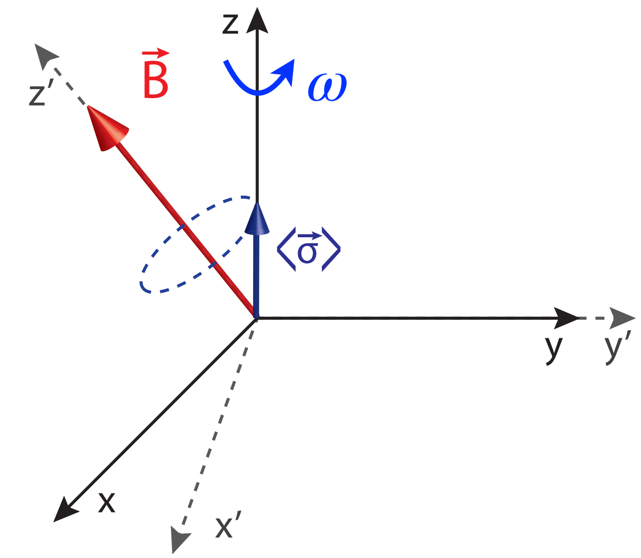

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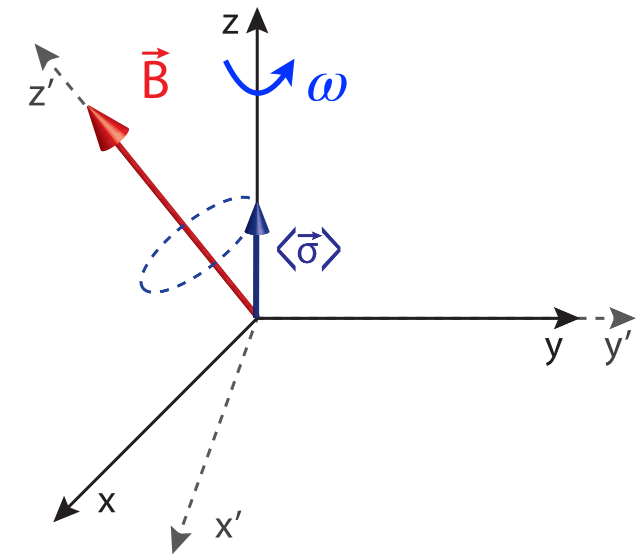
- **Floquet Hamiltonian:** $H_F = P^\dagger(t)H(t)P(t) - iP^\dagger(t)\dot{P}(t) = (\Delta - \omega)S^z + gS^x$

Spin-1 particle in a circularly polarized magnetic field



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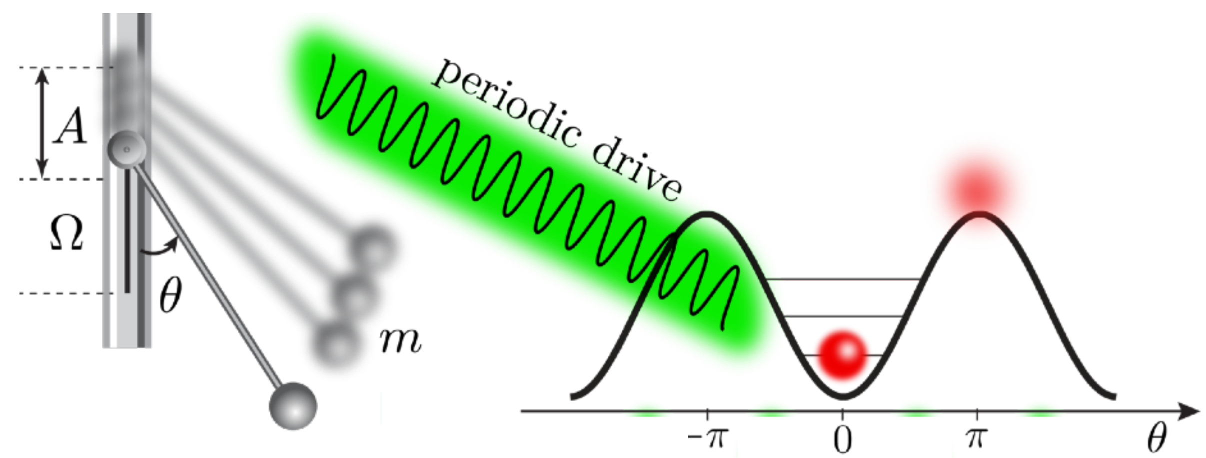
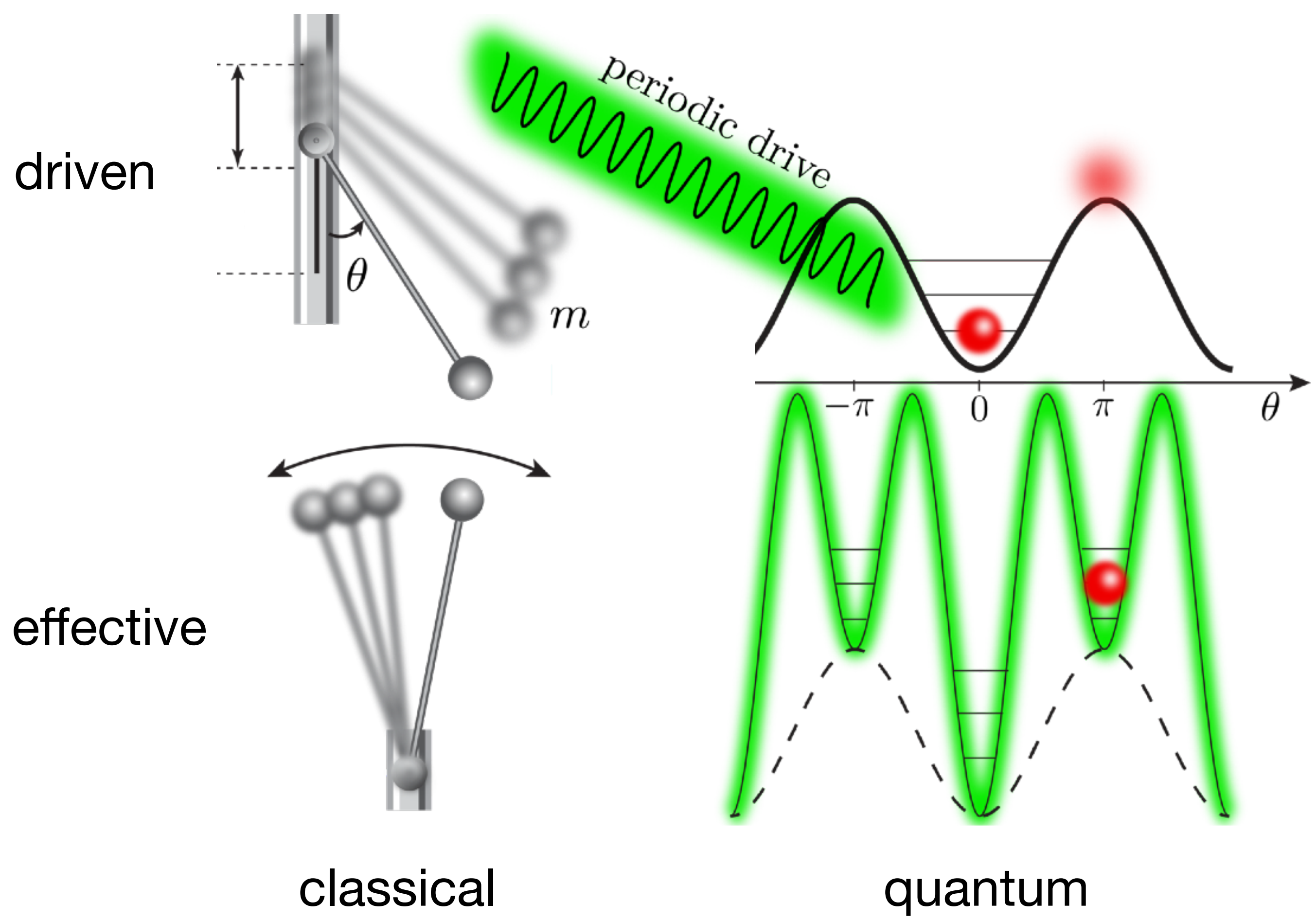
- **note: very few exactly solvable models**

Outline



- Examples
  - quantum Kapitza oscillator
  - artificial gauge fields

Quantum Kapitza oscillator



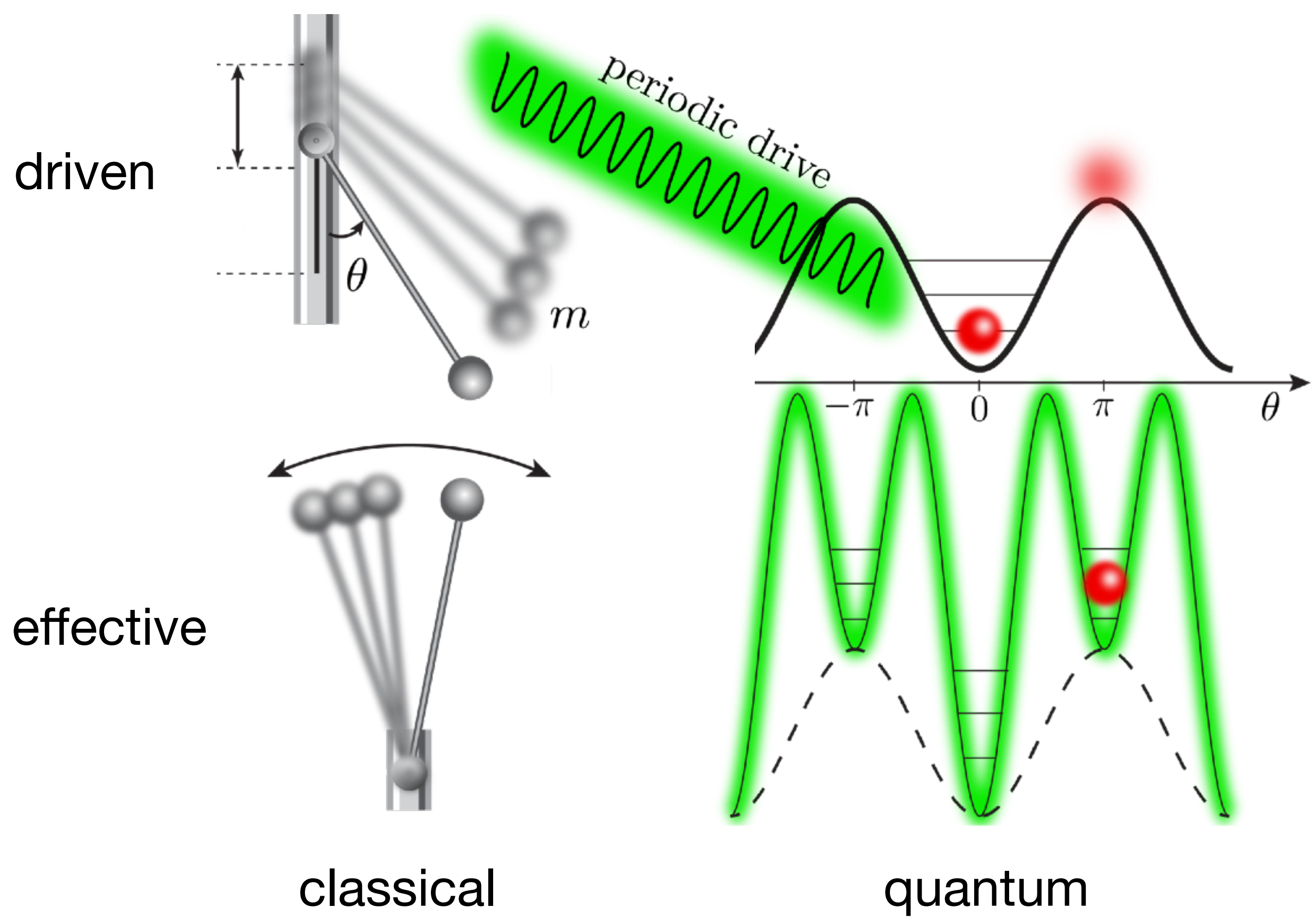
Quantum Kapitza oscillator

- **Hamiltonian** $$H(t) = \frac{p^2}{2} - \left(\omega_0^2 + A\omega\cos\omega t\right)\cos\theta$$

  note: simplified units

  - stable inverted equilibrium for $A \gg \omega_0$

How large is large enough?



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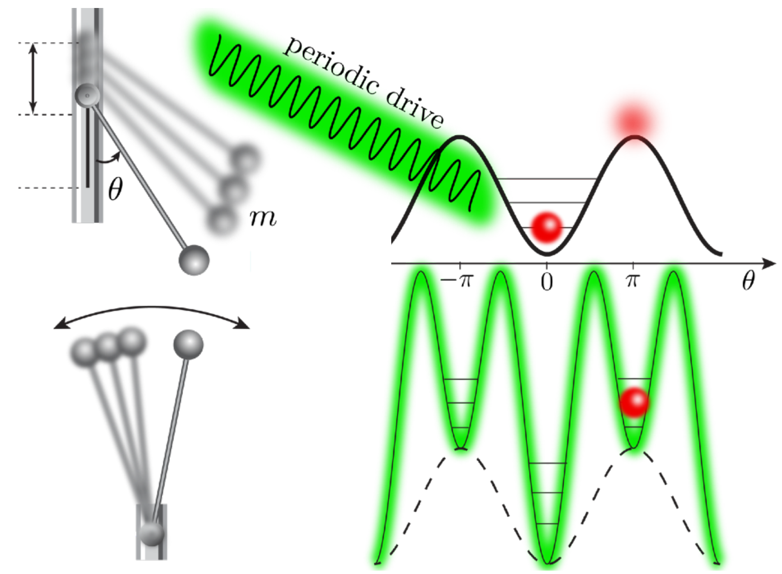

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  - usually given by time-average Hamiltonian, …but: $H_{\text{ave}} = T^{-1}\int_0^T \mathrm{d}t\, H(t) = p^2/2 - \omega_0^2\cos\theta$

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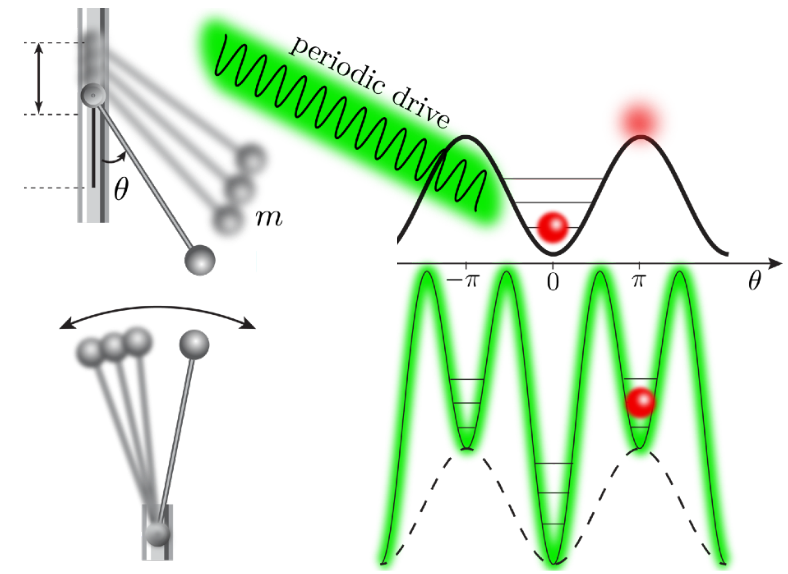

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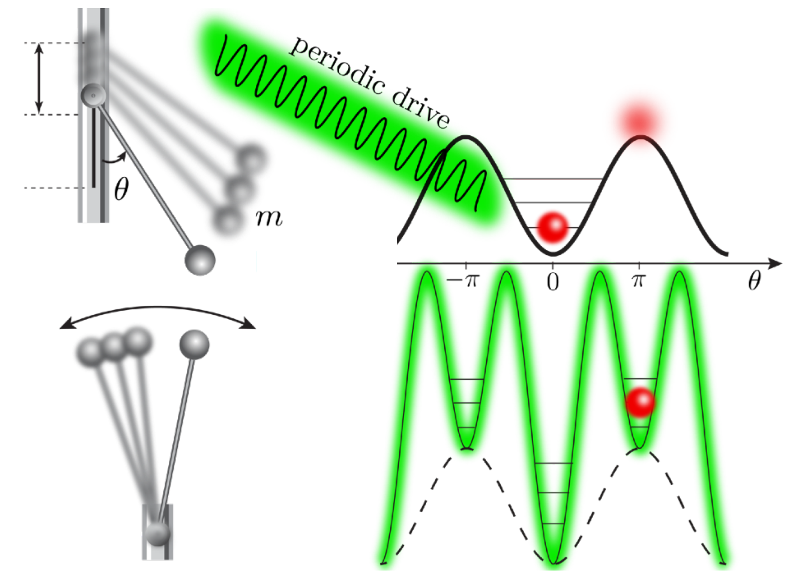
Quantum Kapitza oscillator



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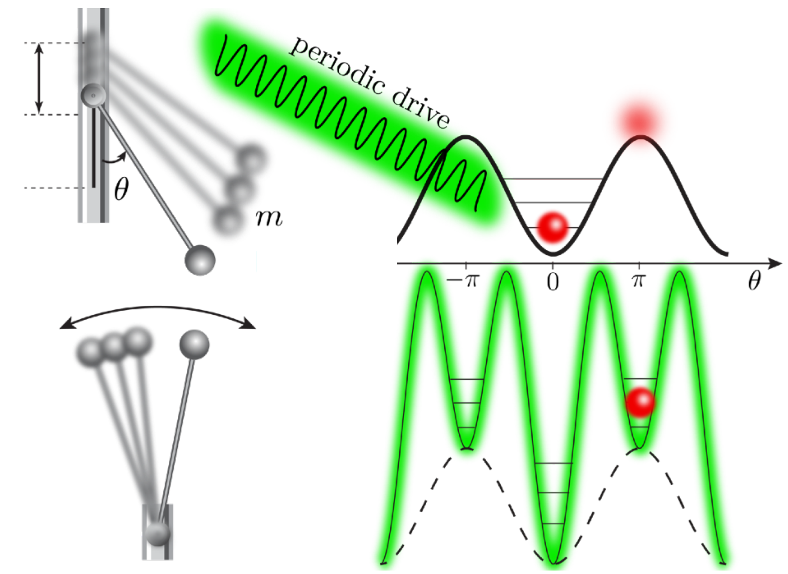
Quantum Kapitza oscillator



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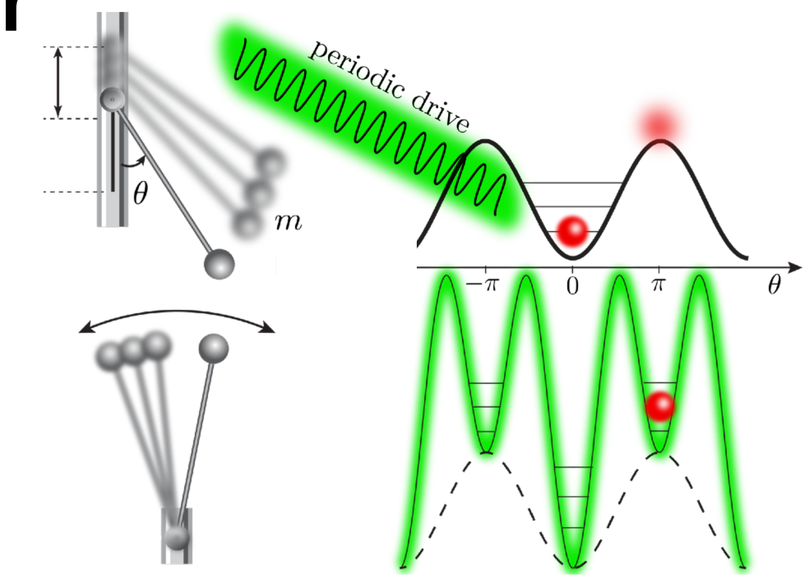

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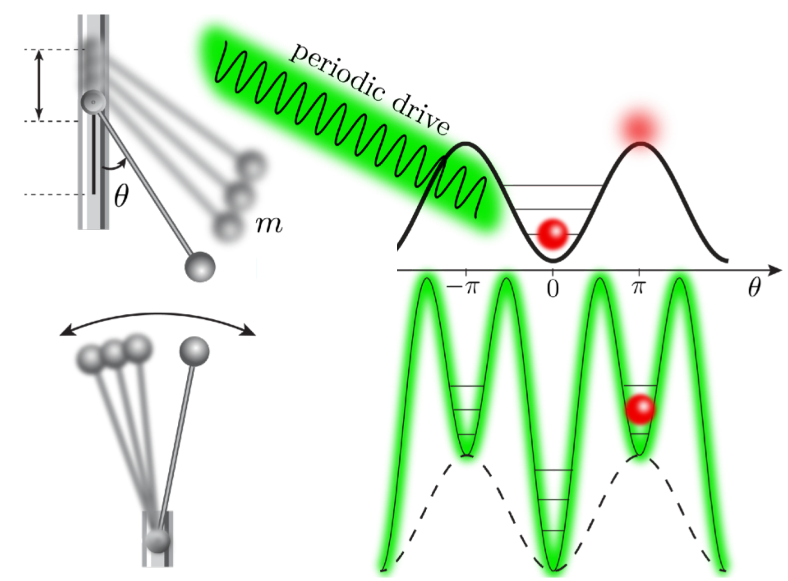

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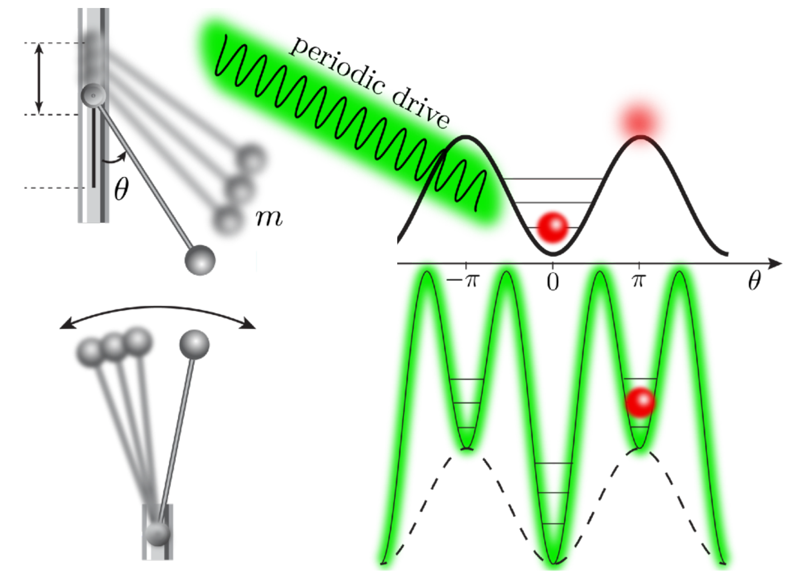
  - intermediate result: $H_{\mathrm{rot}}(t) = \mathscr{P}^\dagger(t)\frac{p^2}{2}\mathscr{P}(t) - \omega_0^2 \ \cos\theta = \frac{1}{2}\left(\mathscr{P}^\dagger(t) p \mathscr{P}(t)\right)^2 - \omega_0^2 \ \cos\theta$

Quantum Kapitza oscillator



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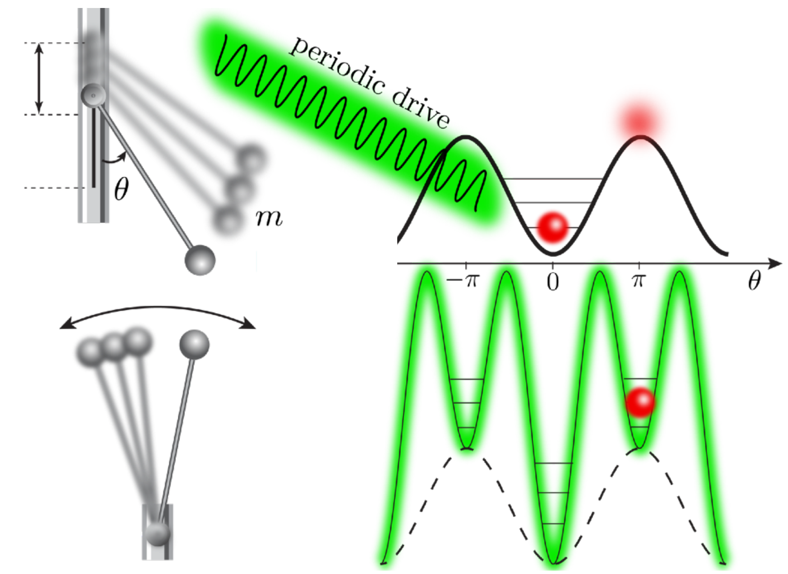
HW: $\mathscr{P}^\dagger(t)p\mathscr{P}(t) = p - i\partial_\theta\left(iA\sin\omega t\;\cos\theta\right) = p - A\sin\omega t\;\sin\theta$

Quantum Kapitza oscillator



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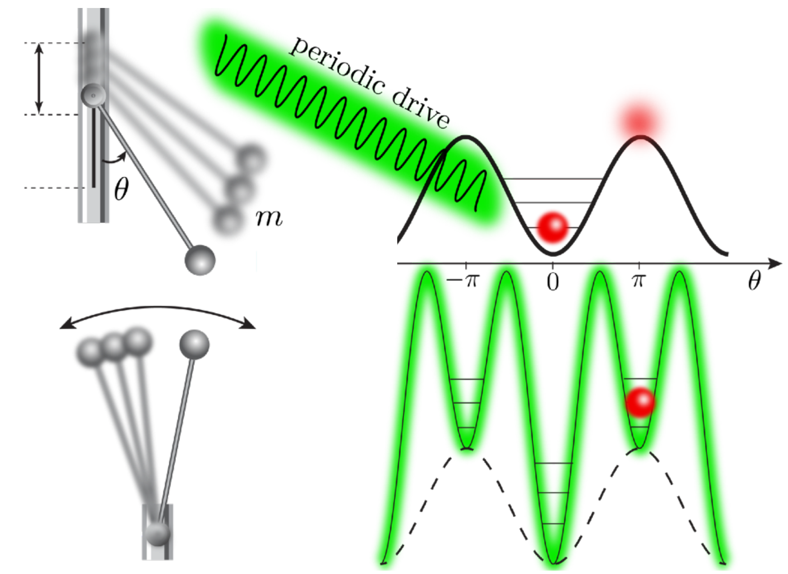
$$H_{\mathrm{rot}}(t) = \frac{1}{2}\left(p - A\sin\omega t\ \sin\theta\right)^2 - \omega_0^2\ \cos\theta$$
$$= \frac{p^2}{2} + \frac{A^2}{2}\sin^2\omega t\ \sin^2\theta - \omega_0^2\ \cos\theta - \frac{A}{2}\sin\omega t\ \{p, \sin\theta\}_+$$

no more diverging terms as $\omega \rightarrow \infty$

Quantum Kapitza oscillator



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$$
\begin{aligned}
H_{\rm rot}(t) &= \frac{1}{2}\left(p - A\sin\omega t\ \sin\theta\right)^2 - \omega_0^2\ \cos\theta \\
&= \frac{p^2}{2} + \frac{A^2}{2}\underbrace{\sin^2\omega t}_{\to 1/2}\ \sin^2\theta - \omega_0^2\ \cos\theta - \frac{A}{2}\sin\omega t\ \{p, \sin\theta\}_+
\end{aligned}
$$

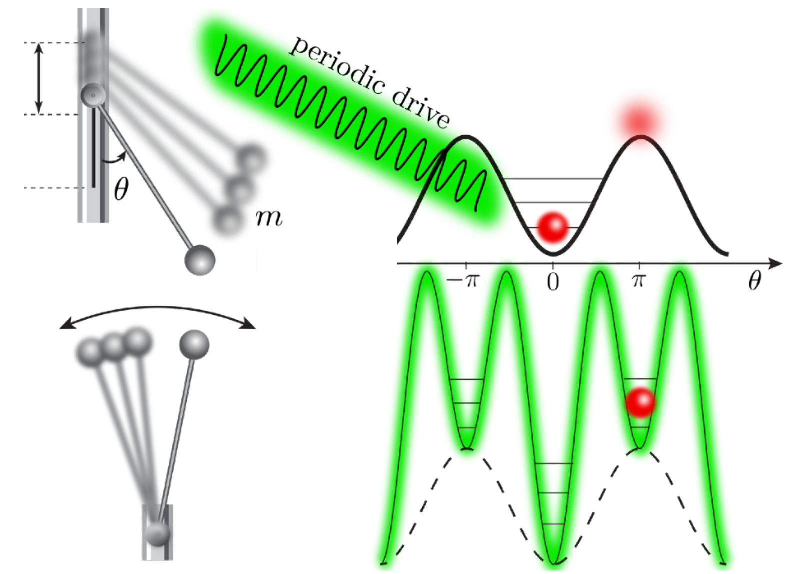
no more diverging terms as $\omega \to \infty$

  - compute period-average

effective potential $V_{\rm eff}(\theta)$

$$
H_F^{(0)} = \frac{1}{T}\int_0^T H_{\rm rot}(t)\,{\rm d}t = \frac{p^2}{2} + \frac{A^2}{4}\ \sin^2\theta - \omega_0^2\ \cos\theta
$$

Quantum Kapitza oscillator



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- **idea: use Galilean 'force potential' in rot frame to cancel strong periodic drive**

  - leading-order effective Hamiltonian

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effective potential $V_{\rm eff}(\theta)$

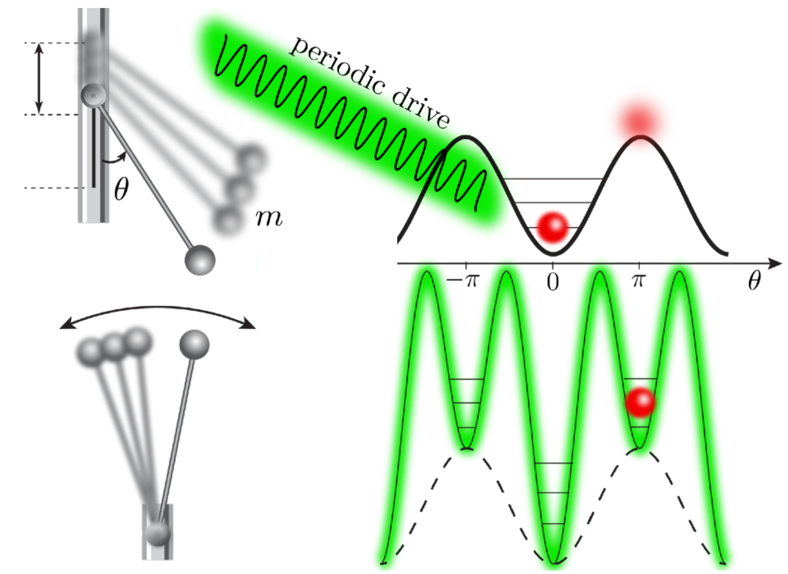
inverse-frequency expansions

$$H_F = \sum_{n=0}^{\infty}\omega^{-n}H_F^{(n)}$$

$$P(t) = \prod_{n=0}^{\infty}P^{(n)}(t)$$

$$P^{(0)}(t) = \mathscr{P}(t)$$

Quantum Kapitza oscillator



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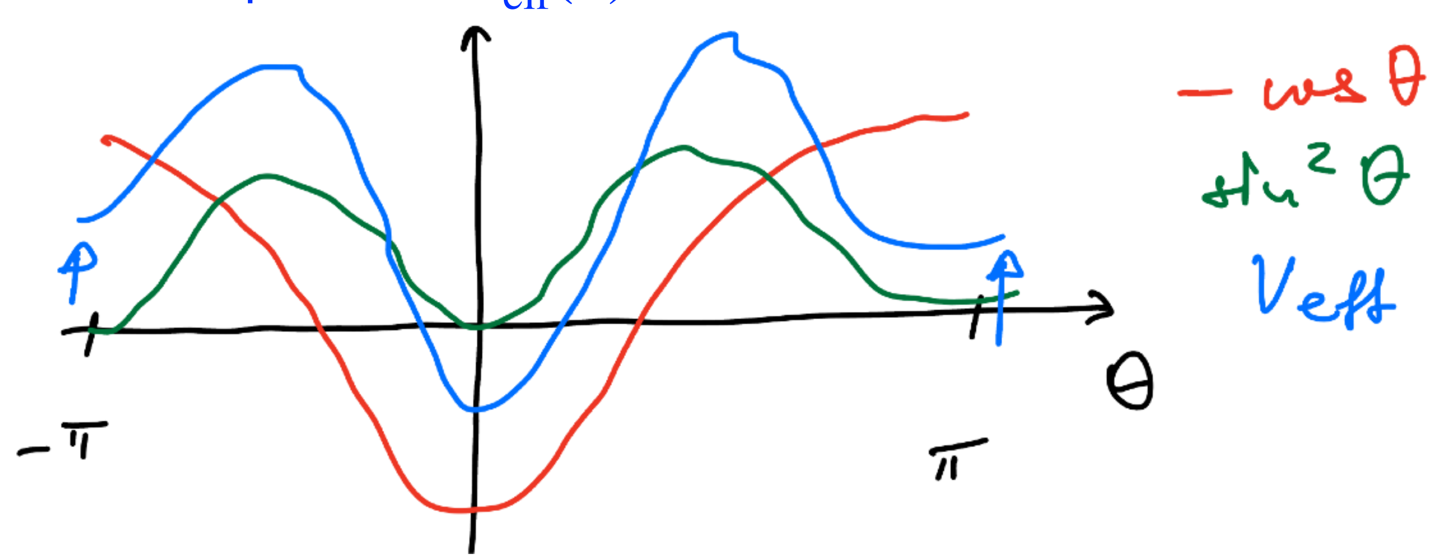
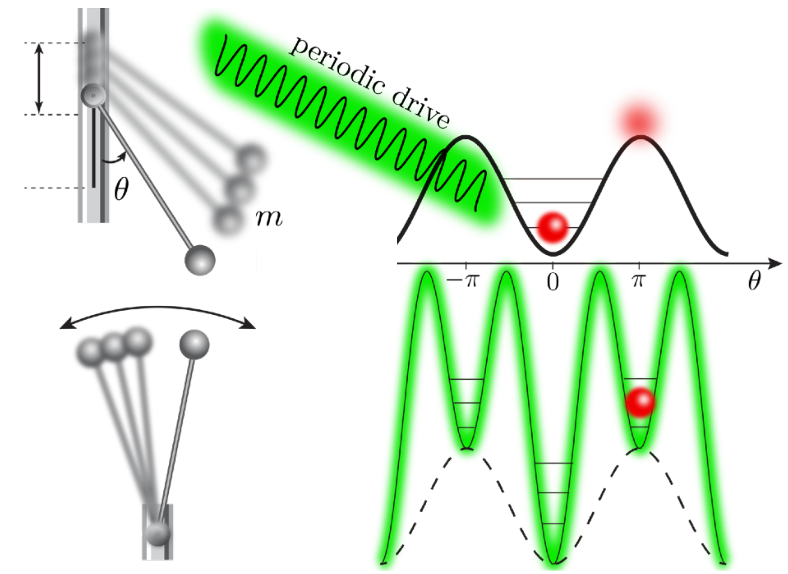
- **analyze stability at $\theta = \pm\pi$**



dynamical stabilization

Quantum Kapitza oscillator



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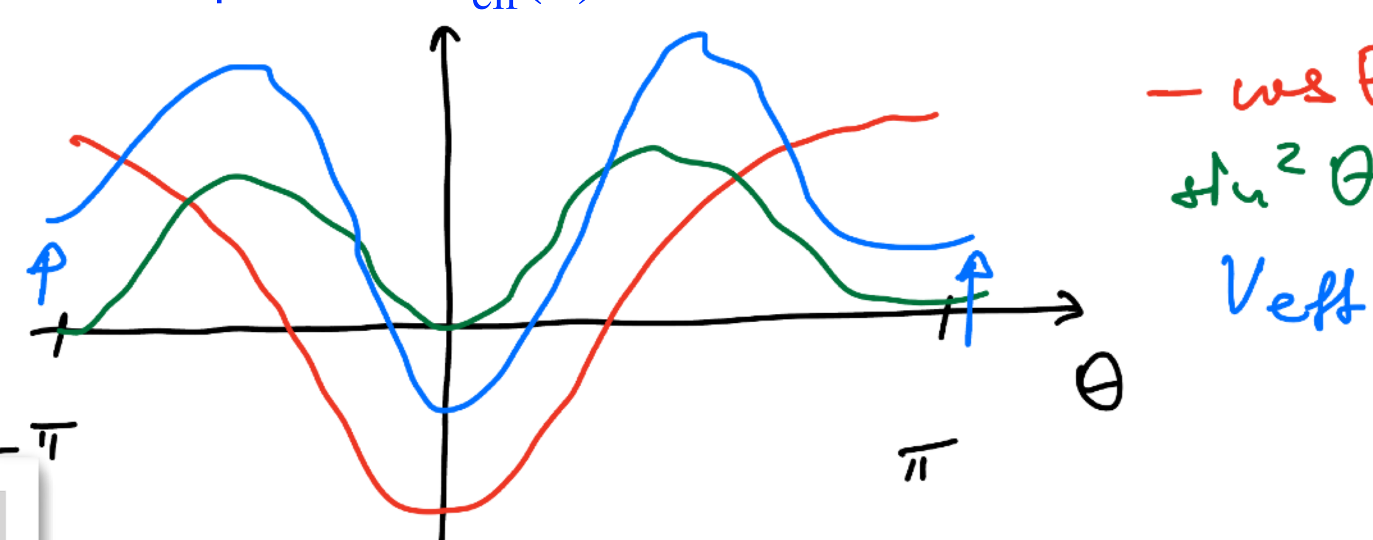
$$\partial_\theta^2 V_{\text{eff}}(\theta) = \omega_0^2\cos\theta + \frac{A^2}{2}\cos 2\theta$$

$$\underset{\theta=\pi}{=} -\omega_0^2 + \frac{A^2}{2} \overset{!}{>} 0$$

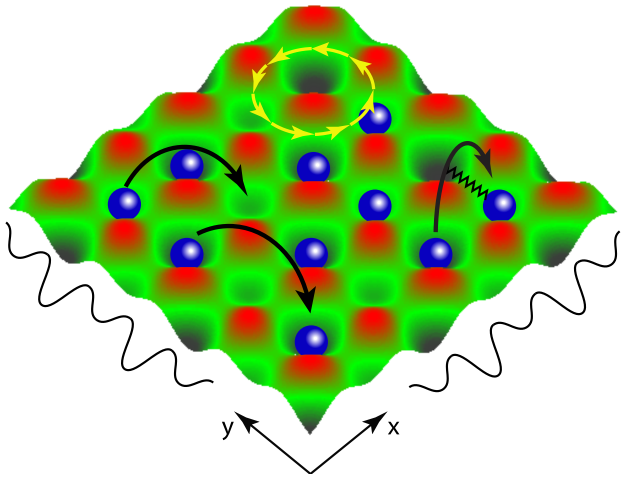
$$\Rightarrow \boxed{A_c > \sqrt{2}\,\omega_0}$$



dynamical stabilization

Outline

- Examples
  - artificial gauge fields



Artificial gauge fields

- **compare:** $H_{\text{rot}}(t) = \frac{1}{2}\left(p - A\sin\omega t\ \sin\theta\right)^2 - \omega_0^2\ \cos\theta$ vs. $H = \frac{1}{2}\left(p - A(x)\right)^2 + V(x)$

Kapitza pendulum in rotating frame

particle in magnetic field?

Artificial gauge fields

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  - gauge potential but no magnetic field in 1d!

particle in magnetic field?

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rotating Bose-Einstein condensate

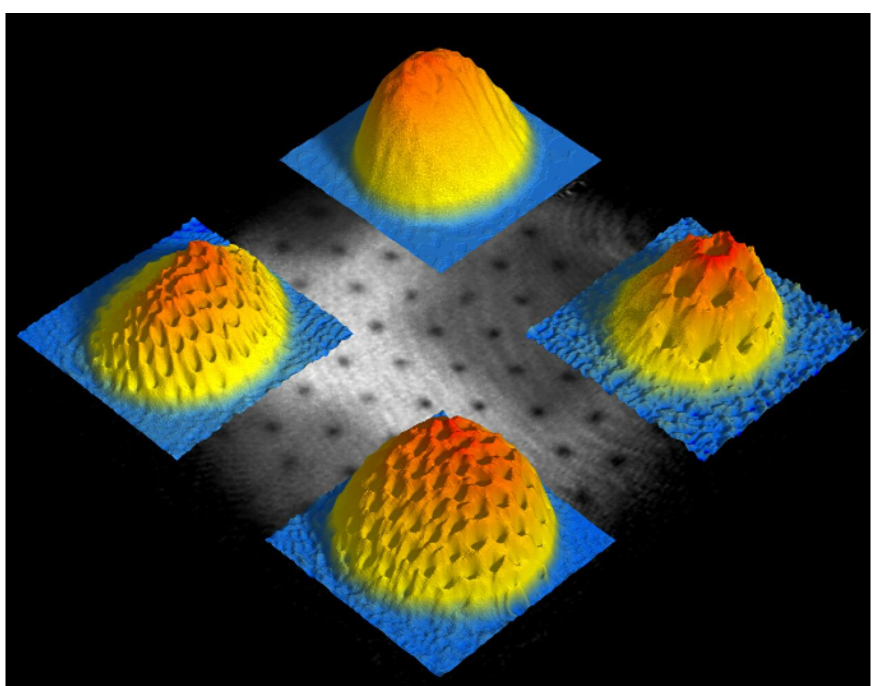
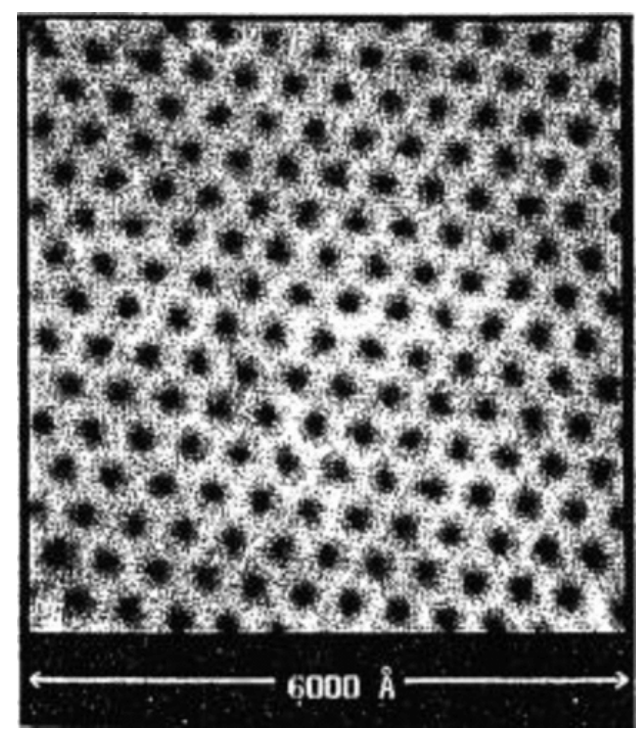


image: MIT

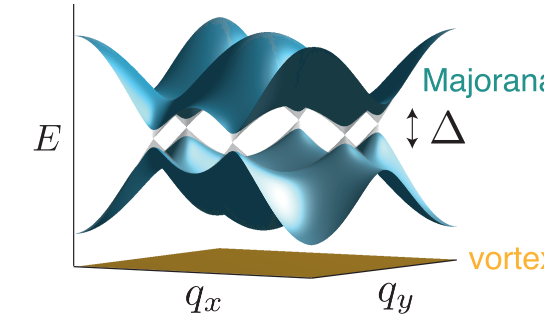
superconductor: Abrikosov vortex lattice



$NbSe_2$
type-II superconductor

scanning tunneling microscopy (STM)

Abrikosov, Nobel Lecture, Rev Mod Phys 76 975 (2004)



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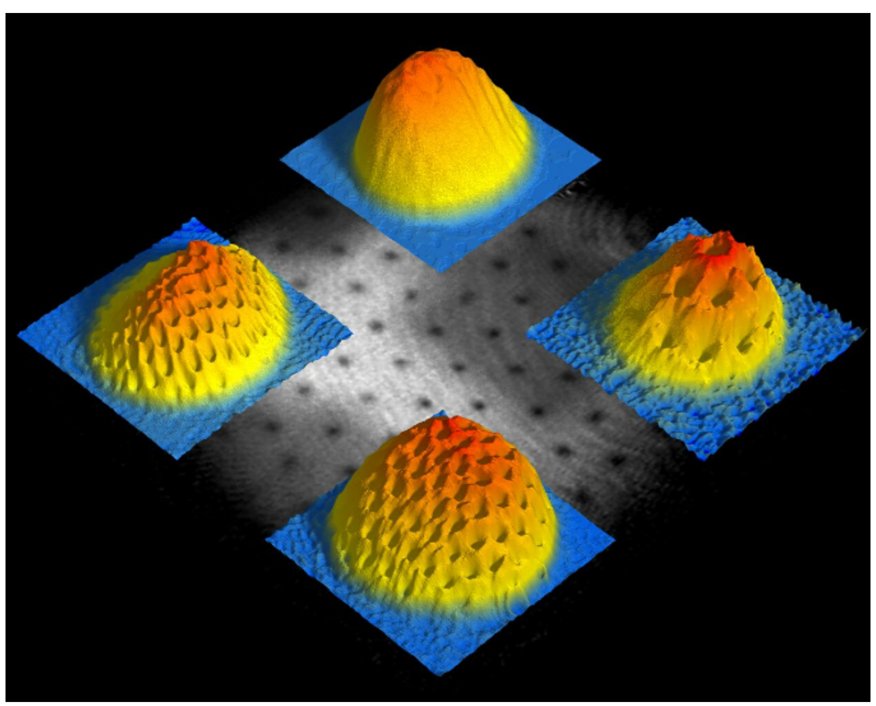
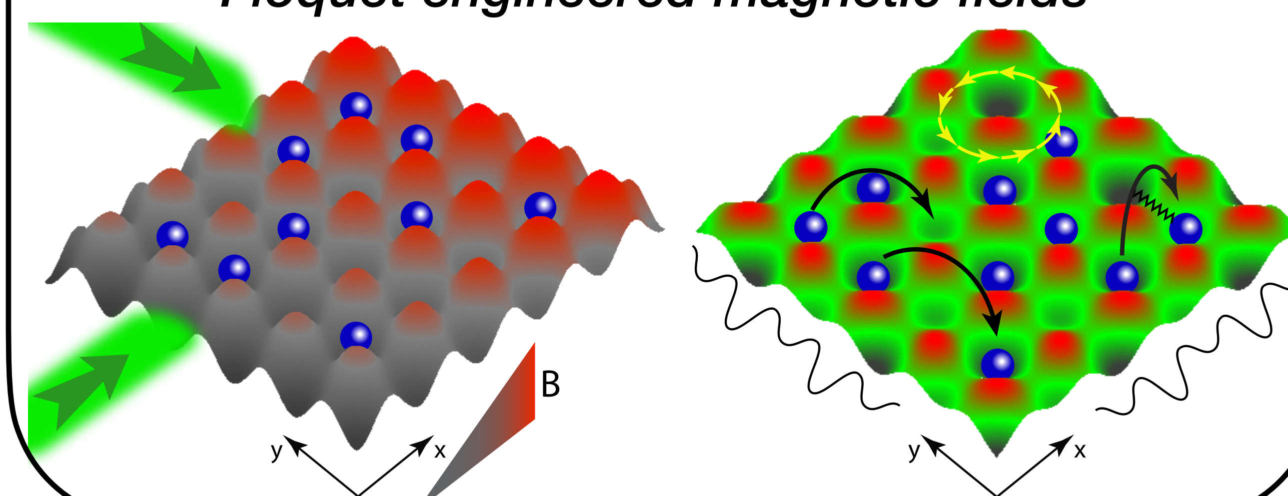


image: MIT

ultracold atoms in optical lattices

- quantum simulation of topological insulators
- *but:* no orbital B-field effects for neutral atoms

Floquet engineered magnetic fields



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Floquet engineering
for
quantum simulation

MPI-PKS, Dresden

- Floquet engineering: periodic drives ascribe new properties to physical systems
  - dynamical stabilization
  - artificial gauge fields (topological insulators, etc.)

key idea: design fictitious forces in rotating reference frame

- caveat: driven systems may absorb energy (heat death)

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